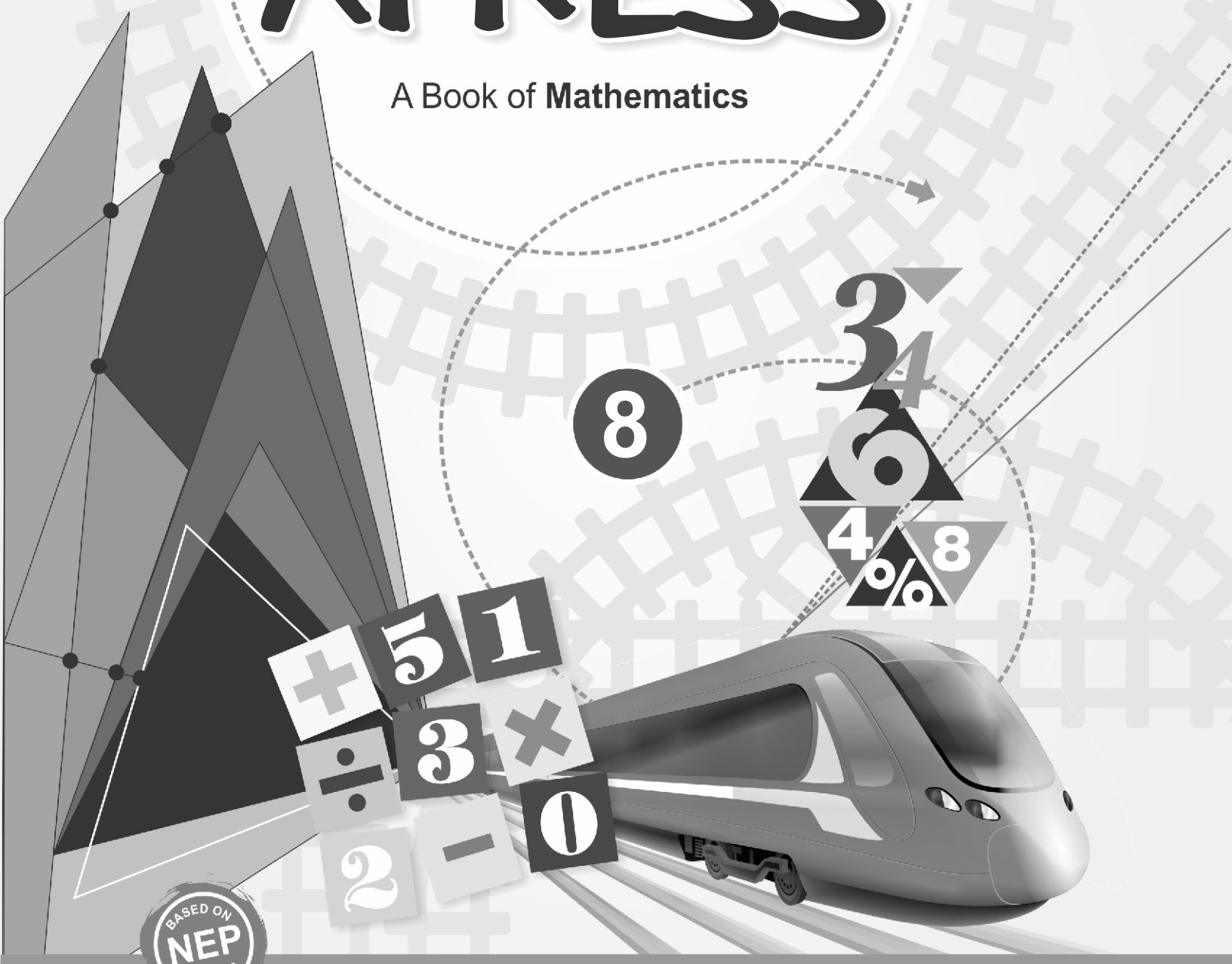
MATHS XPRESS

A Book of **Mathematics**



Ritu Books

BASED ON NEP GUIDELINES

1. Rational Numbers

Exercise 1.1

1. (i) False, (ii) True, (iii) False.

2. (i) $\frac{12}{23}$, Numerator = 12 (ii) $\frac{6}{-5}$, Numerator = 6

(iii) $\frac{-15}{41}$, Numerator = -15 (iv) $\frac{-67}{-167}$, Numerator = -67

3. (i) $\frac{-5}{7}$, Denominator = 7 (ii) $\frac{22}{49}$, Denominator = 49

(iii) $\frac{-7}{-9}$, Denominator = -9 (iv) $\frac{5}{-9}$, Denominator = -9

4. (i) $\frac{5}{20}$ (ii) $\frac{9}{36}$ (iii) $\frac{-20}{-80}$ (iv) $\frac{1000}{4000}$ (v) $\frac{-25}{-100}$

5. (i) $\frac{-32}{70}$, HCF of 32 and 70 is 2.

So, $\frac{-32}{70} = \frac{-32/2}{70/2} \Rightarrow \frac{-32}{70} = \frac{-16}{35}$

(ii) $\frac{150}{-650}$, HCF of 150 and 650 is 50.

So, $\frac{150}{-650} = \frac{150/50}{-650/50} \Rightarrow \frac{150}{-650} = \frac{-3}{13}$

(iii) $\frac{-65}{143}$, HCF of 65 and 143 is 13.

So, $\frac{-65}{143} = \frac{-65/13}{143/13} = \frac{-5}{11}$

(iv) $\frac{-102}{187}$, HCF of 102 and 187 is 17.

So, $\frac{-102}{187} = \frac{-102/17}{187/17} = \frac{-6}{11}$

6. (i) $\frac{2}{10}$, HCF of 2 and 10 is 2.

So, $\frac{2}{10} = \frac{2/2}{10/2} = \frac{1}{5}$

(ii) $\frac{-36}{180}$, HCF of 36 and 180 is 36.

So, $\frac{-36}{180} = \frac{-36/36}{180/36} = \frac{-1}{5}$

(iii) $\frac{91}{364}$, HCF of 91 and 364 is 91.

So, $\frac{91}{364} = \frac{91/91}{364/91} = \frac{1}{4}$

(iv) $\frac{15}{25}$, HCF of 15 and 25 is 5.

So, $\frac{15}{25} = \frac{15/5}{25/5} = \frac{3}{5}$

(v) $\frac{44}{428}$, HCF of 44 and 428 is 4.

So, $\frac{44}{428} = \frac{44/4}{428/4} = \frac{11}{107}$

Exercise 1.2

1. (i) $\frac{7}{15} + 0 = \frac{7}{15}$ (additive identity)

(ii) $\frac{-15}{23} + \frac{13}{42} = \frac{13}{42} + \left(\frac{-15}{23}\right)$ (commutative)

(iii) $\frac{-13}{29} + \frac{13}{29} = 0$ (additive inverse)

(iv) $\frac{5}{6} + \left(\frac{-2}{9} + \frac{1}{4}\right) = \frac{5}{6} + \left(\frac{-2}{9}\right) + \frac{1}{4}$ (associative)

2. (i) $\frac{3}{17} + \frac{-5}{34} + \frac{8}{51} = \frac{3 \times 2 + (-5) \times 1}{34} + \frac{8}{51}$

$= \frac{6 - 5}{34} + \frac{8}{51} = \frac{1}{34} + \frac{8}{51}$

$= \frac{1 \times 3 + 8 \times 2}{102} = \frac{3 + 16}{102} = \frac{19}{102}$

(ii) $\frac{2}{3} + \frac{5}{18} + \frac{-17}{24} = \frac{2 \times 6 + 5 \times 1}{18} - \frac{17}{24}$

$= \frac{12 + 5}{18} - \frac{17}{24} = \frac{17}{18} - \frac{17}{24}$

$= \frac{17 \times 4 - 17 \times 3}{72} = \frac{68 - 51}{72} = \frac{17}{72}$

(iii) $\frac{-9}{22} + \frac{7}{11} + \frac{-8}{33} = \frac{7}{11} - \frac{9}{22} - \frac{8}{33}$

$= \frac{7 \times 2 - 9}{22} - \frac{8}{33} = \frac{14 - 9}{22} - \frac{8}{33}$

$= \frac{5}{22} - \frac{8}{33} = \frac{5 \times 3 - 8 \times 2}{66} = \frac{15 - 16}{66} = \frac{-1}{66}$

3. Do it yourself.

4. Do it yourself.

5. (i) $\frac{4}{7}+0+\frac{-8}{9}+\frac{-13}{7}+\frac{17}{21}$

$=\frac{4}{7}-\frac{13}{7}-\frac{8}{9}+\frac{17}{21}=\frac{4-13}{7}-\frac{8}{9}+\frac{17}{21}$

$=\frac{-9}{7}-\frac{8}{9}+\frac{17}{21}=\frac{-9\times 9-8\times 7}{63}+\frac{17}{21}=\frac{-81-56}{63}+\frac{17}{21}$

$=\frac{-137}{63}+\frac{17}{21}=\frac{-137+17\times 3}{63}=\frac{-137+51}{63}=\frac{-86}{63}$

(ii) $\frac{2}{5}+\frac{8}{3}+\frac{-11}{15}+\frac{4}{5}-\frac{2}{3}$

$=\frac{8}{3}-\frac{2}{3}+\frac{2}{5}+\frac{4}{5}-\frac{11}{15}=\frac{8-2}{3}+\frac{2+4}{5}-\frac{11}{15}$

$=\frac{6}{3}+\frac{6}{5}-\frac{11}{15}=\frac{6\times 5+6\times 3}{15}-\frac{11}{15}=\frac{30+18}{15}-\frac{11}{15}$

$=\frac{48}{15}-\frac{11}{15}=\frac{48-11}{15}=\frac{37}{15}$

6. (i) $0+\frac{7}{11}=\frac{7}{11}$ (ii) $\frac{13}{17}+0=\frac{13}{17}$

(iii) $\frac{3}{5}+0=\frac{3}{5}$ (iv) $\frac{-12}{5}+0=\frac{-12}{5}$

Exercise 1.3

1. (i) False, (ii) True, (iii) True, (iv) False, (v) True, (vi) False.

2. (i) $\frac{-5}{26}$ (ii) $\frac{-9}{14}$ (iii) $\frac{34}{9}$ (iv) $\frac{77}{23}$

3. (i) $\frac{6}{17}-\frac{5}{17}=\frac{6-5}{17}=\frac{1}{17}$

(ii) $\frac{-3}{19}-\frac{15}{38}=\frac{-3\times 2-15}{38}=\frac{-6-15}{38}=\frac{-21}{38}$

(iii) $\frac{-5}{18}-\frac{(-3)}{40}=\frac{-5}{18}+\frac{3}{40}$

$=\frac{-5\times 20+3\times 9}{360}=\frac{-100+27}{360}=\frac{-73}{360}$

4. (i) $2\frac{3}{5}$ from $\frac{-3}{7}=\frac{-3}{7}-\frac{13}{5}=\frac{-3\times 5-13\times 7}{35}$

$=\frac{-15-91}{35}=\frac{-106}{35}=-3\frac{1}{35}$

(ii) $\frac{-4}{9}$ from $3\frac{5}{8}=\frac{29}{8}+\frac{4}{9}=\frac{29\times 9+4\times 8}{72}$

$=\frac{261+32}{72}=\frac{293}{72}=4\frac{5}{72}$

(iii) $-3\frac{1}{5}$ from $-4\frac{7}{9} = \frac{-43}{9} + \frac{16}{5} = \frac{-43 \times 5 + 16 \times 9}{45}$

$= \frac{-215 + 144}{45} = \frac{-71}{45} = -1\frac{26}{45}$

5. (i) $\frac{-2}{3} + \frac{5}{9} - \frac{-7}{6} = \frac{-2}{3} + \frac{5}{9} + \frac{7}{6}$

$= \frac{-2 \times 3 + 5}{9} + \frac{7}{6} = \frac{-6 + 5}{9} + \frac{7}{6}$

$= \frac{-1}{9} + \frac{7}{6} = \frac{-1 \times 2 + 7 \times 3}{18}$

$= \frac{-2 + 21}{18} = \frac{19}{18}$

(ii) $\frac{3}{8} - \frac{-2}{9} + \frac{-1}{36} = \frac{3}{8} + \frac{2}{9} - \frac{1}{36} = \frac{3 \times 9 + 2 \times 8}{72} - \frac{1}{36}$

$= \frac{27 + 16}{72} - \frac{1}{36} = \frac{43}{72} - \frac{1}{36}$

$= \frac{43 \times 1 - 1 \times 2}{72} = \frac{43 - 2}{72} = \frac{41}{72}$

(iii) $\frac{1}{6} + \frac{-2}{5} - \frac{-2}{15} = \frac{1}{6} - \frac{2}{5} + \frac{2}{15} = \frac{1 \times 5 - 2 \times 6}{30} + \frac{2}{15}$

$= \frac{5 - 12}{30} + \frac{2}{15} = \frac{-7}{30} + \frac{2}{15}$

$= \frac{-7 \times 1 + 2 \times 2}{30} = \frac{-1}{10}$

(iv) $\frac{1}{12} + \frac{-5}{18} - \frac{7}{24} = \frac{1}{12} - \frac{5}{18} - \frac{7}{24} = \frac{1 \times 3 - 5 \times 2}{36} - \frac{7}{24}$

$= \frac{3 - 10}{36} - \frac{7}{24} = \frac{-7}{36} - \frac{7}{24}$

$= \frac{-7 \times 2 - 7 \times 3}{72} = \frac{-14 - 21}{72} = \frac{-35}{72}$

6. Let rational number $= x$

$\frac{-3}{22} - x = \frac{-5}{33}$

$x = \frac{5}{33} - \frac{3}{22} = \frac{5 \times 2 - 3 \times 3}{66} = \frac{10 - 9}{66} = \frac{1}{66}$

7. Let rational number $= x$

$\frac{-9}{25} - x = \frac{7}{10} \Rightarrow x = \frac{-9}{25} - \frac{7}{10}$

$= \frac{-9 \times 2 - 7 \times 5}{50} = \frac{-18 - 35}{50} = \frac{-53}{50}$

8. Let rational number = x

$$\frac{-7}{18} + x = \frac{11}{24}$$

$$x = \frac{+7}{18} + \frac{11}{24} = \frac{+7 \times 4 + 11 \times 3}{72} = \frac{+28 + 33}{72} = \frac{61}{72}$$

9. Let rational number = x

$$\frac{-3}{10} + x = -6$$

$$x = -6 + \frac{3}{10} = \frac{-6 \times 10 + 3}{10} = \frac{-60 + 3}{10} = \frac{-57}{10}$$

10. Let rational number = x

$$\frac{8}{57} + x = \frac{-6}{19}$$

$$x = \frac{-6}{19} - \frac{8}{57} = \frac{-6 \times 3 - 8 \times 1}{57} = \frac{-18 - 8}{57} = \frac{-26}{57}$$

Exercise 1.4

1. (i) -4 (ii) $\frac{-63}{108}$ (iii) $\frac{4}{5}, \frac{4}{5}$

(iv) $\frac{-5}{2}$ (v) not defined (vi) 1 and -1

(vii) x^2 (viii) 1

(ix) A negative rational number

2. (i) $\frac{3}{11} \times \frac{2}{5} = \frac{6}{55}$ (ii) $\frac{3}{7} \times \left(\frac{-2}{5}\right) = \frac{-6}{35}$

(iii) $\frac{9}{8} \times \frac{32}{3} = 12$ (iv) $\frac{25}{-9} \times \frac{3}{-10} = \frac{5}{6}$

(v) $\left(\frac{-6}{11}\right) \times \frac{44}{13} = \frac{-24}{13}$ (vi) $\left(\frac{9}{-11}\right) \times \left(\frac{22}{-27}\right) = \frac{2}{3}$

(vii) $\left(\frac{-8}{25}\right) \times \left(\frac{-5}{16}\right) = \frac{1}{10}$ (viii) $\frac{-36}{5} \times \frac{20}{-3} = 48$

3. (i) $\frac{4}{15} \times \frac{9}{5} \times \frac{50}{3} = \frac{4}{15} \times 3 \times 10 = 8$

(ii) $\left(\frac{-3}{2} \times \frac{4}{5}\right) + \left(\frac{9}{5} \times \frac{-10}{3}\right) - \left(\frac{1}{2} \times \frac{3}{4}\right)$

$= \left(\frac{-6}{5}\right) + (-6) - \frac{3}{8} = \frac{-6 \times 8 - 3 \times 5}{40} - 6$

$= \frac{-48 - 15}{40} - 6 = \frac{-63}{40} - 6 = \frac{-63 - 6 \times 40}{40} = \frac{-63 - 240}{40}$

$= \frac{-303}{40} = -7\frac{23}{40}$

(iii) $\frac{23}{5} \times \frac{17}{-22} \times \frac{-11}{69} \times \frac{60}{17} = 2$

(iv) $\frac{4}{5} \times \frac{5}{-9} \times \frac{-18}{-45} \times \frac{7}{4} = \frac{-14}{45}$

(v) $\frac{3}{4} \times \frac{-7}{12} \times \frac{-16}{21} \times \frac{-1}{2} = \frac{-1}{6}$

4. (i) $\left(\frac{12}{8} \times \frac{16}{10}\right) + \left(\frac{-3}{9} \times \frac{18}{-16}\right) = \frac{12}{5} + \frac{3}{8}$

$= \frac{12 \times 8 + 3 \times 5}{40} = \frac{96 + 15}{40} = \frac{111}{40}$

(ii) $\left(\frac{16}{15} \times \frac{-20}{4}\right) + \left(\frac{20}{15} \times \frac{-6}{5}\right) = \frac{-16}{3} - \frac{8}{5}$

$= \frac{-16 \times 5 - 8 \times 3}{15} = \frac{-104}{15}$

(iii) $\left(\frac{-4}{15} \times \frac{-5}{-8}\right) - \left(\frac{3}{5} \times \frac{6}{-15}\right) + \left(\frac{5}{-8} \times \frac{16}{15}\right)$

$= \left(\frac{-1}{6}\right) + \frac{6}{25} - \frac{2}{3} = \frac{-1}{6} - \frac{2}{3} + \frac{6}{25}$

$= \frac{-1 - 2 \times 2}{6} + \frac{6}{25} = \frac{-1 - 4}{6} + \frac{6}{25} = \frac{-5}{6} + \frac{6}{25}$

$= \frac{-5 \times 25 + 6 \times 6}{150} = \frac{-125 + 36}{150} = \frac{-89}{150}$

(iv) $1 - \frac{3}{8} \times \frac{18}{-35} \times \frac{7}{-9} = 1 - \frac{42}{280} = \frac{280 - 42}{280} = \frac{238}{280} = \frac{17}{20}$

5. (i) $\frac{2}{5} \times \frac{-3}{7} - \frac{1}{14} - \frac{3}{7} \times \frac{3}{5}$

$= \frac{-6}{35} - \frac{1}{14} - \frac{9}{35} = \frac{-6}{35} - \frac{9}{35} - \frac{1}{14}$

$= \frac{-15}{35} - \frac{1}{14} = \frac{-15 \times 2 - 5 \times 1}{70} = \frac{-30 - 5}{70} = \frac{-35}{70} = \frac{-1}{2}$

(ii) $\frac{8}{9} \times \frac{4}{5} + \frac{5}{6} - \frac{9}{5} \times \frac{8}{9}$

$= \frac{32}{45} - \frac{72}{45} + \frac{5}{6} = \frac{32 - 72}{45} + \frac{5}{6}$

$= \frac{-40}{45} + \frac{5}{6} = \frac{-40 \times 2 + 5 \times 15}{90} = \frac{-80 + 75}{90} = \frac{-5}{90} = \frac{-1}{18}$

(iii) $\frac{-3}{7} \times \frac{14}{15} \times \frac{7}{12} \times \left(\frac{-30}{35}\right) = \frac{-14}{60} \times \frac{-30}{35} = \frac{7}{35} = \frac{1}{5}$

6. Do it yourself.

7. Given : Average speed of train $= 65\frac{1}{3}$ km/h $= \frac{196}{3}$ km/h

Time $= 6$ hours

Let distance $= x$

We know that : Speed $= \frac{\text{Distance}}{\text{Time}} \Rightarrow \frac{196}{3} = \frac{x}{6} \Rightarrow x = 392$ km

8. Given : Milk $= 3\frac{5}{7}$ litre $= \frac{26}{7}$ litre ; Per litre cost $= ₹16\frac{1}{2} = ₹\frac{33}{2}$

$$\text{Total cost} = \frac{26}{7} \times \frac{33}{2} = \frac{429}{7} = ₹61\frac{2}{7}$$

9. Do it yourself. **10.** Do it yourself.

Exercise 1.5

1. Do it yourself.

2. (i) $\frac{4}{9} \div \left(\frac{-5}{12}\right) = \frac{4}{9} \times \left(\frac{-12}{5}\right) = \frac{-16}{15}$

(ii) $\frac{-4}{6} \div \frac{3}{2} = \frac{-4}{6} \times \frac{2}{3} = \frac{-4}{9}$

(iii) $\frac{-15}{7} \div (-30) = \frac{-15}{7} \times \frac{-1}{30} = \frac{1}{14}$

(iv) $-25 \div \left(\frac{-5}{6}\right) = -25 \times \frac{-6}{5} = 30$

(v) $\left(\frac{-16}{25}\right) \div \left(\frac{-15}{14}\right) = \frac{-16}{25} \times \frac{-14}{15} = \frac{224}{375}$

(vi) $\left(\frac{-5}{7}\right) \div \left(\frac{-15}{28}\right) = \left(\frac{-5}{7}\right) \times \left(\frac{-28}{15}\right) = \frac{4}{3}$

(vii) $\frac{-72}{35} \div \frac{48}{-49} = \frac{-72}{35} \times \frac{-49}{48} = \frac{21}{10}$

(viii) $\left(\frac{-7}{12}\right) \div \left(\frac{-2}{3}\right) = \left(\frac{-7}{12}\right) \times \left(\frac{-3}{2}\right) = \frac{7}{8}$

3. (i) $\frac{9}{8} \div (\quad\_\_\_\quad) = \frac{6}{5}$

Let value of bracket $= x$

$$\frac{9}{8} \div x = \frac{6}{5} \Rightarrow \frac{9}{8} \times \frac{1}{x} = \frac{6}{5} \Rightarrow x = \frac{45}{48}$$

(ii) $(\quad\_\_\_\quad) \div \left(\frac{-9}{15}\right) = \frac{25}{35}$

Let value of bracket $= x \Rightarrow x \div \left(\frac{-9}{15}\right) = \frac{25}{35}$

$$x \times \frac{-15}{9} = \frac{25}{35} \Rightarrow x = \frac{-3}{7}$$

(iii) $(\quad) \div (-5) = \dfrac{-15}{30}$

Let value of bracket $= x$

$$x \div (-5) = \frac{-15}{30} \Rightarrow x \times \frac{-1}{5} = \frac{-15}{30}$$

$$x = \frac{5}{2}$$

(iv) $(-15) \div (\quad) = \dfrac{-6}{5}$

Let value of bracket $= x$

$$(-15) \div x = \frac{-6}{5} \Rightarrow -15 \times \frac{1}{x} = \frac{-6}{5}$$

$$x = \frac{25}{2}$$

4. Do it yourself.

5. Let

$$x = \frac{\frac{65}{13} + \frac{5}{7}}{\frac{65}{13} - \frac{5}{7}} = \frac{\frac{65 \times 7 + 5 \times 13}{91}}{\frac{65 \times 7 - 5 \times 13}{91}}$$

$$x = \frac{65(7+1)}{65(7-1)} = \frac{8}{6} = \frac{4}{3}$$

6. Let $x = \dfrac{\frac{13}{5} + \frac{-6}{15}}{\frac{-29}{7} \times \frac{1}{-2}} = \dfrac{\frac{13 \times 3 - 6 \times 1}{15}}{\frac{29}{14}}$

$$= \frac{39-6}{15} \times \frac{14}{29} = \frac{33}{15} \times \frac{14}{29} \Rightarrow x = \frac{154}{145}$$

7. Let x is the number $\dfrac{-35}{6} \div x = \dfrac{-15}{2} \Rightarrow \dfrac{-35}{6} \times \dfrac{1}{x} = \dfrac{-15}{2}$

$$x = \frac{7}{9}$$

8. Let other number $= x$

$$x \times -12 = -15 \Rightarrow x = \frac{15}{12} = \frac{5}{4}$$

9. Let other number $= x$;

$$\frac{-4}{3} \times x = \frac{-16}{9} \Rightarrow x = \frac{4}{3}$$

10. Given : Cost of $3\frac{2}{5}$ metres of cloth $= ₹ 65\frac{1}{2}$

$$\frac{17}{5} \text{ metres} = ₹ \frac{131}{2}$$

$$1 \text{ metre } = ₹\frac{131}{2} \times \frac{5}{17} = ₹\frac{655}{34}$$

$$1 \text{ metre } = ₹19\frac{9}{34}$$

11. Given : 25 pairs of shirts of cloth = 60 metres

1 pair of shirts of cloth = $\frac{60}{25}$ metres

1 pair of shirts of cloth = 2.4 metres

Exercise 1.6

1. (i) $\frac{-5}{7}$ and $\frac{6}{13} = \frac{-5}{7}, \frac{6}{13}$ [LCM of 7, 13 = 91]

$$= \frac{-65, 42}{91} = \frac{-65}{91} < \frac{42}{91} \Rightarrow \frac{-5}{7} < \frac{6}{13}$$

(ii) $\frac{-4}{5} > \frac{-5}{6}$ (iii) $\frac{-7}{8} = \frac{21}{-24}$ (iv) $\frac{-9}{-10} > \frac{8}{9}$

2. (i) Do it yourself. (ii) $\frac{-16}{20}$ and $\frac{20}{-25}$

We first write the given numbers so that the denominators are positive

$$\frac{-16}{20} = \frac{-16}{20} \text{ and } \frac{20}{-25} = \frac{-20}{25}$$

Now, $\frac{-16}{20}, \frac{-20}{25} = \frac{-80, -80}{100}$ (LCM of 20 and 25 is 100)

$\therefore$ $\frac{-80}{100}, \frac{-80}{100}$ (Compare the numerators)

$\therefore$ $\frac{-16}{20}$ and $\frac{20}{-25}$ are equal.

(iii) and (iv) Do it yourself.

3. (i) $\frac{-4}{11}, \frac{3}{11} \Rightarrow \frac{-4}{11} < \frac{3}{11}$

$\therefore$ Greater rational number = $\frac{3}{11}$

(ii) $\frac{-5}{8}, \frac{-3}{4}$ LCM of 8, 4 is 8,

then $\frac{-5}{8}, \frac{-3 \times 2}{4 \times 2} = \frac{-5}{8}, \frac{-6}{8} \Rightarrow \frac{-5}{8} > \frac{-6}{8}$

$\therefore$ Greater rational number = $-\frac{5}{8}$

(iii), (iv) Do yourself.

4. (i) $\frac{-4}{7}, \frac{5}{-7}$

LCM of 7, 7 is 7.

$\frac{-4}{7} > \frac{-5}{7}$

$\therefore$ Smaller rational number $= -\frac{5}{7}$

(ii) $\frac{6}{13}, \frac{-7}{-13}$

LCM of 13, 13 is 13.

$\frac{6}{13} < \frac{7}{13}$

$\therefore$Smaller rational number $= \frac{6}{13}$

(iii), (iv) Do it yourself.

5. (i) $\frac{3}{-2}, \frac{3}{8}, \frac{-7}{4}, \frac{1}{10}$

Arrange in ascending order $= \frac{-7}{4}, \frac{3}{-2}, \frac{1}{10}, \frac{3}{8}$

(ii) $\frac{3}{5}, \frac{-7}{10}, \frac{1}{2}, \frac{8}{-15}$

Arrange in ascending order $= \frac{-7}{10}, \frac{8}{-15}, \frac{1}{2}, \frac{3}{5}$

6. (i) $\frac{-7}{2}, \frac{2}{-3}, \frac{5}{6}, \frac{2}{3}$

Arrange in descending order $= \frac{5}{6}, \frac{2}{3}, \frac{2}{-3}, \frac{-7}{2}$

(ii) $\frac{-15}{27}, \frac{8}{9}, 0, \frac{13}{45}$

Arrange in descending order $= \frac{8}{9}, \frac{13}{45}, 0, \frac{-15}{27}$

Word Problems

1. Quantity of paint that Rahul had $= \frac{3}{4}$ L

Quantity of paint used by Rahul for a room $= \frac{1}{2}$ L

Quantity of left paint $= \frac{3}{4}\text{ L} - \frac{1}{2}\text{ L} = \frac{3\text{ L} - 2\text{ L}}{4} = \frac{1}{4}$ L

Let *A* L paint need to made it $= \frac{4}{5}$ L

Then, $\frac{1}{4} + A = \frac{4}{5}$

$$A = \frac{4}{5} - \frac{1}{4} = \frac{16-5}{20} = \frac{11}{20}\text{ L}$$

2. Total quantity of sugar with Neha $= \frac{7}{9}$ cup

(i) Quantity of sugar that she used altogether

$$= \frac{1}{2} + \frac{1}{4} = \frac{2+1}{4} = \frac{3}{4}\text{ cup}$$

(ii) Quantity of sugar she had left $= \frac{7}{9} - \frac{3}{4} = \frac{28-27}{36} = \frac{1}{36}$ cup

3. The total weight of fruits $= 20$ kg

$$\text{Weight of oranges} = 7\frac{1}{6}\text{ kg} = \frac{43}{6}\text{ kg}$$

$$\text{Weight of apples} = 8\frac{2}{3}\text{ kg} = \frac{26}{3}\text{ kg}$$

$$\text{Weight of grapes} = 20 - \frac{43}{6} - \frac{26}{3} = 20 - \frac{43\times 1 - 26\times 2}{6}$$

$$= 20 - \frac{43-52}{6} = 20 - \frac{95}{6}$$

$$= \frac{20\times 6 - 95}{6} = \frac{120-95}{6} = \frac{25}{6}\text{ kg}$$

$$\text{Weight of grapes} = 4\frac{1}{6}\text{ kg.}$$

4. The population of a city $= 663432$

$$\text{Adult males} = \frac{1}{2}\text{ of population} = \frac{1}{2}\times 663432$$

$$= 331716$$

$$\text{Adult females} = \frac{1}{3}\text{ of population} = \frac{1}{3}\times 663432$$

$$= 221144$$

Number of children in city $= 663432 - 331716 - 221144$

$$= 110572.$$

5. Ankur spent a total time $= \frac{49}{6}$ h

Visit to old person's $= \frac{24}{7}$ h

Remaining time $= \frac{49}{6} - \frac{24}{7} = \frac{343-144}{42}$

blind persons $= \frac{199}{42}$ hrs

Longer time difference between blind persons and old person

$$= \frac{199}{42} - \frac{24}{7} = \frac{199-144}{42} = \frac{55}{42} \text{ hrs}$$

6. (i) Total time spent by Pulkit

$$= \frac{1}{4} \text{ hrs} + \frac{2}{5} \text{ hrs} = \frac{5\text{ h} + 8\text{ h}}{20} = \frac{13}{20} \text{ hrs.}$$

(ii) Longer time spent by him in cleaning the second classroom

$$= \frac{2}{5} \text{ hrs} - \frac{1}{4} \text{ hrs} = \frac{8\text{ hrs} - 5\text{ hrs}}{20} = \frac{3}{20} \text{ hrs}$$

7. (i) Bill of flour $= \frac{3}{4}\text{ kg} \times ₹\frac{21}{2} = ₹\frac{63}{8}$

(ii) Bill of sugar $= \frac{2}{5}\text{ kg} \times ₹\frac{31}{2} = ₹\frac{62}{10}$

(iii) Bill of salt $= \frac{1}{4}\text{ kg} \times ₹\frac{20}{3} = ₹\frac{20}{12}$

Total bill $= ₹\frac{63}{8} + ₹\frac{62}{10} + ₹\frac{20}{12}$

$$= \frac{63 \times 15 + 62 \times 12 + 20 \times 10}{120}$$

$$= \frac{945 + 744 + 200}{120} = ₹\frac{1889}{120}$$

8. Cost of 1 L milk $= ₹\frac{4}{5}$

Cost of $\frac{15}{28}$ L milk $= \frac{4}{5} \times \frac{15}{28} = ₹\frac{3}{7}$

9. Fraction of plank that is sawn off $= \frac{1}{5}$

Remaining part of plank $= 1 - \frac{1}{5} = \frac{4}{5}$

Fraction of plank that thrown away $= \frac{4}{5} \times \frac{3}{8} = \frac{3}{10}$

Fraction of the original plank remained

$$= \frac{4}{5} - \frac{3}{10} = \frac{8-3}{10} = \frac{5}{10} = \frac{1}{2}$$

10. Let other number $= x$

Product of two number $= \frac{6}{5} \Rightarrow \frac{1}{5} \times x = \frac{6}{5} \Rightarrow x = 6$

(i) Sum of two number $= \frac{1}{5} + 6 = \frac{1+30}{5} = \frac{31}{5}$

(ii) Difference of two number $= 6 - \frac{1}{5} = \frac{30-1}{5} = \frac{29}{5}$

11. Rohit had a rope $= 325\frac{4}{5}\text{ m} = \frac{1629}{5}\text{ m}$

Rohit cut off $= 150\frac{3}{5}\text{ m} = \frac{753}{5}\text{ m}$

Rest rope $= \frac{1629}{5} - \frac{753}{5} = \frac{876}{5}$

Length of each part $= \frac{\frac{876}{5}}{3} = \frac{876}{5 \times 3} = \frac{292}{5}\text{ m} = 58\frac{2}{5}\text{ m}$

12. $3\frac{1}{2}$ L of petrol cost $= ₹ 270\frac{3}{8}$

1 L of petrol cost $= ₹ \frac{2163}{8} \times \frac{2}{7} = ₹ \frac{309}{4}$

4 L of petrol cost $= ₹ \frac{309}{4} \times 4$

4 L of petrol cost $= ₹ 309$

13. Rajesh earns per month = ₹ 40,000

He spend $\frac{3}{8}$ of income on food $= \frac{40000 \times 3}{8} = ₹ 15000$

Rest amount of Rajesh = 25000

He spend $\frac{1}{5}$ of rest on LIC $= \frac{25000}{5} = 5000$

Rest amount of Income = ₹ 20000

He spend $\frac{1}{2}$ of rest on expenses $= \frac{1}{2} \times 20000 = 10000$

Rest amount of Income = ₹ 10000.

14. Let the bill $= x$

P paid $\frac{1}{2}$ of bill $= \frac{x}{2}$

Q paid $\frac{1}{5}$ of bill $= \frac{x}{5}$

Rest bill $= x - \frac{x}{2} - \frac{x}{5} = \frac{2x - x}{2} - \frac{x}{5}$

$= \frac{x}{2} - \frac{x}{5} = \frac{5x - 2x}{10} = \frac{3x}{10}$

Rest bill divide by R, S, T.

Fraction of bill paid by each $= \frac{3x}{10} \times \frac{1}{3} = \frac{x}{10}$

Fraction value $= \frac{1}{10}$.

15. Let total number of students = x

Number of students come by car = $\frac{2x}{5}$

Number of students come by bus = $\frac{x}{4}$

$$\text{Remaining students} = x - \frac{2x}{5} - \frac{x}{4} = \frac{5x - 2x}{5} - \frac{x}{4} = \frac{3x}{5} - \frac{x}{4}$$

$$= \frac{12x - 5x}{20} = \frac{7x}{20}$$

$\frac{1}{3}$ of remaining students walk = $\frac{7x}{20} \times \frac{1}{3} = 224$

$$\frac{7x}{60} = 224 \Rightarrow \frac{x}{60} = 32$$

$\therefore \quad x = 1920$

16. Given : $F + S_1 + S_2 = ₹\,60000$ ...(i)

$$S_1 = \frac{3}{8} \text{ of } F = \frac{3}{8}F \quad \text{and} \quad S_2 = \frac{1}{2}F = \frac{F}{2}$$

$$F + \frac{3}{8}F + \frac{F}{2} = ₹\,60000 \qquad \text{[from eq. (i)]}$$

$$\frac{8F + 3F}{8} + \frac{F}{2} = ₹\,60000 \Rightarrow \frac{11F}{8} + \frac{F}{2} = ₹\,60000$$

$$\frac{11F + 4F}{8} = ₹\,60000 \Rightarrow \frac{15F}{8} = ₹\,60000$$

(i) $F = 32000$

(ii) $S_1 = 32000 \times \frac{3}{8} = 12000$

(iii) $S_2 = 32000 \times \frac{1}{2} = 16000$

Revision Exercise

1. Do it yourself.

2. (i) Let x

$$\frac{2}{5} = \frac{x}{135} \Rightarrow x = 2 \times 27 \Rightarrow x = 54$$

(ii) $\frac{5}{x} = \frac{90}{216} \Rightarrow x = \frac{216}{18} \Rightarrow x = 12$

(iii) $\frac{25}{35} = \frac{5}{x} \Rightarrow x = \frac{35}{5} \Rightarrow x = 7$

(iv) $\frac{49}{56} = \frac{7}{x} \Rightarrow x = \frac{56}{7} \Rightarrow x = 8$

(v) $\frac{72}{81} = \frac{8}{x} \Rightarrow x = \frac{81}{9} \Rightarrow x = 9$

3. (i) $\frac{-144}{-504} \Rightarrow$ Standard form $= \frac{2}{7}$.

(ii) $\frac{140}{490} \Rightarrow$ Standard form $= \frac{2}{7}$.

(iii) $\frac{240}{-840} \Rightarrow$ Standard form $= \frac{-2}{7}$.

(iv) $\frac{225}{625} \Rightarrow$ Standard form $= \frac{9}{25}$

4. (i) $\frac{6}{11}$ and $\frac{-9}{11} \Rightarrow$ Addition $= \frac{6}{11} - \frac{9}{11} = \frac{6-9}{11} = \frac{-3}{11}$

(ii) $\frac{5}{-26}$ and $\frac{8}{39}$

Addition $= \frac{-5}{26} + \frac{8}{39} = \frac{-5 \times 3 + 8 \times 2}{78} = \frac{-15+16}{78} = \frac{1}{78}$

(iii) $\frac{5}{-6}$ and $\frac{2}{3}$

Addition $= \frac{-5}{6} + \frac{2}{3} = \frac{-5 \times 1 + 2 \times 2}{6} = \frac{-5+4}{6} = \frac{-1}{6}$

(iv) -2 and $\frac{2}{5}$

Addition $= -2 + \frac{2}{5} = \frac{-2 \times 5 + 2}{5} = \frac{-10+2}{5} = \frac{-8}{5}$

5. (i) $\frac{5}{9} + \frac{-7}{6} = \frac{5}{9} - \frac{7}{6} = \frac{5 \times 2 - 7 \times 3}{18} = \frac{10-21}{18} = \frac{-11}{18}$

(ii) $4 + \frac{3}{-5} = 4 - \frac{3}{5} = \frac{4 \times 5 - 3}{5} = \frac{20-3}{5} = \frac{17}{5}$

(iii) $\frac{1}{-15} + \frac{5}{-12} = \frac{-1}{15} - \frac{5}{12} = \frac{-1 \times 4 - 5 \times 5}{60} = \frac{-4-25}{60} = \frac{-29}{60}$

(iv) $\frac{5}{9} + \frac{3}{-4} = \frac{5}{9} - \frac{3}{4} = \frac{5 \times 4 - 3 \times 9}{36} = \frac{20-27}{36} = \frac{-7}{36}$

6. (i) $\frac{3}{7} + \frac{-4}{9} + \frac{-11}{7} + \frac{7}{9} = \frac{3}{7} - \frac{4}{9} - \frac{11}{7} + \frac{7}{9}$

$= \frac{3 \times 9 - 4 \times 7}{63} - \left(\frac{11 \times 9 - 7 \times 7}{63} \right) = \frac{27-28}{63} - \frac{(99-49)}{63}$

$= \frac{-1}{63} - \frac{50}{63} = \frac{-1-50}{63} = \frac{-51}{63} = \frac{-17}{21}$

(ii) $\frac{2}{3} + \frac{-4}{5} + \frac{1}{3} + \frac{2}{5} = \frac{2}{3} - \frac{4}{5} + \frac{1}{3} + \frac{2}{5}$

$= \frac{2}{3} + \frac{1}{3} + \frac{2}{5} - \frac{4}{5} = \frac{3}{3} - \frac{2}{5} = 1 - \frac{2}{5} = \frac{3}{5}$

(iii) $\frac{4}{7}+0+\frac{-8}{9}+\frac{-13}{7}+\frac{17}{9}$

$=\frac{4}{7}-\frac{8}{9}-\frac{13}{7}+\frac{17}{9}=\frac{4\times9-8\times7}{63}-\left(\frac{13\times9-17\times7}{63}\right)$

$=\frac{36-56}{63}-\left(\frac{117-119}{63}\right)=\frac{-20}{63}+\frac{2}{63}=\frac{-20+2}{63}=\frac{-18}{63}=\frac{-2}{7}$

(iv) $\frac{3}{8}+\frac{-5}{12}+\frac{3}{7}+\frac{3}{12}+\frac{-5}{8}+\frac{-2}{7}=\frac{3}{8}-\frac{5}{12}+\frac{3}{7}+\frac{3}{12}-\frac{5}{8}-\frac{2}{7}$

$=\frac{3\times3-5\times2}{24}+\frac{3\times12+3\times7}{84}-\left(\frac{5\times7+2\times8}{56}\right)$

$=\frac{9-10}{24}+\frac{36+21}{84}-\left(\frac{35+16}{56}\right)$

$=\frac{-1}{24}+\frac{57}{84}-\frac{51}{56}=\frac{-1\times7+57\times2}{168}-\frac{51}{56}=\frac{-7+114}{168}-\frac{51}{56}$

$=\frac{107}{168}-\frac{51}{56}=\frac{107-51\times3}{168}=\frac{107-153}{168}=\frac{-46}{168}=\frac{-23}{84}$

7. (i) $\frac{-3}{8}-\frac{5}{8}=\frac{-3-5}{8}=\frac{-8}{8}=-1$ (ii) $\frac{4}{11}-\left(\frac{-8}{11}\right)=\frac{4}{11}+\frac{8}{11}=\frac{12}{11}$

(iii) $\frac{-5}{9}-\frac{4}{9}=\frac{-5-4}{9}=\frac{-9}{9}=-1$

(iv) $\frac{-3}{8}-\frac{1}{4}=\frac{-3\times1-1\times2}{8}=\frac{-3-2}{8}=\frac{-5}{8}$

8. (i) $\frac{2}{3}-\frac{4}{5}=\frac{2\times5-4\times3}{15}=\frac{10-12}{15}=\frac{-2}{15}$

(ii) $\frac{-4}{9}-\frac{2}{-3}=\frac{-4}{9}+\frac{2}{3}=\frac{-4\times1+2\times3}{9}=\frac{-4+6}{9}=\frac{2}{9}$

(iii) $-1-\frac{4}{9}=\frac{-1\times9-4\times1}{9}=\frac{-9-4}{9}=\frac{-13}{9}$

(iv) $\frac{-2}{7}-\frac{3}{-14}=\frac{-2\times2+3\times1}{14}=\frac{-4+3}{14}=\frac{-1}{14}$

9. (i) $\frac{2}{3}+\frac{-4}{5}-\frac{1}{3}-\frac{2}{5}=\frac{2}{3}-\frac{4}{5}-\frac{1}{3}-\frac{2}{5}$

$=\frac{2\times5-4\times3}{15}-\left(\frac{1\times5+2\times3}{15}\right)$

$=\frac{10-12}{15}-\left(\frac{5+6}{15}\right)$

$=\frac{-2}{15}-\frac{11}{15}=\frac{-2-11}{15}=\frac{-13}{15}$

(ii) $\frac{4}{7}-\frac{-8}{9}-\frac{-13}{7}+\frac{17}{9}=\frac{4}{7}+\frac{8}{9}+\frac{13}{7}+\frac{17}{9}$

$=\frac{4\times 9+8\times 7}{63}+\frac{13\times 9+17\times 7}{63}$

$=\frac{36+56}{63}+\frac{117+119}{63}$

$=\frac{92}{63}+\frac{236}{63}=\frac{92+236}{63}=\frac{328}{63}=5\frac{13}{63}$

10. Given : Sum of two rational number $=\frac{9}{20}$ and one number $=\frac{2}{5}$

Let other number $= x$

$x+\frac{2}{5}=\frac{9}{20}$

$x=\frac{9}{20}-\frac{2}{5}=\frac{9\times 1-2\times 4}{20}=\frac{9-8}{20}=\frac{1}{20}$

11. Let rational number $= x$

$\frac{-5}{6}-x=\frac{4}{9}$

$x=\frac{-5}{6}-\frac{4}{9}=\frac{-5\times 3-4\times 2}{18}=\frac{-15-8}{18}=\frac{-23}{18}$

12. Let other rational number $= x$

Sum of rational number $=\frac{-2}{3}$

$\frac{-8}{15}+x=\frac{-2}{3}$

$x=\frac{-2}{3}+\frac{8}{15}=\frac{-2\times 5+8\times 1}{15}=\frac{-10+8}{15}=\frac{-2}{15}$

13. (i) Let the other rational number $= x$

$-2-x=\frac{3}{8}$

$x=-2-\frac{3}{8}=\frac{-2\times 8-3}{8}=\frac{-16-3}{8}=\frac{-19}{8}$

(ii) Let the other rational number $= x$

$-2+x=\frac{3}{8}$

$x=\frac{3}{8}+2=\frac{3\times 1+2\times 8}{8}=\frac{3+16}{8}=\frac{19}{8}$

14. (i) $\frac{-8}{22}\times\frac{11}{16}\times\frac{3}{10}=\frac{-1}{2}\times\frac{1}{2}\times\frac{3}{10}=\frac{-3}{40}$

(ii) $\frac{-19}{50}\times\frac{75}{38}\times\frac{-12}{25}=\frac{-1}{50}\times\frac{3}{2}\times -12=\frac{9}{25}$

(iii) $\frac{11}{12} \times \frac{-7}{8} \times \frac{-16}{22} = \frac{1}{12} \times \frac{-7}{1} \times \frac{-2}{2} = \frac{7}{12}$

15. (i) $\frac{-3}{8} \times \left(\frac{5}{9} + \frac{-7}{18}\right) = \frac{-3}{8} \times \left(\frac{5}{9} - \frac{7}{18}\right) = \frac{-3}{8} \times \left(\frac{5 \times 2 - 7 \times 1}{18}\right)$

$= \frac{-3}{8} \times \left(\frac{10 - 7}{18}\right) = \frac{-3}{8} \times \frac{3}{18} = \frac{-1}{16}$

(ii) $\frac{5}{7} \times \left(\frac{-14}{25} + \frac{21}{50}\right) = \frac{5}{7} \times \left(\frac{-14 \times 2 + 21 \times 1}{50}\right)$

$= \frac{5}{7} \times \left(\frac{-28 + 21}{50}\right) = \frac{5}{7} \times \frac{-7}{50} = \frac{-1}{10}$

(iii) $\frac{-5}{8} \times \frac{7}{9} + \frac{5}{12} \times \frac{1}{40} + \frac{7}{9} \times \frac{-3}{5} = \frac{-35}{72} + \frac{5}{480} + \frac{(-21)}{45}$

$= \frac{-35 \times 20 + 5 \times 3 - 21 \times 32}{1440}$

$= \frac{-700 + 15 - 672}{1440} = \frac{-1357}{1440}$

16. Do it yourself.

17. (i) $\frac{-5}{12} \div \frac{1}{16} = \frac{-5}{12} \times 16 = \frac{-20}{3}$

(ii) $\frac{-21}{26} \div \left(\frac{-7}{8}\right) = \frac{-21}{26} \times \frac{8}{-7} = \frac{12}{13}$

(iii) $0 \div \left(\frac{-4}{7}\right) = 0 \times \left(\frac{7}{-4}\right) = 0$

(iv) $\frac{8}{-5} \div \frac{24}{25} = \frac{8}{-5} \times \frac{25}{24} = \frac{1}{-5} \times \frac{25}{3} = \frac{-5}{3}$

18. (i) -2 by $\left(\frac{-1}{2}\right) = -2 \div \left(\frac{-1}{2}\right) = -2 \times -2 = 4$

(ii) 0 by $\frac{7}{-9} = 0 \div \left(\frac{-7}{9}\right) = 0 \times \left(\frac{9}{-7}\right) = 0$

(iii) $\frac{-5}{8}$ by $\frac{1}{4} = \frac{-5}{8} \div \frac{1}{4} = \frac{-5}{8} \times 4 = \frac{-5}{2}$

(iv) $\frac{-3}{4}$ by $\frac{-9}{16} = \frac{-3}{4} \div \frac{-9}{16} = \frac{-3}{4} \times \frac{-16}{9} = \frac{4}{3}$

19. $\left[\frac{3}{7} + \left(\frac{-5}{14}\right)\right] \div \frac{-1}{2} = \left(\frac{3}{7} - \frac{5}{14}\right) \times -2 = \left(\frac{3 \times 2 - 5 \times 1}{14}\right) \times -2$

$= \left(\frac{6 - 5}{14}\right) \times -2 = \frac{1}{14} \times -2 = \frac{-1}{7}$

20. (i) $m=\frac{2}{3}$ and $n=\frac{3}{2}$

$$(m+n)\div(m-n)=\left(\frac{2}{3}+\frac{3}{2}\right)\div\left(\frac{2}{3}-\frac{3}{2}\right)$$
$$=\left(\frac{4+9}{6}\right)\div\left(\frac{2\times 2-3\times 3}{6}\right)$$
$$=\frac{13}{6}\div\left(\frac{4-9}{6}\right)=\frac{13}{6}\div\left(\frac{-5}{6}\right)$$
$$=\frac{13}{6}\times\frac{-6}{5}=\frac{-13}{5}$$

(ii) $m=\frac{3}{4}$ and $n=\frac{4}{3}$

$$(m+n)\div(m-n)=\left(\frac{3}{4}+\frac{4}{3}\right)\div\left(\frac{3}{4}-\frac{4}{3}\right)$$
$$=\left(\frac{3\times 3+4\times 4}{12}\right)\div\left(\frac{3\times 3-4\times 4}{12}\right)$$
$$=\left(\frac{9+16}{12}\right)\div\left(\frac{9-16}{12}\right)$$
$$=\frac{25}{12}\div\left(\frac{-7}{12}\right)=\frac{25}{12}\times\frac{-12}{7}=\frac{-25}{7}$$

(iii) $m=\frac{4}{5}$ and $n=\frac{-3}{10}$

$$(m+n)\div(m-n)=\left(\frac{4}{5}+\left(\frac{-3}{10}\right)\right)\div\left(\frac{4}{5}-\left(\frac{-3}{10}\right)\right)$$
$$=\left(\frac{4}{5}-\frac{3}{10}\right)\div\left(\frac{4}{5}+\frac{3}{10}\right)$$
$$=\left(\frac{4\times 2-3\times 1}{10}\right)\div\left(\frac{4\times 2+3\times 1}{10}\right)$$
$$=\frac{(8-3)}{10}\div\left(\frac{8+3}{10}\right)=\frac{5}{10}\times\frac{10}{11}=\frac{5}{11}$$

21. $\left(\frac{5}{8}+\left(\frac{-11}{12}\right)\right)\div\left(\frac{3}{7}-\frac{5}{14}\right)=\left(\frac{5}{8}-\frac{11}{12}\right)\div\left(\frac{3}{7}-\frac{5}{14}\right)$

$$=\left(\frac{5\times 3-11\times 2}{24}\right)\div\left(\frac{3\times 2-5\times 1}{14}\right)$$
$$=\left(\frac{15-22}{24}\right)\div\left(\frac{6-5}{14}\right)$$
$$=\frac{-7}{24}\times\frac{14}{1}=\frac{-7}{12}\times 7=\frac{-49}{12}$$

22. Given : Dimension of a hall $= 8$ m by $\frac{11}{2}$ m.

Area of hall $=$ Length $\times$ Width $= 8 \times \frac{11}{2} = \frac{88}{2} = 44\ \text{m}^2$

Required cost of $1\ \text{m}^2 = ₹\frac{21}{4}$

Required cost of $44\ \text{m}^2 = ₹\frac{21}{4} \times 44 = ₹\,231 = ₹\,231$

23. A car cover distance in $4\frac{1}{5}$ L petrol $= 82$ km

$$\frac{21}{5}\text{ L petrol} = 82\text{ km}$$

$$1\text{ L petrol} = \frac{82 \times 5}{21}\text{ km} = \frac{410}{21}\text{ km} = 19\frac{11}{21}\text{ km}$$

24. The cost of 12 balls $= ₹\,205\frac{1}{7} = ₹\frac{1436}{7}$

The cost of 1 ball $= ₹\frac{1436}{12 \times 7} = ₹\frac{359}{3 \times 7} = ₹\frac{359}{21} = ₹\,17\frac{2}{21}$.

25. Length of wire $= 50$ m

Two pieces of wire cut off $= 10\frac{3}{4} + 22\frac{1}{5}$

Length of remaining wire $= 50 - 10\frac{3}{4} - 22\frac{1}{5} = 50 - \frac{43}{4} - \frac{111}{5}$

$$= 50 - \left(\frac{43 \times 5 + 111 \times 4}{20}\right) = 50 - \frac{(215 + 444)}{20}$$

$$= 50 - \frac{659}{20} = \frac{50 \times 20 - 659}{20}$$

Length of remaining wire $= \frac{1000 - 659}{20} = \frac{341}{20} = 17\frac{1}{20}$ m

❑

2. Exponents and Radicals

Exercise 2.1

1. (i) $5^{1/4} \Rightarrow$ Radical form $= \sqrt[4]{5}$

(ii) $21^{2/3} \Rightarrow$ Radical form $= \sqrt[3]{21^2}$

(iii) $2^{5/6} \Rightarrow$ Radical form $= \sqrt[6]{2^5}$

(iv) $\left(\frac{5}{17}\right)^{1/9} \Rightarrow$ Radical form $= \sqrt[9]{\left(\frac{5}{17}\right)}$

(v) $\left(\frac{17}{21}\right)^{2/5} \Rightarrow$ Radical form $= \sqrt[5]{\left(\frac{17}{21}\right)^2}$

(vi) $(-215)^{1/7} \Rightarrow$ Radical form $= \sqrt[7]{(-215)}$

2. (i) $\sqrt[4]{37} \Rightarrow$ Exponential form $= (37)^{1/4}$

(ii) $\sqrt[5]{27} \Rightarrow$ Exponential form $= (27)^{1/5}$

(iii) $\sqrt[7]{(29)^2} \Rightarrow$ Exponential form $= (29)^{2/7}$

(iv) $\sqrt[6]{\frac{8}{9}} \Rightarrow$ Exponential form $= \left(\frac{8}{9}\right)^{1/6}$

(v) $\sqrt[3]{\left(\frac{2}{3}\right)^2} \Rightarrow$ Exponential form $= \left(\frac{2}{3}\right)^{2/3}$

(vi) $\sqrt[3]{2^{-6}} \Rightarrow$ Exponential form $= 2^{-6/3} = 2^{-2}$

3. (i) $3 \times 16^{3/4} = 3 \times (2^4)^{3/4} = 3 \times (2)^{\frac{4 \times 3}{4}}$

$= 3 \times 2^3 = 3 \times 8 = 24$

(ii) $2 \times (27)^{-2/3} = 2 \times (3^3)^{-2/3} = 2 \times (3)^{\frac{-3 \times 2}{3}}$

$= 2 \times 3^{-2} = 2 \times \frac{1}{3^2} = \frac{2}{9}$

(iii) $5^2 \times 5^{-4} = 5^2 \times \frac{1}{5^4} = \frac{1}{5^4 \times 5^{-2}}$

$= \frac{1}{5^{4-2}} = \frac{1}{5^2} = \frac{1}{25}$

(iv) $[(8)^{2/3}]^{-3/2} = [(2)^{\frac{3 \times 2}{3}}]^{-3/2}$

$= [2^2]^{-3/2} = 2^{\frac{2 \times -3}{2}} = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

4. (i) $(0.04)^{3/2} = \left(\frac{004}{100}\right)^{3/2} = \left(\frac{4}{100}\right)^{3/2} = \left(\frac{2^2}{10^2}\right)^{3/2}$

$= \frac{(2)^{\frac{2 \times 3}{2}}}{(10)^{\frac{2 \times 3}{2}}} = \frac{2^3}{10^3} = \frac{8}{1000} = 0.008$

(ii) $(0.008)^{2/3} = \left(\frac{8}{1000}\right)^{2/3} = \left(\frac{2^3}{10^3}\right)^{2/3}$

$= \frac{(2^3)^{2/3}}{(10^3)^{2/3}} = \frac{2^{\frac{3\times 2}{3}}}{10^{\frac{3\times 2}{3}}} = \frac{2^2}{10^2} = \frac{4}{100} = 0.04$

(iii) $(0.000064)^{5/6} = \left(\frac{64}{1000000}\right)^{5/6} = \left(\frac{2^6}{10^6}\right)^{5/6} = \frac{(2^6)^{5/6}}{(10^6)^{5/6}}$

$= \frac{2^{\frac{6\times 5}{6}}}{10^{\frac{6\times 5}{6}}} = \frac{2^5}{10^5} = \frac{32}{100000} = 0.00032$

(iv) $(6.25)^{3/2} = \left(\frac{625}{100}\right)^{3/2} = \left[\frac{(25)^2}{(10)^2}\right]^{3/2}$

$= \frac{(25)^{\frac{2\times 3}{2}}}{(10)^{\frac{2\times 3}{2}}} = \frac{(25)^3}{(10)^3} = \frac{15625}{1000} = 15.625$

5. (i) $64^{1/2} \times (64^{1/2} + 1)$

$= (8)^{\frac{2\times 1}{2}} \times [(8^2)^{1/2} + 1] = (8)^{\frac{2\times 1}{2}} \times [(8)^{\frac{2\times 1}{2}} + 1]$

$= 8 \times (8 + 1) = 8 \times 9 = 72$

(ii) $27^{-1/3} \times (27^{1/3} - 27^{2/3})$

$= (3^3)^{-1/3} \times [(3^3)^{1/3} - (3^3)^{2/3}]$

$= (3)^{\frac{3\times -1}{3}} \times [(3)^{\frac{3\times 1}{3}} - (3)^{\frac{3\times 2}{3}}]$

$= (3)^{-1} \times [(3)^1 - (3)^2]$

$= \frac{1}{(3)} \times [3 - 9] = \frac{1}{3} \times [-6] = \frac{-6}{3} = -2$

(iii) $4 \times (81)^{-1/2} \times [(81)^{1/2} + (81)^{3/2}]$

$= 4 \times (9^2)^{-1/2} \times [(9^2)^{1/2} + (9^2)^{3/2}]$

$= 4 \times 9^{\frac{2\times -1}{2}} \times [9^{\frac{2\times 1}{2}} + 9^{\frac{2\times 3}{2}}]$

$= 4 \times 9^{-1} \times [9 + 9^3] = \frac{4\times 1}{9} \times [9 + 729]$

$= \frac{4}{9} \times [738] = \frac{4}{9} \times 738 = 4 \times 82 = 328$

(iv) $\dfrac{(36)^{7/2}-(36)^{9/2}}{(36)^{5/2}}=\dfrac{(6^2)^{7/2}-(6^2)^{9/2}}{(6^2)^{5/2}}$

$=\dfrac{6^{\frac{2\times7}{2}}-6^{\frac{2\times9}{2}}}{6^{\frac{2\times5}{2}}}=\dfrac{6^7-6^9}{6^5}=\dfrac{6^7[1-6^2]}{6^5}$

$=6^7\times6^{-5}[1-6^2]=6^{7-5}[1-36]$

$=6^2[-35]=36\times-35=-1260$

6. (i) $[(2)^{-1}+(4)^{-1}+(3)^{-1}]^{-1}$

$=\left[\dfrac{1}{(2)}+\dfrac{1}{(4)}+\dfrac{1}{3}\right]^{-1}=\left[\dfrac{1}{2}+\dfrac{1}{4}+\dfrac{1}{3}\right]^{-1}$

$=\left[\dfrac{1\times6+1\times3+1\times4}{12}\right]^{-1}$

$=\left[\dfrac{6+3+4}{12}\right]^{-1}=\left(\dfrac{13}{12}\right)^{-1}=\dfrac{1}{\left(\frac{13}{12}\right)}=\left(\dfrac{12}{13}\right)$

(ii) $[(4)^{-1}-(5)^{-1}]^2\times\left(\dfrac{5}{8}\right)^{-1}=\left[\dfrac{1}{4}-\dfrac{1}{5}\right]^2\times\dfrac{1}{\left(\frac{5}{8}\right)}=\left[\dfrac{5-4}{20}\right]^2\times\left(\dfrac{8}{5}\right)$

$=\left(\dfrac{1}{20}\right)^2\times\left(\dfrac{8}{5}\right)=\dfrac{1}{20}\times\dfrac{1}{20}\times\dfrac{8}{5}$

$=\dfrac{1}{10}\times\dfrac{1}{5}\times\dfrac{1}{5}=\dfrac{1}{250}$

(iii) $[4^0+4^2-2^3]\times3^{-2}=[1+16-8]\times\dfrac{1}{3^2}=\dfrac{(9)\times1}{3^2}=\dfrac{9}{9}=1$

(iv) $\left[(5)^2-\left(\dfrac{1}{4}\right)^{-2}\right]\times\left(\dfrac{3}{4}\right)^{-2}=\left[25-\dfrac{1}{(4)^{-2}}\right]\times\dfrac{1}{\left(\frac{3}{4}\right)^2}$

$=[25-(4)^2]\times\dfrac{1}{\frac{9}{16}}$

$=[25-16]\times\dfrac{16}{9}=9\times\dfrac{16}{9}=16$

7. (i) $(x^{-4})^3$ Positive exponent $=(x)^{-4\times3}=(x)^{-12}=\dfrac{1}{x^{12}}$

(ii) $2(x)^{1/6} \times 2(x)^{-7/6}$

Positive exponent $= 4(x)^{1/6} \times (x)^{-7/6}$

$= 4[(x)^{1/6 - 7/6}] = 4[(x)^{\frac{-6}{6}}]$

$= 4[(x)^{-1}] = 4\left(\frac{1}{x}\right) = \frac{4}{x}$

(iii) $x^{-7} \times y^{-7} = \frac{1}{x^7} \times \frac{1}{y^7} = \frac{1}{x^7 y^7} = \frac{1}{(xy)^7}$

(iv) $x^{5/7} \div x^{12/7} = \frac{(x)^{5/7}}{(x)^{12/7}} = (x)^{5/7} \times (x)^{-(12/7)}$

$= (x)^{\left(\frac{5}{7} - \frac{12}{7}\right)} = (x)^{\left(\frac{-7}{7}\right)} = (x)^{-1} = \frac{1}{x}$

8. (i) $(3^2 + 4^2)^{1/2} = (9 + 16)^{1/2} = (25)^{1/2}$

$= (5^2)^{1/2} = 5^{\frac{2 \times 1}{2}} = 5$

(ii) $(5^2 + 12^2)^{1/2} = (25 + 144)^{1/2}$

$= (169)^{1/2} = (13^2)^{1/2} = (13)^{\frac{2 \times 1}{2}} = 13$

(iii) $(8^2 + 15^2)^{1/2} = (64 + 225)^{1/2}$

$= (289)^{1/2} = (17^2)^{1/2} = (17)^{\frac{2 \times 1}{2}} = 17$

(iv) $(1^3 + 2^3 + 3^3)^{1/2} = (1 + 8 + 27)^{1/2}$

$= (36)^{1/2} = (6^2)^{1/2} = 6^{\frac{2 \times 1}{2}} = 6$

9. (i) $5^3 \times \left(\frac{4}{5}\right)^3 = (5^3) \times \frac{4^3}{(5)^3} = 4^3$

Negative exponent $= \frac{1}{4^{-3}}$

(ii) $\left[\left(\frac{3}{7}\right)^{-2}\right]^{-3} = \left[\frac{(3)^{-2}}{(7)^{-2}}\right]^{-3} = \left[\frac{(3)^{-2 \times -3}}{(7)^{-2 \times -3}}\right] = \left[\frac{3^6}{7^6}\right] = \left(\frac{3}{7}\right)^6$

Negative exponent $= \left(\frac{7}{3}\right)^{-6}$

(iii) $\left(\frac{5}{9}\right)^{-2} \times \left(\frac{5}{3}\right)^{2} \div \left(\frac{1}{5}\right)^{-2} = \left(\frac{5}{9}\right)^{-2} \times \left(\frac{5}{3}\right)^{2} \times \frac{1}{\left(\frac{1}{5}\right)^{-2}}$

$= \left(\frac{5}{9}\right)^{-2} \times \left(\frac{5}{3}\right)^{2} \times \left(\frac{1}{5}\right)^{2} = \frac{(5)^{-2}}{(3^2)^{-2}} \times \frac{(5)^2}{(3)^2} \times \frac{1}{(5)^2}$

$= \frac{(5)^{-2}}{(3^{-4}) \times 3^2} = \frac{(5)^{-2} \times 1}{3^{-2}} = \frac{5^{-2}}{3^{-2}} = \left(\frac{5}{3}\right)^{-2}$

Negative exponent $= \left(\frac{5}{3}\right)^{-2}$.

(iv) $2^{-1}\left[\left(\frac{5}{3}\right)^{4} + \left(\frac{3}{5}\right)^{-2}\right] \div \frac{17}{9} = \frac{1}{2}\left[\left(\frac{5}{3}\right)^{4} + \frac{1}{\left(\frac{3}{5}\right)^{2}}\right] \div \frac{17}{9}$

$= \frac{1}{2}\left[\left(\frac{5}{3}\right)^{4} + \left(\frac{5}{3}\right)^{2}\right] \div \frac{17}{9} = \frac{1}{2}\left(\frac{5}{3}\right)^{2}\left[\left(\frac{5}{3}\right)^{2} + 1\right] \div \frac{17}{9}$

$= \frac{1}{2} \times \frac{25}{9}\left[\frac{25}{9} + 1\right] \div \frac{17}{9} = \frac{25}{18}\left[\frac{25+9}{9}\right] \times \frac{9}{17}$

$= \frac{25}{18} \times \frac{34}{9} \times \frac{9}{17} = \frac{25}{18} \times 2 = \frac{25}{9} = \left(\frac{5}{3}\right)^{2} = \frac{1}{\left(\frac{5}{3}\right)^{-2}}$

Negative exponent $= \left(\frac{3}{5}\right)^{-2}$.

(v) $(-7)^3 \times \left(\frac{1}{-7}\right)^{-9} \div (-7)^{10} = (-7)^3 \frac{1}{(-7)^{-9}} \times \frac{1}{(-7)^{10}}$

$= (-7)^3 \times (-7)^9 \times \frac{1}{(-7)^{10}} = (-7)^{3+9} \times (-7)^{-10}$

$= (-7)^{12} \times (-7)^{-10} = (-7)^{12-10} = (-7)^2 = (7^2)$

Negative exponent $= \frac{1}{(7)^{-2}}$.

10. $\left(\frac{-2}{3}\right)^{-13} \times \left(\frac{3}{-2}\right)^{8} = \left(\frac{-2}{3}\right)^{-2x+1}$

$= \frac{(-2)^{-13}}{(3)^{-13}} \times \frac{(3)^8}{(-2)^8} = \frac{(-2)^{-2x+1}}{(3)^{-2x+1}}$

$$\frac{(-2)^{-2x+1}}{(3)^{-2x+1}} = \frac{(-2)^{-13} \times (-2)^{-8}}{(3)^{-13} \times (3)^{-8}}$$

$$\frac{(-2)^{-2x+1}}{(3)^{-2x+1}} = \frac{(-2)^{-21}}{(3)^{-21}}$$

Compare both side : $-21 = -2x + 1$

$x = 11$

Exercise 2.2

1. (i) 6250000000

Number in standard form $= 6.25 \times 10^9$.

(ii) 71960000000000000

Number in standard form $= 7.196 \times 10^{16}$.

(iii) $0.0000000213 = \frac{0000000213}{10^{10}} = 213 \times 10^{-10}$

Number in standard form $= 2.13 \times 10^{-8}$.

(iv) $0.0000000925 = \frac{0000000925}{10^{10}} = \frac{925}{10^{10}} = 925 \times 10^{-10}$

Number in standard form $= 9.25 \times 10^{-8}$.

2. (i) 9.67×10^5

Number in usual form $= \frac{967}{10^2} \times 10^5 = 967 \times 10^{5-2}$

$= 967 \times 10^3 = 967000$

(ii) 8.37×10^8

Number in usual form $= \frac{837}{10^2} \times 10^8$

$= 837 \times 10^{8-2} = 837 \times 10^6$

$= 837000000$

(iii) 9.42×10^{-4}

Number in usual form $= \frac{942}{10^2} \times 10^{-4} = 942 \times 10^{-4} \times 10^{-2}$

$= 942 \times 10^{-6} = \frac{942}{10^6} = 0.000942$

(iv) 6.75×10^{-7}

Number in usual form $= \frac{675}{10^2} \times 10^{-7} = 675 \times 10^{-7} \times 10^{-2}$

$= 675 \times 10^{-9} = \frac{675}{10^9} = 0.000000675$

3. (i) Thickness of a piece of paper $= 0.0016$ cm. $= \frac{16}{10^4}$

$$= 16 \times 10^{-4} = 16 \times 10^{-4} \times \frac{10}{10}$$

Thickness of paper in standard form

$$= 1.6 \times 10^{-4+1} = 1.6 \times 10^{-3} \text{ cm.}$$

(ii) A helium atom has diameter

$$= \frac{22}{100000000000} = 22 \times 10^{-11}$$

$$= 22 \times 10^{-11} \times \frac{10}{10} = 2.2 \times 10^{-11+1}$$

Atom diameter in standard form $= 2.2 \times 10^{-10}$ m.

(iii) Mass of molecule of hydrogen

$$= 0.00000000000000000000334$$

$$= \frac{334}{10^{23}} = 334 \times 10^{-23}$$

$$= 334 \times 10^{-23} \times \frac{10^2}{10^2} = 3.34 \times 10^{-23+2}$$

Hydrogen mass in standard form $= 3.34 \times 10^{-21}$ tons.

(iv) Speed of light $= 300{,}000{,}000$ m/sec

Speed of light in standard form $= 3 \times 10^8$ m/sec.

(v) 3 years in seconds

$= 3$ years $\times$ 365 days $\times$ 24 hours $\times$ 60 minutes $\times$ 60 seconds

$$= 3 \times 365 \times 24 \times 60 \times 60 = 94608000 = 94608000 \times \frac{10^7}{10^7}$$

3 years in second in standard form $= 9.46 \times 10^7$ sec.

(vi) A sugar factory has annual sales $= 3$ billion 720 million kg

$$= 3000000000 + 720000000$$

$$= 3720000000 = 372 \times 10^7$$

Standard form of sugar $= 372 \times 10^7 \times \frac{100}{100}$

$$= 3.72 \times 10^7 \times 10^2$$

$$= 3.72 \times 10^{7+2} = 3.72 \times 10^9 \text{ kg}$$

Word Problems

1. Distance of moon from earth in standard form

$$= 384{,}467{,}000 = 384467000 \times \frac{10^8}{10^8}$$

$$= 3.84 \times 10^8 \text{ m}$$

2. Size of bacterium in standard form

$$= 0.0000005 = \frac{5}{10^7} = 5 \times 10^{-7} \text{ m}$$

3. Mass of sun in standard form $= 1.989 \times 10^{30}$ kg

= 1,989, 000, 000, 000, 000, 000, 000, 000, 000, 000, 000 kg.

Revision Exercise

1. (i) $16^{1/2} \Rightarrow$ Radical form $= \sqrt{16}$

(ii) $(125)^{1/3} \Rightarrow$ Radical form $= \sqrt[3]{125}$

(iii) $\left(\frac{6}{17}\right)^{1/9} \Rightarrow$ Radical form $= \sqrt[9]{\left(\frac{6}{17}\right)}$

(iv) $\left(\frac{11}{17}\right)^{1/11} \Rightarrow$ Radical form $= \sqrt[11]{\left(\frac{11}{17}\right)}$

(v) $\left(\frac{61}{325}\right)^{1/17} \Rightarrow$ Radical form $= \sqrt[17]{\left(\frac{61}{325}\right)}$

2. (i) $\sqrt{5} \Rightarrow$ Exponential form $= (5)^{1/2}$

(ii) $\sqrt[3]{7} \Rightarrow$ Exponential form $= (7)^{1/3}$

(iii) $\sqrt[9]{1100} \Rightarrow$ Exponential form $= (1100)^{1/9}$

(iv) $\sqrt[4]{\frac{3}{4}} \Rightarrow$ Exponential form $= \left(\frac{3}{4}\right)^{1/4}$

(v) $\sqrt[8]{\frac{61}{1123}} \Rightarrow$ Exponential form $= \left(\frac{61}{1123}\right)^{1/8}$

3. (i) $(3^{-1} \times 9^{-1}) \div 3^{-2} = \left(\frac{1}{3} \times \frac{1}{9}\right) \times \frac{1}{3^{-2}}$

$$= \frac{1}{3} \times \frac{1}{9} \times 3^2 = \frac{1}{3} \times \frac{1}{9} \times 9 = \frac{1}{3}$$

(ii) $(3^{-1} \times 4^{-1}) \div 6^{-1} = \left(\frac{1}{3} \times \frac{1}{4}\right) \times \frac{1}{6^{-1}}$

$$= \left(\frac{1}{3} \times \frac{1}{4}\right) \times 6 = \frac{1}{3} \times \frac{1}{2} \times \frac{1}{2} \times 6 = \frac{1}{2}$$

(iii) $(2^{-1} + 3^{-1})^3 = \left(\frac{1}{2} + \frac{1}{3}\right)^3 = \left(\frac{3+2}{6}\right)^3 = \left(\frac{5}{6}\right)^3 = \frac{125}{216}$

(iv) $(3^{-1} \div 4^{-1})^2 = \left(\frac{1}{3} \times \frac{1}{4^{-1}}\right)^2 = \left(\frac{1}{3} \times 4\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9} = 1\frac{7}{9}$

4. (i) $\left(\frac{-3}{5}\right)^4 \times \left(\frac{5}{6}\right)^2 = \frac{(-3)^4}{(5)^4} \times \frac{(5)^2}{(3)^2 \times (2)^2}$

$= \frac{(3)^{4-2}}{(5)^{4-2} \times (2)^2} = \frac{3^2}{5^2} \times \frac{1}{2^2}$

$= \frac{9}{25 \times 4} = \frac{9}{100}$

(ii) $\left(\frac{-4}{7}\right)^5 \times \left(\frac{4}{-7}\right)^{-6} \times \left(\frac{4}{-7}\right)^2 = \frac{(-4)^5}{(7)^5} \times \frac{(4)^{-6}}{(-7)^{-6}} \times \frac{(4)^2}{(-7)^2}$

$= \frac{(-4)^5 \times 4^{-6+2}}{(7)^5 \times (-7)^{-6+2}} = \frac{(-4)^5 \times 4^{-4}}{(7)^5 \times (-7)^{-4}}$

$= \frac{(-4)^{5-4}}{(7)^{5-4}} = \left(\frac{-4}{7}\right)$

(iii) $\left(\frac{8}{7}\right)^{-6} \div \left(\frac{8}{7}\right)^5 = \left(\frac{8}{7}\right)^{-6} \times \frac{1}{\left(\frac{8}{7}\right)^5} = \left(\frac{8}{7}\right)^{-6} \times \left(\frac{8}{7}\right)^{-5}$

$= \left(\frac{8}{7}\right)^{-6-5} = \left(\frac{8}{7}\right)^{-11} = \frac{1}{\left(\frac{8}{7}\right)^{11}} = \left(\frac{7}{8}\right)^{11}$

(iv) $\left\{\left(\frac{3}{7}\right)^2\right\}^{-2} \times \left\{\left(\frac{3}{7}\right)^{-3}\right\}^2 = \left(\frac{3}{7}\right)^{2 \times -2} \times \left(\frac{3}{7}\right)^{-3 \times 2}$

$= \left(\frac{3}{7}\right)^{-4} \times \left(\frac{3}{7}\right)^{-6} = \left(\frac{3}{7}\right)^{-4-6} = \left(\frac{3}{7}\right)^{-10} = \frac{1}{\left(\frac{3}{7}\right)^{10}} = \left(\frac{7}{3}\right)^{10}$

5. (i) $(3^{-1} + 4^{-1}) + (4^{-1} + 5^{-1}) = \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{4} + \frac{1}{5}\right)$

$= \left(\frac{4+3}{12}\right) + \left(\frac{5+4}{20}\right) = \frac{7}{12} + \frac{9}{20}$

$= \frac{7 \times 5 + 9 \times 3}{60} = \frac{35+27}{60} = \frac{62}{60} = \frac{31}{30} = 1\frac{1}{30}$

(ii) $(6^{-1} - 7^{-1})^{-1} + (6^{-1} + 7^{-1})^{-1}$

$= \left(\frac{1}{6} - \frac{1}{7}\right)^{-1} + \left(\frac{1}{6} + \frac{1}{7}\right)^{-1} = \left(\frac{7-6}{42}\right)^{-1} + \left(\frac{7+6}{42}\right)^{-1}$

$$= \left(\frac{1}{42}\right)^{-1} + \left(\frac{13}{42}\right)^{-1} = \frac{1}{\left(\frac{1}{42}\right)^{1}} + \frac{1}{\left(\frac{13}{42}\right)^{1}}$$

$$= \frac{42}{1} + \frac{42}{13} = \frac{(42 \times 13 + 42 \times 1)}{13} = \frac{546 + 42}{13} = \frac{588}{13}$$

(iii) $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-3} + \left(\frac{1}{4}\right)^{-4} = \frac{1}{\left(\frac{1}{2}\right)^{2}} + \frac{1}{\left(\frac{1}{3}\right)^{3}} + \frac{1}{\left(\frac{1}{4}\right)^{4}}$

$$= (2)^2 + (3)^3 + (4)^4$$

$$= 4 + 27 + 256 = 287$$

(iv) $\left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{-6} \div \left(\frac{2}{5}\right)^{4} = \left(\frac{2}{5}\right)^{-3} \times \left(\frac{2}{5}\right)^{-6} \times \frac{1}{\left(\frac{2}{5}\right)^{4}}$

$$= \frac{(2)^{-3}}{(5)^{-3}} \times \frac{(2)^{-6}}{(5)^{-6}} \times \frac{(5)^4}{(2)^4}$$

$$= (2)^{-3-6-4} \times (5)^{4+6+3}$$

$$= (2)^{-13} \times (5)^{13} = \frac{(5)^{13}}{(2)^{13}} = \left(\frac{5}{2}\right)^{13}$$

6. (i) $\left(\frac{3}{5}\right)^{2x} \times \left(\frac{3}{5}\right)^{x+8} = \left\{\left(\frac{3}{5}\right)^{2}\right\}^{3} \Rightarrow \left(\frac{3}{5}\right)^{2x+x+8} = \left(\frac{3}{5}\right)^{2\times 3}$

$$\left(\frac{3}{5}\right)^{3x+8} = \left(\frac{3}{5}\right)^{6}$$

Compare both side :

$$3x + 8 = 6 \quad \Rightarrow \quad 3x = 6 - 8$$

$$3x = -2 \quad \Rightarrow \quad x = \frac{-2}{3}$$

(ii) $\frac{2^{2x}}{2^{3x-4}} = 4^{-2}$

$$2^{2x} \times (2)^{-(3x-4)} = (2^2)^{-2}\,(2)^{(2x-(3x-4))} = 2^{2\times -2}$$

$$(2)^{(2x-3x+4)} = 2^{-4}\,(2)^{-x+4} = 2^{-4}$$

Compare both side :

$$-x + 4 = -4 \quad \Rightarrow \quad x = 4 + 4 = 8$$

7. (i) $(64)^{-5/6} = (2^6)^{-5/6} = 2^{\frac{6\times -5}{6}} = 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

(ii) $(3125)^{-2/5} = (5^5)^{-2/5} = (5)^{\frac{5\times -2}{5}}$

$= (5)^{-2} = \frac{1}{(5)^2} = \frac{1}{25}$

(iii) $\left(\frac{1331}{343}\right)^{-2/3} = \left[\left(\frac{11}{7}\right)^3\right]^{-2/3} = \left(\frac{11}{7}\right)^{\frac{3\times -2}{3}}$

$= \left(\frac{11}{7}\right)^{-2} = \frac{1}{\left(\frac{11}{7}\right)^2} = \left(\frac{7}{11}\right)^2 = \frac{49}{121}$

(iv) $\left(\frac{625}{81}\right)^{-1/4} = \left(\frac{5^4}{3^4}\right)^{-1/4} = \left(\frac{5}{3}\right)^{4\times -1/4} = \left(\frac{5}{3}\right)^{-1} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$

8. Let number $= x$

$$\left(\frac{-4}{5}\right)^3 \times x = \left(\frac{-4}{5}\right)^{-4} \quad \Rightarrow \quad x = \frac{\left(\frac{-4}{5}\right)^{-4}}{\left(\frac{-4}{5}\right)^3}$$

$$x = \left(\frac{-4}{5}\right)^{-4} \times \left(\frac{-4}{5}\right)^{-3} = \left(\frac{-4}{5}\right)^{-4-3} = \left(\frac{-4}{5}\right)^{-7}$$

9. Let number $= x$

$$\left\{\left(\frac{2}{9}\right)^{-3}\right\}^2 \div x = \left(\frac{9}{2}\right)^4 \quad \Rightarrow \quad \left(\frac{2}{9}\right)^{-3\times 2} \times \frac{1}{x} = \left(\frac{9}{2}\right)^4$$

$$\frac{(2)^{-6}}{(9)^{-6}} \times \frac{1}{x} = \frac{(9)^4}{(2)^4}$$

$$x = \frac{(2)^{-6} \times (2)^4}{(9)^{-6} \times (9)^4} = \frac{(2)^{-6+4}}{(9)^{-6+4}} = \frac{(2)^{-2}}{(9)^{-2}} = \left(\frac{2}{9}\right)^{-2}$$

10. (i) 6500000000

Standard form $= 65 \times 10^8 = 65 \times 10^8 \times \frac{10}{10} = 6.5 \times 10^9$

(ii) 39120000000000

Standard form $= 3912 \times 10^{10} = \frac{3912 \times 10^{10} \times 10^3}{10^3}$

$= 3.912 \times 10^{13}$

(iii) $0.00000825 = \dfrac{0.00000825 \times 10^8}{10^8} = 825 \times 10^{-8}$

$= 825 \times 10^{-8} \times \dfrac{10^2}{10^2} = 8.25 \times 10^{-6}$

(iv) $0.000000694 = 0.000000694 \times \dfrac{10^9}{10^9}$

$= 694 \times 10^{-9} \times \dfrac{10^2}{10^2}$

$= 6.94 \times 10^{-9+2} = 6.94 \times 10^{-7}$

11. (i) $6.32 \times 10^4 = \dfrac{632 \times 10^4}{10^2} = 632 \times 10^{4-2}$

$= 632 \times 10^2 = 63200$

(ii) $5.96 \times 10^{-4} = \dfrac{596}{10^2} \times 10^{-4} = 596 \times 10^{-4-2}$

$= \dfrac{596}{10^6} = 0.000596$

(iii) $3.94 \times 10^{-8} = \dfrac{394}{10^2} \times 10^{-8} = 394 \times 10^{-8-2}$

$= \dfrac{394}{10^{10}} = 0.0000000394$

(iv) $6.24 \times 10^7 = \dfrac{624}{10^2} \times 10^7 = 624 \times 10^{7-2}$

$= 624 \times 10^5 = 62400000$ ❑

3. Squares and Square Roots

Exercise 3.1

1. Do it yourself.

2. 100

Factorising 100 by division method,

We get, $100 = 2 \times 2 \times 5 \times 5$

Which can be grouped into pairs of equal factors and no factors left.

Therefore, 100 is a perfect square.

2	100
2	50
5	25
5	5
	1

1000

Factorising 1000 by division method.

We get,

$$1000 = \underline{2 \times 2} \times 2 \times \underline{5 \times 5} \times 5$$

Which can be grouped into pairs of equal factors, we find that 2, 5 is left.

Therefore, 1000 is not a perfect square.

2	1000
2	500
2	250
5	125
5	25
5	5
	1

330550

Factorising 330550 by division method.

We get,

$$330550 = 2 \times \underline{5 \times 5} \times 11 \times 601$$

Which can be grouped into equal factor.

We find that 2, 11, 601 is left.

Therefore, 330550 is not a perfect square.

2	330550
5	165275
5	33055
11	6611
601	601
	1

12345600000

Do yourself.

3. Factorising 121 by division method.

We get,

$$121 = \underline{11 \times 11}$$

Which can be grouped into equal factor and no factors left.

Therefore, 121 is a perfect square.

11	121
11	11
	1

4. 121, 225, 256, 1296, 6561

Even number of perfect square is

$$256 = \underline{16 \times 16}$$

$$1296 = \underline{36 \times 36}$$

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

5. (i) 425

Square of 425 $= (425)^2 = 425 \times 425 = 180625$

The unit place of square is 5.

(ii) 637

Square of $637 = (637)^2 = 637 \times 637 = 405769$

The unit place of square = 9.

(iii) 896

Square of $896 = (896)^2 = 896 \times 896 = 802816$

The unit place of square = 6.

(iv) 1741

Square of $1741 = (1741)^2 = 1741 \times 1741 = 3031081$

The unit place of square = 1.

6. (i) 529

Factorising 529 by division method.

We get, $529 = \underline{23 \times 23}$

23	529
23	23
	1

Therefore, 529 is a odd number perfect square.

(ii) 361

Factorising 361 by division method.

We get, $361 = 19 \times 19$

19	361
19	19
	1

Therefore, 361 is a odd number perfect square.

7. (i) $35^2 - 34^2$

Using formula : $(n+1)^2 - n^2 = 2n + 1$

$(34+1)^2 - 34^2 = 2 \times 34 + 1 = 68 + 1 = 69$

(ii) $54^2 - 53^2$

Using formula : $(n+1)^2 - n^2 = 2n + 1$

$(53+1)^2 - 53^2 = 2 \times 53 + 1 = 106 + 1 = 107$

(iii) $68^2 - 67^2$

Using formula : $(n+1)^2 - n^2 = 2n + 1$

$(67+1)^2 - 67^2 = 2 \times 67 + 1 = 134 + 1 = 135$

(iv) $84^2 - 83^2$

Using formula : $(n+1)^2 - n^2 = 2n + 1$

$(83+1)^2 - 83^2 = 2 \times 83 + 1 = 166 + 1 = 167$

(v) $78^2 - 77^2$

Using formula : $(n+1)^2 - n^2 = 2n + 1$

$(77+1)^2 - 77^2 = 2 \times 77 + 1 = 154 + 1 = 155$

(vi) $106^2 - 105^2$

Using formula : $(n+1)^2 - n^2 = 2n + 1$

$(105+1)^2 - 105^2 = 2 \times 105 + 1 = 210 + 1 = 211$

(vii) $208^2 - 207^2$

Using formula : $(n+1)^2 - n^2 = 2n + 1$

$(207+1)^2 - 207^2 = 2 \times 207 + 1 = 414 + 1 = 415$

(viii) $311^2 - 310^2$

Using formula : $(n+1)^2 - n^2 = 2n + 1$

$(310+1)^2 - 310^2 = 2 \times 310 + 1 = 620 + 1 = 621$

8. (i) $1 + 3 + 5 + 7 + 9 + 11 + 13$

Sum of digit without adding = Square of total number

$= (7)^2 = 49$

(ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$

Sum of digit without adding = Square of total number

$= (9)^2 = 81$

(iii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21$

Sum of digit without adding = Square of total number

$= (11)^2 = 121$

(iv) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27$

Sum of digit without adding = Square of total number

$= (14)^2 = 196$

9. Do it yourself.

10. (i) 6 $\Rightarrow$ Pythagorean triplet is $(2m, m^2 - 1, m^2 + 1)$

Let $2m = 6 \quad \Rightarrow \quad m = 3$

So, $m^2 - 1 = (3)^2 - 1 = 9 - 1 = 8$

and $m^2 + 1 = (3)^2 + 1 = 9 + 1 = 10$

Pythagorean triplet is (6, 8, 10).

(ii) 12

Pythagorean triplet is $(2m, m^2 - 1, m^2 + 1)$

Let $2m = 12 \quad \Rightarrow \quad m = 6$

So, $m^2 - 1 = (6)^2 - 1 = 36 - 1 = 35$

and $m^2 + 1 = (6)^2 + 1 = 36 + 1 = 37$

Pythagorean triplet is (12, 35, 37).

(iii) 18

Pythagorean triplet is $(2m, m^2 - 1, m^2 + 1)$

Let $2m = 18 \quad \Rightarrow \quad m = 9$

So, $m^2 - 1 = (9)^2 - 1 = 81 - 1 = 80$

And $m^2 + 1 = (9)^2 + 1 = 81 + 1 = 82$

Pythagorean triplet is (18, 80, 82).

(iv) 30

Pythagorean triplet is $(2m, m^2 - 1, m^2 + 1)$

Let $2m = 30 \Rightarrow m = 15$

So, $m^2 - 1 = (15)^2 - 1 = 225 - 1 = 224$

And $m^2 + 1 = (15)^2 + 1 = 225 + 1 = 226$

Pythagorean triplet is (30, 224, 226).

11. (i) 1296

Factorising 1296 by division method.

We get, $1296 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{3 \times 3}$

Which can be grouped into pairs of equal factors and no factors left.

Therefore, 1296 is a perfect square.

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

(ii) 2401

Factorising 2401 by division method.

We get, $2401 = \underline{7 \times 7} \times \underline{7 \times 7}$

Which can be grouped into pairs of equal factor and no factors left.

Therefore, 2401 is a perfect square.

7	2401
7	343
7	49
7	7
	1

(iii) 9216

Factorising 9216 by division method.

We get, 9216

$= \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$

Which can be grouped pairs of equal factors and no factors left.

Therefore, 9216 is perfect square.

2	9216
2	4608
2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

(iv), (v), (vi), (vii) Do yourself.

(viii) 9801

Factorising 9801 by division method.

We get, $9801 = \underline{3 \times 3} \times \underline{3 \times 3} \times \underline{11 \times 11}$

Which can be grouped pairs of equal factor and no factors left.

Therefore, 9801 is perfect square.

3	9801
3	3267
3	1089
3	363
11	121
11	11
	1

12. (i) (15, 36, 39)

Pythagorean triplets

$\Rightarrow \quad (39)^2 = (36)^2 + (15)^2$

$\Rightarrow \quad 1521 = 1296 + 225 \Rightarrow 1521 = 1521$

It's proof that triplet Pythagorean.

(ii) Do it yourself.

13. Square of all natural number between 80 and 90.

81, 82, 83, 84, 85, 86, 87, 88, 89

$(81)^2 = 6561; \quad (82)^2 = 6724; \quad (83)^2 = 6889;$

$(84)^2 = 7056; \quad (85)^2 = 7225; \quad (86)^2 = 7396;$

$(87)^2 = 7569; \quad (88)^2 = 7744; \quad (89)^2 = 7921;$

Exercise 3.2

1. Do it yourself.

2. (i) 676

Factorising 676 by division method.

We get, $676 = \underline{2 \times 2} \times \underline{13 \times 13}$

$= \overline{2 \times 13} = \overline{26}$

So, square root of 676 is 26.

2	676
2	338
13	169
13	13
	1

(ii) 784

Factorising 784 by division method.

We get, $784 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7}$

$= \overline{2 \times 2} \times \overline{7} = 28$

So, square root of 784 is 28.

Then similarly,

2	784
2	392
2	196
2	98
7	49
7	7
	1

(iii) Square root of 1849 = 43

(iv) Square root of 3025 = 55

(v) Square root of 3364 = 58

(vi) Square root of 4096 = 64

(vii) Square root of 9216 = 96

(viii) Square root of 7396 = 86

Word Problems

1. The smallest square number which is exactly divisible by 10, 12, 18 and 24.

First we will find the LCM of 10, 12, 18, 24.

So, LCM $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

To be a perfect square, it should be having prime factors.

Therefore, multiplication by 10 is necessary.

Thus, $360 \times 10 = 3600$

2	10, 12, 18, 24
2	5, 6, 9, 12
2	5, 3, 9, 6
3	5, 3, 9, 3
3	5, 1, 3, 1
5	5, 1, 1, 1
	1, 1, 1, 1

2. 451634

The least number must be subtracted from 451634 to make it a perfect square.

Hence, 451634 lies between $(672)^2$ and $(673)^2$.

To make it a perfect square the least number that should be subtracted is $2734 - 2684 = 50$.

	672
6	45 16 34
	36
127	916
	889
1342	2734
	2684
	50

3. 131023

Hence, 131023 lies between $(361)^2$ and $(362)^2$.

To make it a perfect square the least number that should be added is

$1444 - 1423 = 21$

	362
3	13 10 23
	9
66	410
	396
722	1423
	−1444
	21

4. 66029

Hence, 66029 lies between $(256)^2$ and $(257)^2$.

To make it a perfect square the least number that should be added is $3549 - 3529 = 20$.

	257
2	6 60 29
	4
45	260
	225
507	3529
7	3549
	20

5. 9408

We get, $9408 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times 3 \times \underline{7 \times 7}$

2	9408
2	4704
2	2352
2	1176
2	588
2	294
3	147
7	49
7	7
	1

Therefore, divided by 3.

Then , number $= \frac{9408}{3} = 3136$

Square root of 3136 = 56

6. A school collected fees = ₹ 2304

Factors of $2304 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3}$

2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Square root of $2304 = 2 \times 2 \times 2 \times 2 \times 3 = 48$

Hence, students in school = 48.

Exercise 3.3

1. (i) $\sqrt{\frac{841}{1521}} = \sqrt{\frac{29 \times 29}{3 \times 3 \times 13 \times 13}} = \sqrt{\frac{29 \times 29}{39 \times 39}} = \frac{29}{39}$

(ii) $\sqrt{8\frac{257}{529}} = \sqrt{\frac{4489}{529}} = \sqrt{\frac{529 \times 8 + 257}{529}} = \sqrt{\frac{67 \times 67}{23 \times 23}} = \frac{67}{23} = 2\frac{21}{23}$

(iii) $\sqrt{16\frac{169}{441}} = \sqrt{\frac{441 \times 16 + 169}{441}} = \sqrt{\frac{7225}{441}} = \frac{85}{21} = 4\frac{1}{21}$

(iv) $\sqrt{\frac{16}{25}} = \sqrt{\frac{4 \times 4}{5 \times 5}} = \frac{4}{5}$

(v) $\sqrt{(0.09)} = \sqrt{\frac{9}{100}} = \sqrt{\frac{3 \times 3}{10 \times 10}} = \frac{3}{10} = 0.3$

(vi) $\sqrt{(0.0004)} = \sqrt{\frac{4}{10000}} = \sqrt{\frac{2 \times 2}{100 \times 100}} = \frac{2}{100} = 0.02$

(vii) $\sqrt{\frac{169}{289}} = \sqrt{\frac{13 \times 13}{17 \times 17}} = \frac{13}{17}$

(viii) $\sqrt{\frac{121}{10000}} = \sqrt{\frac{11 \times 11}{100 \times 100}} = \frac{11}{100}$

(ix) $\sqrt{0.0625} = \sqrt{\frac{625}{10000}} = \sqrt{\frac{25 \times 25}{100 \times 100}} = \frac{25}{100} = 0.25$

(x) $\sqrt{0.0324} = \sqrt{\frac{324}{10000}} = \sqrt{\frac{18 \times 18}{100 \times 100}} = \frac{18}{100} = 0.18$

2. (i) $\sqrt{\frac{625}{1296}} = \sqrt{\frac{25 \times 25}{36 \times 36}} = \frac{25}{36}$

(ii) $\sqrt{4\frac{29}{49}} = \sqrt{\frac{49 \times 4 + 29}{49}} = \sqrt{\frac{225}{49}} = \sqrt{\frac{15 \times 15}{7 \times 7}} = \frac{15}{7} = 2\frac{1}{7}$

(iii) $\sqrt{2\frac{137}{196}} = \sqrt{\frac{529}{196}} = \sqrt{\frac{23 \times 23}{14 \times 14}} = \frac{23}{14} = 1\frac{9}{14}$

(iv) $\sqrt{23\frac{26}{121}} = \sqrt{\frac{121 \times 23 + 26}{121}} = \sqrt{\frac{2809}{121}}$

$= \sqrt{\frac{53 \times 53}{11 \times 11}} = \frac{53}{11} = 4\frac{9}{11}$

(v) $\sqrt{52\frac{857}{2116}} = \sqrt{\frac{2116 \times 52 + 857}{2116}}$

$= \sqrt{\frac{110889}{2116}} = \sqrt{\frac{333 \times 333}{46 \times 46}} = \frac{333}{46} = 7\frac{11}{46}$

(vi) $\sqrt{75\frac{46}{49}} = \sqrt{\frac{49 \times 75 + 46}{49}} = \sqrt{\frac{3721}{49}} = \sqrt{\frac{61 \times 61}{7 \times 7}} = \frac{61}{7} = 8\frac{5}{7}$

(vii) $\sqrt{5.774409} = \sqrt{\frac{5774409}{1000000}} = \sqrt{\frac{2403 \times 2403}{1000 \times 1000}} = \frac{2403}{1000} = 2.403$

(viii) $\sqrt{0.00053361} = \sqrt{\frac{53361}{100000000}} = \sqrt{\frac{231 \times 231}{10000 \times 10000}}$

$= \frac{231}{10000} = 0.0231$

(ix) $\sqrt{804609} = \sqrt{897 \times 897} = 897$

(x) $\sqrt{\frac{1}{4}} = \sqrt{\frac{1}{2 \times 2}} = \frac{1}{2}$

Exercise 3.4

1. $\sqrt{4489} = 67$

	67
6	44 89
	36
127	889
	889
	×

2. $\sqrt{9801} = 99$

	99
9	98 01
	81
189	1701
	1701
	×

3. $\sqrt{44100} = 210$

	210
2	44 100
	4
41	41
	41
	×

4. $\sqrt{54756} = 234$

	234
2	5 47 56
	4
43	147
	129
464	1856
	1856
	×

5. $\sqrt{99856} = 316$

	316
3	9 98 56
	9
61	98
	61
626	3756
	3756
	×

6. $\sqrt{390625} = 625$

	625
6	39 06 25
	36
122	306
	244
1245	6225
	6225
	×

7. $\sqrt{1234321} = 1111$

	1111
1	1 23 43 21
	1
21	23
	21
221	243
	221
2221	2221
	2221
	×

8. $\sqrt{21224449} = 4607$

	4607
4	21 22 44 49
	16
86	522
	516
9207	64449
7	64449
	×

9. $\sqrt{82264900} = 9070$

	9070
9	82 26 49 00
	81
1807	12649
	12649
	×

10. $\sqrt{62504836} = 7906$

	7906
7	62 50 48 36
	49
149	1350
	1341
15806	94836
	94836
	×

11. $\sqrt{3915380329} = 62573$

	62573
6	39 15 38 03 29
	36
122	315
	244
1245	7138
	6225
12507	91303
	87549
125143	375429
	375429
	×

12. $\sqrt{3226694416} = 56804$

	56804
5	3226694416
	25
106	726
	636
1128	9069
	9024
113604	454416
	454416
	×

Word Problems

1. 2361

	48
4	23 61
	16
88	761
	704
	57

Hence, 2361 lies between $(48)^2$ and $(49)^2$.

To make it a perfect square the least number that should be subtracted is $761 - 704 = 57$

2. 4931

	71
7	49 31
	49
141	31
	141
	–110

Hence, 4931 lies between $(70)^2$ and $(71)^2$.

To make it a perfect square the least number that should be added is $141 - 31 = 110$.

3. 18265

	135
1	1 82 65
	1
23	82
	69
265	1365
	1325
	40

Hence, 18265 lies between $(135)^2$ and $(136)^2$.

To make it a perfect square the least number that should be subtracted is $1365 - 1325 = 40$.

4. 4515600

	2125
2	4 51 56 00
	4
41	51
	41
422	1056
	844
4245	21200
	–21225
	–25

Hence, 4515600 lies between $(2124)^2$ and $(2125)^2$.

To make it a perfect square the least number that should be added is $21225 - 21200 = 25$.

5. Least number of four digits = 1000

	31
3	10 00
	9
61	100
	61
	39

	32
3	10 00
	9
62	100
	124
	–24

The difference of 124 – 100 = 24

Hence, the smallest number of four digits which is a perfect square is 1000 + 24 = 1024.

6. The greatest number of six digits is 999999.

	999
9	99 99 99
	81
189	1899
	1701
1989	19899
	17901
	1998

Hence, the greatest number of six digits which is a perfect square is 999999 – 1998 = 998001.

Exercise 3.5

1. (i) $\sqrt{\frac{361}{625}} = \sqrt{\frac{19 \times 19}{25 \times 25}} = \frac{19}{25}$

(ii) $\sqrt{5\frac{19}{25}} = \sqrt{\frac{25 \times 5 + 19}{25}} = \sqrt{\frac{144}{25}} = \sqrt{\frac{12 \times 12}{5 \times 5}} = \frac{12}{5} = 2\frac{2}{5}$

(iii) $\sqrt{34\frac{15}{49}} = \sqrt{\frac{49 \times 34 + 15}{49}} = \sqrt{\frac{1681}{49}} = \sqrt{\frac{41 \times 41}{7 \times 7}} = \frac{41}{7} = 5\frac{6}{7}$

(iv) $\sqrt{84\frac{37}{121}} = \sqrt{\frac{84 \times 121 + 37}{121}} = \sqrt{\frac{10201}{121}}$

$= \sqrt{\frac{101 \times 101}{11 \times 11}} = \frac{101}{11} = 9\frac{2}{11}$

(v) $\sqrt{23\frac{394}{729}} = \sqrt{\frac{729 \times 23 + 394}{729}} = \sqrt{\frac{17161}{729}}$

$$= \sqrt{\frac{131 \times 131}{27 \times 27}} = \frac{131}{27} = 4\frac{23}{27}$$

(vi) $\sqrt{21\frac{51}{169}} = \sqrt{\frac{169 \times 21 + 51}{169}} = \sqrt{\frac{3600}{169}}$

$$= \sqrt{\frac{60 \times 60}{13 \times 13}} = \frac{60}{13} = 4\frac{8}{13}$$

(vii) $\sqrt{10\frac{151}{225}} = \sqrt{\frac{225 \times 10 + 151}{225}} = \sqrt{\frac{2401}{225}}$

$$= \sqrt{\frac{49 \times 49}{15 \times 15}} = \frac{49}{15} = 3\frac{4}{15}$$

(viii) $\sqrt{332\frac{61}{169}} = \sqrt{\frac{169 \times 332 + 61}{169}} = \sqrt{\frac{56169}{169}}$

$$= \sqrt{\frac{237 \times 237}{13 \times 13}} = \frac{237}{13} = 18\frac{3}{13}$$

2. (i) $\sqrt{27.3529} = 5.23$

	5.23
5	27 . 35 29
	25
102	235
	204
1043	3129
	3129
	×

(ii) $\sqrt{40.5769} = 6.37$

	6.37
6	40 . 57 69
	36
123	457
3	369
1267	8869
	8869
	×

(iii) $\sqrt{0.00038809} = \sqrt{\frac{38809}{100000000}}$

$$= \sqrt{\frac{197 \times 197}{10000 \times 10000}} = \frac{197}{10000} = 0.0197$$

	197
1	3 88 09
	1
29	288
	261
387	2709
	2709
	×

(iv) $\sqrt{3873.8176} = 62.24$

	62.24
6	38 73 . 81 76
	36
122	273
	244
1242	2981
	2484
12444	49776
	49776
	×

(v) $\sqrt{0.00059049} = \sqrt{\frac{59049}{100000000}} = \frac{243}{10000} = 0.0243$

	243
2	5 90 49
	4
44	190
	176
483	1449
	1449
	×

(vi) $\sqrt{170.0416} = 13.04$

	13.04
1	170 . 04 16
	1
23	70
	69
2604	10416
	10416
	×

(vii) $\sqrt{7286.3296} = 85.36$

	85.36
8	72 86 . 32 96
	64
165	886
	825
1703	6132
	5109
17066	102396
	102396
	×

(viii) $\sqrt{4003.0929} = 63.27$

	63.27
6	40 03 . 09 29
	36
123	403
	369
1262	3409
	2524
12647	88529
	88529
	×

3. (i) $\dfrac{\sqrt{462.25} - \sqrt{33.64}}{\sqrt{462.25} + \sqrt{33.64}} = \dfrac{\sqrt{21.5 \times 21.5} - \sqrt{5.8 \times 5.8}}{\sqrt{21.5 \times 21.5} + \sqrt{5.8 \times 5.8}}$

$= \dfrac{21.5 - 5.8}{21.5 + 5.8} = \dfrac{15.7}{27.3} = \dfrac{157}{273}$

(ii) $\dfrac{\sqrt{1281.64} + \sqrt{166.41}}{\sqrt{1281.64} - \sqrt{166.41}} = \dfrac{\sqrt{35.8 \times 35.8} + \sqrt{12.9 \times 12.9}}{\sqrt{35.8 \times 35.8} - \sqrt{12.9 \times 12.9}}$

$= \dfrac{35.8 + 12.9}{35.8 - 12.9} = \dfrac{48.7}{22.9} = \dfrac{487}{229}$

Word Problems

1. A decimal fraction $= x$

Let multiplied by the decimal fraction $= x$

Product $= 251953.8025$

$x \times x = 251953.8025$

$x^2 = 251953.8025$

$x = \sqrt{251953.8025}$

$= \sqrt{501.95 \times 501.95} = 501.95$

Therefore, fraction value is 501.95.

2. Let fraction $= x$

Let multiplied by the fraction $= x$

Product $= 227.798649$

$x \times x = 227.798649$

$x^2 = 227.798649$

$x = \sqrt{227.798649}$

$= \sqrt{15.093 \times 15.093} = 15.093$

Therefore, fraction is 15.093.

3. A decimal fraction $= x$

Let multiplied by the decimal fraction $= x$

$$\text{Product} = 0.00431649$$
$$x \times x = 0.00431649$$
$$x^2 = 0.00431649$$
$$x = \sqrt{0.00431649} = \sqrt{\frac{431649}{100000000}}$$
$$= \sqrt{\frac{657 \times 657}{10000 \times 10000}} = \frac{657}{10000} = 0.0657$$

4. The decimal fraction $= x$

Let multiplied by the decimal fraction $= x$

$$\text{Product} = 0.00279841$$
$$x \times x = 0.00279841$$
$$x^2 = 0.00279841$$
$$x = \sqrt{0.00279841} = \sqrt{\frac{279841}{100000000}}$$
$$= \sqrt{\frac{529 \times 529}{10000 \times 10000}} = \frac{529}{10000} = 0.0529$$

Revision Exercise

1. Do it yourself.

2. Do it yourself.

3. (i) 81

Square root of $81 = \sqrt{81} = 9$

Number of digits in square root $= 1$

(ii) 169

Square root of $169 = \sqrt{169} = 13$

Number of digits in square root $= 2$

(iii) 4761

Square root of $4761 = \sqrt{4761} = 69$

Number of digits in square root $= 2$

(iv) 27889

Square root of $27889 = \sqrt{27889} = 167$

Number of digits in square root $= 3$

(v) 525625

Square root of $525625 = \sqrt{525625} = 725$

Number of digits in square root $= 3$

4. (i) 2304

Ones place of digit of square root = $\sqrt{04} = 2$

Square root of 2304 = $\sqrt{2304} = 48$

Ones place digit of 48 = 8

Therefore, answer = (2, 8)

(ii) 1225

Ones place of digit of square root = $\sqrt{25} = 5$

Square root of 1225 = $\sqrt{1225} = 35$

Ones place digit of 35 = 5

Therefore, answer = (5, 5)

(iii) 8649

Ones place of digit of square root = $\sqrt{49} = 7$

Square root of 8649 = $\sqrt{8649} = 93$

Ones place of 93 = 3

Therefore, answer = (3, 7)

(iv) 9801

Ones place of digit of sqaure root = $\sqrt{01} = 1$

Square root of 9801 = $\sqrt{9801} = 99$

Ones place of 99 = 9

Therefore, answer = (1, 9)

5. (i) 2401

	49
4	2401
	16
89	801
	801
	×

Square root of 2401 = 49

(ii) 4489

	67
6	4489
	36
127	889
	889
	×

Square root of 4489 = 67

(iii) 106929

	327
3	106929
	9
62	169
	124
647	4529
	4529
	×

Square root of 106929 = 327

(iv) 167281

	409
4	167281
	16
809	7281
	7281
	×

Square root of 167281 = 409

(v) 53824

	232
2	5 38 24
	4
43	138
	129
462	924
	924
	×

Square root of 53824 = 232

(vi) 213444

	462
4	213444
	16
86	534
	516
922	1844
	1844
	×

Square root of 213444 = 462

6. (i) 51.84

	7.2
7	51. 84
	49
142	284
	284
	×

Square root of 51.84 = 7.2

(ii) 42.25

	6.5
6	42. 25
	36
125	625
	625
	×

Square root of 42.25 = 6.5

(iii) 18.4041

	4.29
4	18. 4041
	16
82	240
	164
849	7641
	7641
	×

Square root of 18.4041 = 4.29

(iv) 5.774409

	2.403
2	5. 77 4409
	4
44	177
	176
4803	14409
	14409
	×

Square root of 5.774409 = 2.403

7. (i) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21

Total number = 11

Sum of the number = Square of total number

Sum of the number = $(11)^2 = 121$

(ii) 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25

Total number = 13

Sum of the number = Square of total number

Sum of the number = $(13)^2 = 169$

8. 46080

Factorising 46080 by division method.

We get, $46080 = \underline{2\times 2} \times \underline{2\times 2} \times \underline{2\times 2}$
$\times \underline{2\times 2} \times \underline{2\times 2} \times \underline{3\times 3} \times 5$

Which can be grouped into pairs of equal factor and 5 is left.

Therefore, $= 46080 \div 5 = 9216$

Square root of $9216 = 96$

2	46080
2	23040
2	11520
2	5760
2	2880
2	1440
2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

9. 6328

Hence, 6328 lies between $(79)^2$ and $(80)^2$.

To make it a perfect square the least number that should be added is

$6400 - 6328 = 72$

	79
7	6328
	49
149	1428
	1341
	87

10. Cost of grassing a square park = ₹ 89,520.64

At rate of ₹ 2.56 m^2

$$\text{Area of square field} = \frac{\text{Total cost}}{\text{Per } m^2 \text{ cost}} = \frac{89{,}520.64}{2.56} = 34969 \text{ m}^2$$

Since Area of square field $= \text{side}^2$

$\text{Side}^2 = 34969$

$\text{Side} = \sqrt{34969} = 187$ m

Perimeter of square field $= 4 \times \text{side} = 4 \times 187 = 748$ m

11. Money collected by the students of class VIII for picnic = ₹ 2304

Let number of students be $= x$

Each student collected rupees $= x$

Then, $x \times x = ₹\ 2304$

$x^2 = 2304$

$x = \sqrt{2304} = 48$

Hence, Number of students in class = 48

❑

4. Cubes and Cube Roots

Exercise 4.1

1. (i) $7 \Rightarrow$ Cube of $7 = 7 \times 7 \times 7 = 49 \times 7 = 343$

(ii) $12 \Rightarrow$ Cube of $12 = 12 \times 12 \times 12 = 144 \times 12 = 1728$

(iii) $21 \Rightarrow$ Cube of $21 = 21 \times 21 \times 21 = 441 \times 21 = 9261$

(iv) $100 \Rightarrow$ Cube of $100 = 100 \times 100 \times 100$

$= 10000 \times 100 = 10{,}00{,}000$

(v) $302 \Rightarrow$ Cube of $302 = 302 \times 302 \times 302$

$= 91204 \times 302 = 27543608$

(vi) $15 \Rightarrow$ Cube of $15 = 15 \times 15 \times 15 = 225 \times 15 = 3375$

(vii) $-18 \Rightarrow$ Cube of $-18 = -18 \times -18 \times -18$

$= +324 \times -18 = -5832$

(viii) $\frac{3}{11} \Rightarrow$ Cube of $\frac{3}{11} = \frac{3}{11} \times \frac{3}{11} \times \frac{3}{11} = \frac{9}{121} \times \frac{3}{11} = \frac{27}{1331}$

(ix) $\frac{-8}{17} \Rightarrow$ Cube of $\frac{-8}{17} = \frac{-8}{17} \times \frac{-8}{17} \times \frac{-8}{17}$

$= \frac{64}{289} \times \frac{-8}{17} = \frac{-512}{4913}$

2. (i) 64

Factorising 64 by division method.

$64 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

Grouping the factors in triplets of equal factors.

Therefore, 64 is a perfect cube.

2	64
2	32
2	16
2	8
2	4
2	2
	1

(ii) 216

Factorising 216 by division method.

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Grouping the factors in triplets of equal factors.

Therefore, 216 is a perfect cube.

2	216
2	108
2	54
3	27
3	9
3	3
	1

(iii) 3375

Factorising 3375 by division method

$3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5$

Grouping the factors in triplets of equal factors.

Therefore, 3375 is a perfect cube.

3	3375
3	1125
5	375
5	75
5	15
3	3
	1

(iv) 3675

Factorising 3675 by division method.

$3675 = 3 \times 5 \times 5 \times 7 \times 7$

Grouping pair are not in triplet form.

Therefore, 3675 is not perfect cube.

3	3675
5	1225
5	245
7	49
7	7
	1

(v) 42875

Factorising 42875 by division method.

$42875 = 5 \times 5 \times 5 \times 7 \times 7 \times 7$

Grouping the factors in triplets of equal factors.

Therefore, 42875 is a perfect cube.

5	42875
5	8575
5	1715
7	343
7	49
7	7
	1

(vi) 4096

Factorising 4096 by division method

$4096 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Grouping the factors in triplets of equal factors.

Therefore, 4096 is a perfect cube.

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

(vii) Do it yourself.

(viii) 1728

Factorising 1728 by division method.

$1728 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

Grouping the factors in triplets of equal factors.

Therefore, 1728 is a perfect cube.

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

(ix) Do it yourself.

Word Problems

1. Do it yourself.

2. Do it yourself.

3. Do it yourself.

4. Do it yourself.

5. 5184

Factorising 5184 by division method.

$5184 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times 3$

Grouping the factors in equal pairs and 3 is left.

Then, 9 is multiple. Hence, answer is 9.

$5184 \times 9 = 46656$

Cube root of $46656 = 36$

2	5184
2	2592
2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

6. 197568

Factorising 197568 by division method.

$197568 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 3 \times 3 \times \underline{7 \times 7 \times 7}$

Grouping factors is equal pairs and 9 is left.

Therefore, 9 is divided to get perfect cube.

2	197568
2	98784
2	49392
2	24696
2	12348
2	6174
3	3087
3	1029
7	343
7	49
7	7
	1

7. 392

Factorising 392 by division method.

$392 = \underline{2 \times 2 \times 2} \times 7 \times 7$

Grouping factors is equal pairs and 49 is left.

Therefore, 7 is multiplied to get perfect cube.

2	392
2	196
2	98
7	49
7	7
	1

8. Given : One side of cube $= 13$ m

Volume of cube $= (\text{Side})^3 = (13)^3$

$= 13 \times 13 \times 13 = 169 \times 13$

$= 2197\ \text{m}^3$

9. 8640

Factorising 8640 by division method.

$8640 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times 5$

Grouping factors is equal pairs and 5 is left.

Therefore, 5 is divided to get perfect cube.

2	8640
2	4320
2	2160
2	1080
2	540
2	270
3	135
3	45
3	15
5	5
	1

Exercise 4.2

1. We have, $1000 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

Cube root of $1000 = 2 \times 5 = 10$

2	1000
2	500
2	250
5	125
5	25
5	5
	1

2. We have, $2744 = \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$

Cube root of $2744 = 2 \times 7 = 14$

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

3. We have, $32768 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

Cube root of $32768 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

2	32768
2	16384
2	8192
2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

4. 13824

$= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

Cube root of $13824 = 2 \times 2 \times 2 \times 3 = 24$

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

5. $54872 = \underline{2 \times 2 \times 2} \times \underline{19 \times 19 \times 19}$

Cube root of $54872 = 2 \times 19 = 38$

2	54872
2	27436
2	13718
19	6859
19	361
19	19
	1
	1

6. $74088 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$

Cube root of $74088 = 2 \times 3 \times 7 = 42$

2	74088
2	37044
2	18522
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

7. $125000 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$

Cube root of $125000 = 2 \times 5 \times 5 = 10 \times 5 = 50$

2	125000
2	62500
2	31250
5	15625
5	3125
5	625
5	125
5	25
5	5
	1

8. 884736

$= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$\times \underline{12 \times 12 \times 12}$

Cube root of $884736 = 2 \times 2 \times 2 \times 12 = 96$

2	884736
2	442368
2	221184
2	110592
2	55296
2	27648
2	13824
2	6912
2	3456
12	1728
12	144
12	12
	1

9. 48228544

$= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7} \times \underline{13 \times 13 \times 13}$

Cube root of 48228544 $= 2 \times 2 \times 7 \times 13 = 364$

2	48228544
2	24114272
2	12057136
2	6028568
2	3014284
2	1507142
7	753571
7	107653
7	15379
13	2197
13	169
13	13
	1

10. 74088000

$= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} \times \underline{7 \times 7 \times 7}$

Cube root of 74088000 $= 2 \times 2 \times 3 \times 5 \times 7 = 420$

2	74088000
2	37044000
2	18522000
2	9261000
2	4630500
2	2315250
3	1157625
3	385875
3	128625
5	42875
5	8575
5	1715
7	343
7	49
7	7
	1

Word Problems

1. 137592

Factorising 137592 by division method.

$$137592 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times 7 \times 7 \times 13$$

Which have factors in equal pairs and 49×13 is left.

Hence, multiplied $= 7 \times 13 \times 13 = 1183$

Perfect cube $= \sqrt[3]{137592 \times 1183} = 546$

2	137592
2	68796
2	34398
7	17199
7	2457
13	351
3	27
3	9
3	3
	1

2. 26244

Factorising 26244 by division method.

$$26244 = 2 \times 2 \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times 3 \times 3$$

Grouping the factors in equal pairs and 4×9 is not a pair.

Hence, divide by 36.

Therefore, perfect cube $= \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

Perfect cube root $= 3 \times 3 = 9$

There will be answer is (36, 9)

2	26244
2	13122
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

3. Given : Volume of cube $= 512\ m^3$

Length of side of cube $= \sqrt[3]{512} = \sqrt[3]{8 \times 8 \times 8} = 8$ m

Length of side of cube $= 8$ m

4. Given : Volume of cube $= 10648\ cm^3$

We know that,

Length of edge $= \sqrt[3]{\text{Volume of cube}} = \sqrt[3]{10648} = \sqrt[3]{22 \times 22 \times 22}$

Length of edge $= 22$ cm.

5. Cube root of 216 $= \sqrt[3]{216}$

$= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}$

$= 2 \times 3 = 6$

2	216
2	108
2	54
3	27
3	9
3	3
	1

Exercise 4.3

1. (i) -125

Factorising -125 by division method.

$$-125 = \underline{-5 \times -5 \times -5}$$

Cube root of $-125 = -5$

5	125
5	25
5	5
	1

(ii) -39304

Factorising -39304 by division method

$$-39304 = -\underline{2 \times 2 \times 2} \times \underline{17 \times 17 \times 17}$$

Cube root of $-39304 = -2 \times 17$

$= -34$

2	39304
2	19652
2	9826
17	4913
17	289
17	17
	1

(iii) -46656

Factorising -46656 by division method.

$$-46656 = -\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$$

Cube root of $-46656 = -2 \times 2 \times 3 \times 3 = -36$

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

(iv) -5832

Factorising -5832 by division method

$$-5832 = -(\underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3})$$

Cube root of $-5832 = -(2 \times 3 \times 3) = -18$

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

(v) -17576

Factorising -17576 by division method.

$-17576 = -(\underline{2 \times 2 \times 2} \times \underline{13 \times 13 \times 13})$

Cube root of $-17576 = -2 \times 13 = -26$

2	17576
2	8788
2	4394
13	2197
13	169
13	13
	1

(vi) -2744000

Factorising -2744000 by division method.

$-2744000 = -[\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5} \times \underline{7 \times 7 \times 7}]$

Cube root of $-2744000 = -2 \times 2 \times 5 \times 7 = -140$

2	2744000
2	1372000
2	686000
2	343000
2	171500
2	85750
5	42875
5	8575
5	1715
7	343
7	49
7	7
	1

2. Do it yourself.

3. (i) $(8) \times (64)$

2	8
2	4
2	2
	1

2	64
2	32
2	16
2	8
2	4
2	2
	1

$(8) \times (64) = (\underline{2 \times 2 \times 2}) \times (\underline{2 \times 2 \times 2 \times 2 \times 2 \times 2})$

Cube root of $8 \times 64 = (2) \times (2 \times 2) = 8$

(ii) -216×1728

2	216
2	108
2	54
3	27
3	9
3	3
	1

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$(-216) \times (1728) = -(\underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}) \times (\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3})$

Cube root of $-216 \times 1728 = -(2 \times 3) \times (2 \times 2 \times 3)$

$= -6 \times 12 = -72$

(iii) $27 \times (-2744)$

3	27
3	9
3	3
	1

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

$27 \times (-2744) = (3 \times 3 \times 3) \times -(\underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7})$

Cube root of $27 \times (-2744) = -3 \times (2 \times 7)$

$= -3 \times 14 = -42$

(iv) $(-125) \times (-3375)$

5	125
5	25
5	5
	1

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

$(-125) \times (-3375) = -(5 \times 5 \times 5) \times -(3 \times 3 \times 3 \times 5 \times 5 \times 5)$

Cube root of $-(125) \times (-3375) = -5 \times -(3 \times 5)$

$= 15 \times 5 = 75$

(v) $(729) \times (15625)$

3	729
3	243
3	81
3	27
3	9
3	3
	1

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

$(729) \times (15625) = (\underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}) \times (\underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5})$

Cube root of $(729) \times (15625) = 3 \times 3 \times 5 \times 5$

$= 9 \times 5 \times 5 = 225$

(vi) $-456533 = -(\underline{11 \times 11 \times 11} \times \underline{7 \times 7 \times 7})$

Cube root of $-456533 = -11 \times 7 = -77$

11	456533
11	41503
11	3773
7	343
7	49
7	7
	1

(vii) $-474552 = -\underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{13 \times 13 \times 13}$

Cube root of $-474552 = -2 \times 3 \times 13$

$= -6 \times 13$

$= -78$

2	474552
2	237276
2	118638
3	59319
3	19773
3	6591
13	2197
13	169
13	13
	1

(viii) $-5832000 = -\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

Cube root of -5832000,

$= -2 \times 2 \times 3 \times 3 \times 5$

$= -4 \times 9 \times 5 = -180$

2	5832000
2	2916000
2	1458000
2	729000
2	364500
2	182250
3	91125
3	30375
3	10125
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

4. (i) $\sqrt[3]{81} \times \sqrt[3]{72000}$

3	81
3	27
3	9
3	3
	1

2	72000
2	36000
2	18000
2	9000
2	4500
2	2250
3	1125
3	375
5	125
5	25
5	5
	1

$= \sqrt[3]{3 \times 3 \times 3 \times 3} \times \sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 3 \times 3 \times \underline{5 \times 5 \times 5}}$

$= 3 \times \sqrt[3]{3} \times 2 \times 2 \times 5 \times \sqrt[3]{3} \times \times \sqrt[3]{3}$

$= 3 \times 2 \times 2 \times 5 \times 3 = 180$

(ii) $\sqrt[3]{16} \times \sqrt[3]{4000}$

2	16
2	8
2	4
2	2
	1

2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

$= \sqrt[3]{2 \times 2 \times 2 \times 2} \times \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5}$

$= 2 \times \sqrt[3]{2} \times 5 \times 2 \times \sqrt[3]{2} \times \sqrt[3]{2}$

$= 2 \times 2 \times 5 \times 2 = 40$

(iii) $\sqrt[3]{968} \times \sqrt[3]{1375} = \sqrt[3]{2 \times 2 \times 2 \times 11 \times 11} \times \sqrt{5 \times 5 \times 5 \times 11}$

2	968
2	484
2	242
11	121
11	11
	1

5	1375
5	275
5	55
11	11
	1

$= 2 \times 5 \times \sqrt[3]{11} \times \sqrt[3]{11} \times \sqrt[3]{11}$

$= 2 \times 5 \times 11 = 110$

(iv) $\sqrt[3]{729} \times \sqrt[3]{1331} = \sqrt[3]{9 \times 9 \times 9} \times \sqrt[3]{11 \times 11 \times 11}$

$= 9 \times 11 = 99$

9	729
9	81
9	9
	1

11	1331
11	121
11	11
	1

5. (i) $\frac{-2197}{-9261} = \frac{2197}{9261} = \frac{13 \times 13 \times 13}{3 \times 3 \times 3 \times 7 \times 7 \times 7}$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

13	2197
13	169
13	13
	1

Cube root of $\frac{2197}{9261} = \frac{13}{3 \times 7} = \frac{13}{21}$

(ii) $\frac{729}{2197} = \frac{9 \times 9 \times 9}{13 \times 13 \times 13}$

13	2197
13	169
13	13
	1

9	729
9	81
9	9
	1

Cube root of $\frac{729}{2197} = \frac{9}{13}$

(iii) $\frac{512}{8000} = \frac{8 \times 8 \times 8}{20 \times 20 \times 20}$

20	8000
20	400
20	20
	1

8	512
8	64
8	8
	1

Cube root of $\frac{512}{8000} = \frac{8}{20}$

(iv) $\frac{1728}{9261} = \frac{12 \times 12 \times 12}{21 \times 21 \times 21}$

21	9261
21	441
21	21
	1

12	1728
12	144
12	12
	1

Cube root of $\frac{1728}{9261} = \frac{12}{21}$

6. (i) $32.768 = \frac{32768}{1000} = \frac{32 \times 32 \times 32}{10 \times 10 \times 10}$

10	1000
10	100
10	10
	1

32	32768
32	1024
32	32
	1

Cube root of $32.768 = \frac{32}{10} = 3.2$

(ii) $12.167 = \frac{12167}{1000} = \frac{23 \times 23 \times 23}{10 \times 10 \times 10}$

10	1000
10	100
10	10
	1

23	12167
23	529
23	23
	1

Cube root of $12.167 = \frac{23}{10} = 2.3$

(iii) $10.648 = \frac{10648}{1000} = \frac{22 \times 22 \times 22}{10 \times 10 \times 10}$

10	1000
10	100
10	10
	1

22	10648
22	484
22	22
	1

Cube root of $10.648 = \frac{22}{10} = 2.2$

(iv) $46.656 = \frac{46656}{1000} = \frac{36 \times 36 \times 36}{10 \times 10 \times 10}$

10	1000
10	100
10	10
	1

36	46656
36	1296
36	36
	1

Cube root of $46.656 = \frac{36}{10} = 3.6$

7. Volume of cubical box= $32.768\ m^3$

We know that,

Volume of cubical box = (length of side of box)^3

Let length of side = x

$$x^3 = 32.768$$

$$x = \sqrt[3]{32.768} = 3.2$$

Length of side $= 3.2$ m

Revision Exercise

1. (i) $5 \Rightarrow$ Cube of $5 = (5)^3 = 5 \times 5 \times 5 = 125$

(ii) 11

Cube of $11 = (11)^3 = 11 \times 11 \times 11 = 121 \times 11 = 1331$

(iii) 16

Cube of $16 = (16)^3 = 16 \times 16 \times 16 = 256 \times 16 = 4096$

(iv) 23

Cube of $23 = (23)^3 = 23 \times 23 \times 23 = 529 \times 23 = 12167$

(v) 31

Cube of $31 = (31)^3 = 31 \times 31 \times 31 = 961 \times 31 = 29791$

(vi) 40

Cube of $40 = (40)^3 = 40 \times 40 \times 40 = 1600 \times 40 = 64000$

2. Do it yourself.

3. (i) $243 \Rightarrow$ Factorising 243 by division method.

3	243
3	81
3	27
3	9
3	3
	1

$$243 = \underline{3 \times 3 \times 3} \times 3 \times 3$$

Complete pair to multiply by 3,

We get, $= \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$

$$= 729 = (9)^3$$

Therefore 3 is multiplied.

(ii) $3072 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2 \times 3$

2	3072
2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

Complete pair to multiply by $2 \times 2 \times 3 \times 3$

We get $3072 \times 4 \times 9 = (110592)$

$$= (48)^3$$

Therefore 36 is multiplied.

(iii) $11979 = 3 \times 3 \times \underline{11 \times 11 \times 11}$

Complete pair to multiply by 3.

We get, $35937 = (33)^3$

Therefore, 3 is multiplied.

3	11979
3	3993
11	1331
11	121
11	11
	1

(iv) $19652 = 2 \times 2 \times 17 \times 17 \times 17$

Complete pair to multiply by 2,

$= 19652 \times 2$

$= 39304 = (34)^3$

Therefore, 2 is multiplied.

2	19652
2	9826
17	4913
17	289
17	17
	1

4. (i) $1536 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 3$

Complete pair to divide by 3,

$= \dfrac{1536}{3}$

$= 512$

$= (8)^3$

Therefore, 3 is divided.

2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

(ii) $10985 = 5 \times \underline{13 \times 13 \times 13}$

Complete pair to divide by 5,

$= \dfrac{10985}{5} = 2197 = (13)^3$

Therefore, 5 is divided.

5	10985
13	2197
13	169
13	13
	1

(iii) $28672 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 7$

Complete pair to divide by 7,

$$= \frac{28672}{7} = 4096$$

Therefore, 7 is divided.

2	28672
2	14336
2	7168
2	3584
2	1792
2	896
2	448
2	224
2	112
2	56
2	28
2	14
7	7
	1

(iv) $13718 = 2 \times 19 \times 19 \times 19$

Complete pair to divide by 2,

$$= \frac{13718}{2} = 6859 = (19)^3$$

Therefore, 2 is divided.

2	13718
19	6859
19	361
19	19
	1

5. Assume, we need x cuboids.

Volume of cuboid $= 3 \times 3 \times 5 = 45$

Volume of x cuboid $= 45 \times x = 45x$

$45x$ is a perfect cube.

Therefore, we know that $45x$ is multiply of 5 and 3.

Hence, $45x = (15)^3 = 3375$

$45x = 3375 \Rightarrow x = 75$ (cuboids)

6. Given : Surface area of cubical box $= 486\ \text{cm}^2$

$6a^2 = 486$; $a^2 = 81$ and $a = 9\ \text{cm}$

Volume of cubical box $= a^3 = (9)^3 = 9 \times 9 \times 9$

$= 81 \times 9 = 729\ \text{cm}^3$

7. (i) $27 \Rightarrow$ Cube of the number $= 27$

$a^3 = 27 \Rightarrow a = \sqrt[3]{27} = 3$

3 is odd natural number.

(ii) $216 \Rightarrow$ Cube of the number $= 216$

$a^3 = 216 \Rightarrow a = \sqrt[3]{216} = 6$

6 is even natural number.

(iii) 1000 $\Rightarrow$ Cube of the number = 1000

$$a^3 = 1000 \Rightarrow a = \sqrt[3]{1000} = 10$$

10 is even natural number.

(iv) 2197 $\Rightarrow$ Cube of the number = 2197

$$a = \sqrt[3]{2197} = 13$$

13 is odd natural number.

(v) 4096 $\Rightarrow$ Cube of the number = 4096

$$a^3 = 4096 \Rightarrow a = \sqrt[3]{4096} = 16$$

16 is even natural number.

(vi) 6859 $\Rightarrow$ Cube of the number = 6859

$$a^3 = 6859 \Rightarrow a = \sqrt[3]{6859} = 19$$

19 is odd natural number.

8. (i) 512

Cube root of 512 $= \sqrt[3]{512}$

$= \sqrt[3]{8 \times 8 \times 8} = 8.$

8	512
8	64
8	8
	1

(ii) 729

Cube root of 729 $= \sqrt[3]{729}$

$= \sqrt[3]{9 \times 9 \times 9} = 9.$

9	729
9	81
9	9
	1

(iii) 1728

Cube root of 1728 $= \sqrt[3]{1728}$

$= \sqrt[3]{12 \times 12 \times 12} = 12.$

12	1728
12	144
12	12
	1

(iv) 4913

Cube root of 4913 $= \sqrt[3]{4913}$

$= \sqrt[3]{17 \times 17 \times 17} = 17.$

17	4913
17	289
17	17
	1

(v) 4096

Cube root of 4096 $= \sqrt[3]{4096}$

$= \sqrt[3]{16 \times 16 \times 16} = 16.$

16	4096
16	256
16	16
	1

(vi) 8000

Cube root of $8000 = \sqrt[3]{8000}$

$= \sqrt[3]{20 \times 20 \times 20} = 20.$

20	8000
20	400
20	20
	1

9. (i) $\frac{-64}{125}$

4	64
4	16
4	4
	1

5	125
5	25
5	5
	1

Cube root of $\frac{-64}{125} = -\sqrt[3]{\frac{64}{125}} = -\sqrt[3]{\frac{4 \times 4 \times 4}{5 \times 5 \times 5}} = \frac{-4}{5}$

(ii) $\frac{-27}{343}$

3	27
3	9
3	3
	1

7	343
7	49
7	7
	1

Cube root of $\frac{-27}{343} = -\sqrt[3]{\frac{27}{343}} = -\sqrt[3]{\frac{3 \times 3 \times 3}{7 \times 7 \times 7}} = \frac{-3}{7}$

(iii) $\frac{-512}{343}$

8	512
8	64
8	8
	1

7	343
7	49
7	7
	1

Cube root of $\frac{-512}{343} = -\sqrt[3]{\frac{8 \times 8 \times 8}{7 \times 7 \times 7}} = \frac{-8}{7}$

(iv) -2197

Cube root of $-2197 = -\sqrt[3]{2197}$

$= -\sqrt[3]{13 \times 13 \times 13}$

$= -13$

13	2197
13	169
13	13
	1

(v) -5822

Cube root of $-5832 = -\sqrt[3]{5832}$

$= -\sqrt[3]{18 \times 18 \times 18}$

$= -18$

18	5832
18	324
18	18
	1

(vi) -2744000

Cube root of -2744000

$= -\sqrt[3]{2744000}$

$= -\sqrt[3]{14 \times 14 \times 14 \times 10 \times 10 \times 10}$

$= -14 \times 10 = -140$

14	2744
14	196
14	14
	1

10. (i) -216×1728

Cube root of -216×1728

$= -\sqrt[3]{216 \times 1728}$

$= -\sqrt[3]{6 \times 6 \times 6 \times 12 \times 12 \times 12}$

$= -6 \times 12 = -72$

(ii) -27×-125

Cube root of $27 \times 125 = \sqrt[3]{27 \times 125}$

$= \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5}$

$= 3 \times 5 = 15$

(iii) $\frac{-27}{64} \Rightarrow$ Cube root of $\frac{-27}{64} = -\sqrt[3]{\frac{27}{64}} = -\sqrt[3]{\frac{3 \times 3 \times 3}{4 \times 4 \times 4}} = \frac{-3}{4}$

(iv) $\frac{729}{-1331}$

Cube root of $-\frac{729}{1331} = -\sqrt[3]{\frac{729}{1331}} = -\sqrt[3]{\frac{9 \times 9 \times 9}{11 \times 11 \times 11}} = -\frac{9}{11}$

(v) 250.047

Cube root of $250.047 = \sqrt[3]{250.047} = \sqrt[3]{6.3 \times 6.3 \times 6.3} = 6.3$

(vi) -166375

Cube root of $-166375 = -\sqrt[3]{166375}$

$= -\sqrt[3]{55 \times 55 \times 55} = -55$

11. (i) $\frac{27}{64} \Rightarrow$ Cube root of $\frac{27}{64} = \sqrt[3]{\frac{27}{64}} = \sqrt[3]{\frac{3 \times 3 \times 3}{4 \times 4 \times 4}} = \frac{3}{4}$

(ii) $\frac{64}{343} \Rightarrow$ Cube root of $\frac{64}{343} = \sqrt[3]{\frac{64}{343}} = \sqrt[3]{\frac{4 \times 4 \times 4}{7 \times 7 \times 7}} = \frac{4}{7}$

(iii) $\frac{343}{512} \Rightarrow$ Cube root of $\frac{343}{512} = \sqrt[3]{\frac{343}{512}} = \sqrt[3]{\frac{7 \times 7 \times 7}{8 \times 8 \times 8}} = \frac{7}{8}$

(iv) 512×729

Cube root of $512 \times 729 = \sqrt[3]{512 \times 729}$

$= \sqrt[3]{8 \times 8 \times 8 \times 9 \times 9 \times 9}$

$= 8 \times 9 = 72$

(v) 64×27 Cube root of $64 \times 27 = \sqrt[3]{64 \times 27}$

$= \sqrt[3]{4 \times 4 \times 4 \times 3 \times 3 \times 3}$

$= 4 \times 3 = 12$

(vi) 1331×8000

Cube root of $1331 \times 8000 = \sqrt[3]{1331 \times 8000}$

$= \sqrt[3]{11 \times 11 \times 11 \times 20 \times 20 \times 20}$

$= 11 \times 20 = 220$

12. (i) 1.728

Cube root of $1.728 = \sqrt[3]{1.728} = \sqrt[3]{1.2 \times 1.2 \times 1.2} = 1.2$

(ii) 9.261

Cube root of $9.261 = \sqrt[3]{9.261} = \sqrt[3]{2.1 \times 2.1 \times 2.1} = 2.1$

(iii) 0.000027

Cube root of $0.000027 = \sqrt[3]{\frac{27}{1000000}} = \sqrt[3]{\frac{3 \times 3 \times 3}{100 \times 100 \times 100}}$

$= \frac{3}{100} = 0.03$

(iv) –0.512

Cube root of $-0.512 = -\sqrt[3]{\frac{512}{1000}} = -\sqrt[3]{\frac{8 \times 8 \times 8}{10 \times 10 \times 10}}$

$= -\frac{8}{10} = -0.8$

(v) –15.625

Cube root of $-15.625 = -\sqrt[3]{15.625}$

$= -\sqrt[3]{2.5 \times 2.5 \times 2.5} = -2.5$

(vi) 125×8000

Cube root of $125 \times 8000 = \sqrt[3]{125 \times 8000}$

$= \sqrt[3]{5 \times 5 \times 5 \times 20 \times 20 \times 20}$

$= 5 \times 20 = 100$

13. 30375

Factorising 30375 by division method

$30375 = \underline{3 \times 3 \times 3} \times 3 \times 3 \times \underline{5 \times 5 \times 5}$

Grouping the factors in triplets of equal factors and 3×3 is left.

Clearly to make it a perfect cube.

It must be multiplied by 3.

3	30375
3	10125
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

14. 26244

Factorising 26244 by division method

$26244 = 2 \times 2 \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times 3 \times 3$

Grouping the factors in triplets of equal factor.

Therefore, to make 26244 perfect cube divided by 36.

2	26244
2	13122
3	6561
3	2187
3	729
3	243
3	81
3	27
3	9
3	3
	1

15. Given : Volume of cubical box $= 21952 \text{ m}^3$

Let length of side of box $= x$

$$x^3 = 21952 = \sqrt[3]{21952}$$

$$x = \sqrt[3]{28 \times 28 \times 28} = 28 \text{ m}$$

16. Given : Three number $= 3 : 4 : 5$

Product of their number $= 480$

$$3x \times 4x \times 5x = 480$$

$$x \times 4x \times 5x = \frac{480}{3} \Rightarrow x \times 20 \times x^2 = 160$$

$$x^3 = 8 \Rightarrow x = \sqrt[3]{8} = 2$$

Therefore hare, three number $= 3 \times 2, 4 \times 2, 5 \times 2 = 6, 8, 10$

17. Two number ratio $= 4 : 5$

Difference of their cubes $= 61$

$$(5x)^3 - (4x)^3 = 61$$

$$125x^3 - 64x^3 = 61 \Rightarrow 61x^3 = 61$$

$$x^3 = 1 \Rightarrow x = \sqrt[3]{1} = 1$$

Two number $= 4 \times 1, 5 \times 1 = 4, 5$ ❑

5. Algebraic Expressions and Their Factorization

Exercise 5.1

1. (i) $8x^2y, 6yx^2, -3x^2y^2, 8y^2x^2, 3x^2y$

$8x^2y + 6yx^2 + (-3x^2y^2) + 8y^2x^2 + 3x^2y$

$= 8x^2y + 6x^2y + 3x^2y - 3x^2y^2 + 8x^2y^2$

$= 17x^2y + 5x^2y^2$

(ii), (iii), (iv) Do yourself.

2. (i) $(2a^2 + 5b^2) + (3a^2 - 6b^2) + (-6a^2 + 8b^2)$

$= 2a^2 + 5b^2 + 3a^2 - 6b^2 - 6a^2 + 8b^2$

$= 2a^2 + 3a^2 - 6a^2 + 5b^2 - 6b^2 + 8b^2$

$= -a^2 + 7b^2$

(ii) $(10x^2 - 3xy + 4y^2) + (2x^2 + 2xy - 6y^2)$

$+ (-3x^2 - 5y^2 + 8xy)$

$= 10x^2 - 3xy + 4y^2 + 2x^2$

$+ 2xy - 6y^2 - 3x^2 - 5y^2 + 8xy$

$= 10x^2 + 2x^2 - 3x^2 - 3xy$

$+ 2xy + 8xy + 4y^2 - 6y^2 - 5y^2$

$= 9x^2 + 7xy - 7y^2$

(iii), (iv) Do yourself.

3. (i)

$$\begin{array}{r} 5x^2 + 3y^2 - 2z^2 \\ 4x^2 + 2y^2 - 6z^2 \\ \hline + 2x^2 - 6y^2 + 4z^2 \\ \hline 11x^2 - y^2 - 4z^2 \end{array}$$

(ii)

$$\begin{array}{r} x^3 + 6x^2 \\ -6x^3 + 4x^2 + 2 \\ \hline + 2x^3 + 8x^2 + 4 \\ \hline -3x^3 + 18x^2 + 6 \end{array}$$

(iii), (iv) Do yourself.

4. (i) $6nm^2 - 5m^2n = 6m^2n - 5m^2n = m^2n$

(ii) $-3xyz - 15xyz = -18xyz$

(iii), (iv) Do yourself.

5. (i) $(2xy - 2x^2 - 5y^2) - (3x^2 - 6xy + 4y^2)$

$= 2xy - 2x^2 - 5y^2 - 3x^2 + 6xy - 4y^2$

$= -5x^2 + 8xy - 9y^2$

(ii) $(3mn - 2m^2 + 4p^2) - (5mn + 6m^2 - 6p^2)$

$= 3mn - 2m^2 + 4p^2 - 5mn - 6m^2 + 6p^2$

$= -2mn - 8m^2 + 10p^2$

(iii), (iv) Do yourself.

6. (i) $3x^2 + 8xy - 2y^2$

$-x^2 + 7xy - 3y^2$

$+ \quad - \quad +$

$4x^2 + xy + y^2$

(ii) $-3a^2 + 8ab + 7b^2$

$9a^2 - 6ab + 5b^2$

$- \quad + \quad -$

$-12a^2 + 14ab + 2b^2$

(iii), (iv) Do yourself.

7. $(8x^2 - 3xy + 6y^2) - [x^2 + 4xy + 3y^2 + (-3x^2 - 2xy + y^2)]$

$= (8x^2 - 3xy + 6y^2) - (x^2 + 4xy + 3y^2 - 3x^2 - 2xy + y^2)$

$= (8x^2 - 3xy + 6y^2) - (-2x^2 + 2xy + 4y^2)$

$= 8x^2 - 3xy + 6y^2 + 2x^2 - 2xy - 4y^2$

$= 10x^2 - 5xy + 2y^2$

8. Do yourself.

Word Problems

1. Perimeter of triangle = Sum of three sides of a triangle

$= (4x + 6y - 3) + (5x + 2) + (2x + y + 4)$

$= 4x + 6y - 3 + 5x + 2 + 2x + y + 4$

$= (11x + 7y + 3)$ cm

2. $[-6ab + 8a^2 + (-4b^2 + 8ab)] - [3ab - 6b^2 + 5a^2 + 2ab]$

$= (-6ab + 8a^2 - 4b^2 + 8ab) - (5ab - 6b^2 + 5a^2)$

$= 2ab + 8a^2 - 4b^2 - 5ab + 6b^2 - 5a^2$

$= -3ab + 3a^2 + 2b^2$

3. Total cloth that she purchased

$= (5x + 9y + 10) + (2x + y + 3)$

$= (7x + 10y + 13)$ m

4. $(-7mnp - 8m^2) - (6m^2 + 8mnp - 3n^2)$

$= -7mnp - 8m^2 - 6m^2 - 8mnp + 3n^2$

$= -14m^2 - 15mnp + 3n^2$

$= 15mnp - 14m^2 + 3n^2$

5. Perimeter of the rectangle = 2 (Length + Breadth)

$= 2\,[4x^2 + 5y - 3 + 3x^2 - 2y + 2]$ cm

$= 2\,[7x^2 + 3y - 1]$ cm $= (14x^2 + 6y - 2)$ cm

6. $(3p^2 + 5pm - 6m^2) - (-3pm - 4p^2 + 8m^2)$

$= 3p^2 + 5pm - 6m^2 + 3pm + 4p^2 - 8m^2$

$= 7p^2 + 8pm - 14m^2$

7. Perimeter of the square $= 4 \times$ Side $= 4 \times (5m + 3n - 4)$ units

$= (20m + 12n - 16)$ units

8. Money left in Sanjay's account $= (5x + 4y - 30) - (2x - 2y + 40)$

$= 5x + 4y - 30 - 2x + 2y - 40$

$= ₹(3x + 6y - 70)$

Exercise 5.2

1. (i) $7x \times 5x^2 = 35x^3$

(ii) $-5x^3 \times 7x^2 = -35x^5$

(iii), (iv), (v) Do yourself.

(vi) $x^{-6} \times x^7 \times (-2x) = -x^{-6} \times x^7 \times 2x$

$= -2x^{-6+7+1} = -2x^2$

2. $(-2x^2) \times (7x^2) \times (6x^3) = -2 \times 7 \times 6 \times x^2 \times x^2 \times x^3$

$= -84x^7$

Verification : we put $x = 1$

$(-2x^2) \times (7x^2) \times (6x^3) = (-2 \times 1^{2)} \times (7 \times 1^2) \times (6 \times 1^3)$

$= -2 \times 7 \times 6 = -84$

$-84x^7 = -84 \times 1^7 = -84$

LHS = RHS

Hence, $(-2x^2) \times (7x^2) \times (6x^3) = -84x^7$

3. and **4.** Do yourself.

5. (i) $a^7 \times a^{10} \times a^{-3} = a^{17} \times a^{-3} = a^{14}$

(ii) Do yourself.

6. (i) $5a(a^2 + a + 3) = 5a \times a^2 + 5a \times a + 5a \times 3$

$= 5a^3 + 5a^2 + 15a$

(ii) $x^5(x^2 + 7x + 9) = x^5 \times x^2 + x^5 \times 7x + x^5 \times 9$

$= x^7 + 7x^6 + 9x^5$

(iii) Do yourself.

7. (i) $(3x + 5) \times 7x = 3x \times 7x + 5 \times 7x = 21x^2 + 35x$

(ii) $(2x - y) \times xy = 2x \times xy - y \times xy = 2x^2y - xy^2$

(iii), (iv), (v), (vi) Do yourself.

8. (i) $(2x + 9) \times (6x + 5) = (2x + 9) \times 6x + (2x + 9) \times 5$

$= 2x \times 6x + 9 \times 6x + 2x \times 5 + 9 \times 5$

$= 12x^2 + 54x + 10x + 45 = 12x^2 + 64x + 45$

(ii), (iii) Do yourself.

(iv) $(2.5a + 2.3b) \times (2.5a - 2.3b) = (2.5a + 2.3b) \times 2.5a$
$+ (2.5a + 2.3b) \times -2.3b$
$= 6.25a^2 + 5.75ab - 5.75ab - 5.29b^2$
$= 6.25a^2 - 5.29$

9. (i) $a(a - b) + b(a - b) = a^2 - ab + ab - b^2 = a^2 - b^2$

(ii) $a^2 - b^2 + a(a + b) = a^2 - b^2 + a(a + b)$
$= (a - b)(a + b) + a(a + b)$
$= (a + b)(a - b + a) = (a + b)(2a - b)$

(iii), (iv) Do it yourself.

10. (i) $(2x - 5)(7 + 4x) = (2x - 5) \times 7 + (2x - 5) \times 4x$
$= 14x - 35 + 8x^2 - 20x$
$= 8x^2 - 6x - 35$

Verification : we put $x = 2$
$(2x - 5)(7 + 4x) = (2 \times 2 - 5) \times (7 + 4 \times 2)$
$= (4 - 5) \times (7 + 8) = -1 \times 15 = -15$
$8x^2 - 6x - 35 = 8 \times 2^2 - 6 \times 2 - 35$
$= 32 - 12 - 35 = -15$
LHS = RHS

(ii), (iii), (iv) Do it yourself.

11. (i) $(2x + 3y)(4x^2y + 5xy^2) = 2x \times (4x^2y + 5xy^2)$
$+ 3y \times (4x^2y + 5xy^2)$
$= 8x^3y + 10x^2y^2 + 12x^2y^2 + 15xy^3$
$= 8x^3y + 22x^2y^2 + 15xy^3$

(ii) $(a^5 + 5)(b^3 + 3) + 4 = a^5(b^3 + 3) = 5(b^3 + 3) + 4$
$= a^5b^3 + 3a^5 + 5b^3 + 15 + 4$
$= a^5b^3 + 3a^5 + 5b^3 + 19$

(iii), (iv) Do it yourself.

12. (i) $(x + y)(x^2 - xy + y^2) = x(x^2 - xy + y^2)$
$+ y(x^2 - xy + y^2)$
$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3 = x^3 + y^3$

(ii) $x^2 + (3x - y)(3x + y + y^2) = x^2 + 3x(3x + y + y^2)$
$- y(3x + y + y^2)$
$= x^2 + 9x^2 + 3xy + 3xy^2 - 3xy - y^2 - y^3$
$= 10x^2 + 3xy^2 - y^2 - y^3$

(iii) Do it yourself.

13. Do it yourself.

Exercise 5.3

1. (i) $25m^2 + 40m + 16 = (5m)^2 + 2 \times 5m \times 4 + (4)^2$

We know that :

$a^2 + b^2 + 2ab = (a + b)^2$

$= (5m + 4)^2 = (5m + 4)\,(5m + 4)$

(ii) $16x^2 + 25 + 40x = (4x)^2 + (5)^2 + 2 \times 5 \times 4x$

$= (4x + 5)^2 = (4x + 5)\,(4x + 5)$

(iii) $16x^2y^2 - 40xyz + 25z^2$

$= (4xy)^2 - 2 \times 4xy \times 5z + (5z)^2$

$= (4xy - 5z)^2$

$[\because a^2 - 2ab + b^2 = (a - b)^2]$

$= (4xy - 5z)\,(4xy - 5z)$

(iv) $49x^4 - 168x^2y^2 + 144y^4$

$= (7x^2)^2 - 2 \times 7x^2 \times 12y^2 + (12y^2)^2$

$= (7x^2 - 12y^2)^2$

$= (7x^2 - 12y^2)\,(7x^2 - 12y^2)$

(v) $1 - 8xy + 16x^2y^2 = 1 - 2 \times 4xy \times 1 + (4xy)^2 = (1 - 4xy)^2$

$= (1 - 4xy)\,(1 - 4xy)$

(vi) $\dfrac{x^2}{4y^2} - \dfrac{1}{3} + \dfrac{y^2}{9x^2} = \left(\dfrac{x}{2y}\right)^2 - \dfrac{y}{3x} \times 2 \times \dfrac{x}{2y} + \left(\dfrac{y}{3x}\right)^2$

$= \left(\dfrac{x}{2y} - \dfrac{y}{3x}\right)^2 = \left(\dfrac{x}{2y} - \dfrac{y}{3x}\right)\left(\dfrac{x}{2y} - \dfrac{y}{3x}\right)$

2. (i) $25x^2 - 16y^2 = (5x)^2 - (4y)^2$

We know that :

$a^2 - b^2 = (a + b)\,(a - b)$

$= (5x - 4y)\,(5x + 4y)$

(ii) $m^2 - \dfrac{n^2}{100} = (m)^2 - \left(\dfrac{n}{10}\right)^2$

$= \left(m - \dfrac{n}{10}\right)\left(m + \dfrac{n}{10}\right)$

(iii) $16m^5 - 144m^3 = 16m^3\,(m^2 - 9)$

$= 16m^3\,(m^2 - 3^2)$

$= 16m^3\,(m + 3)\,(m - 3)$

(iv) $25a^2 - 30ab + 9b^2 - 121$

$= (5a)^2 - 2 \times 5a \times 3b + (3b)^2 - (11)^2$

$= (5a - 3b)^2 - (11)^2$

$= (5a - 3b + 11)\,(5a - 3b - 11)$

(v) $2x^4 - 32 = 2[(x^2)^2 - 4^2]$

$= 2\,(x^2 + 4)\,(x^2 - 4)$

$= 2\,(x^2 + 4)\,(x^2 - 2^2)$

$= 2\,(x^2 + 4)\,(x + 2)\,(x - 2)$

(vi) $(x + y)^2 - (x - y)^2 = (x + y + x - y)\,(x + y - x + y)$

$= (2x)\,(2y) = 4xy$

3. (i) $y^2 + 5y + 6 = y^2 + (2 + 3)y + 6$

$= y^2 + 2y + 3y + 6$

$= y\,(y + 2) + 3\,(y + 2)$

$= (y + 3)\,(y + 2)$

(ii) $x^2 + 7x + 12 = x^2 + (3 + 4)\,x + 12$

$= x^2 + 3x + 4x + 12$

$= x\,(x + 3) + 4\,(x + 3)$

$= (x + 4)\,(x + 3)$

(iii) $x^2 - 4x - 5 = x^2 - (5 - 1)\,x - 5$

$= x^2 - 5x + x - 5$

$= x\,(x - 5) + 1\,(x - 5)$

$= (x - 5)\,(x + 1)$

(iv) $x^2 - 2x - 24 = x^2 - (6 - 4)x + 24$

$= x^2 - 6x + 4x - 24$

$= x\,(x - 6) + 4\,(x - 6)$

$= (x - 6)\,(x + 4)$

(v) $m^2 - 21m + 68 = m^2 - (17 + 4)\,m + 68$

$= m^2 - 17m - 4m + 68$

$= m\,(m - 17) - 4\,(m - 17)$

$= (m - 17)\,(m - 4)$

(vi) $y^2 + 19y - 150 = y^2 + (25 - 6)\,y - 150$

$= y^2 + 25y - 6y - 150$

$= y\,(y + 25) - 6\,(y + 25)$

$= (y + 25)\,(y - 6)$

(vii) $y^2 - 10y + 16 = y^2 - (8 + 2)y + 16$

$= y^2 - 8y - 2y + 16$

$= y(y - 8) - 2(y - 8)$

$= (y - 8)(y - 2)$

(viii) $x^2 - 11x - 102 = x^2 - (17 - 6)x - 102$

$= x^2 - 17x + 6x - 102$

$= x(x - 17) + 6(x - 17)$

$= (x - 17)(x + 6)$

(ix) $48 + 22x - x^2 = 48 + (24 - 2)x - x^2$

$= 48 + 24x - 2x - x^2$

$= 24(2 + x) - x(2 + x)$

$= (x + 2)(24 - x)$

4. (i) $x^2 + 4y^2 + z^2 + 4xy - 2xz - 4yz$

$= (x)^2 + (2y)^2 + (-z)^2 + 2(x \times 2y - 2y \times z - zx)$

$= (x + 2y - z)^2$

$[\because \quad a^2 + b^2 + c^2 + 2(ab + bc + ca) = (a + b + c)^2]$

(ii) $4p^2 + 9q^2 + 4r^2 + 12pq + 12qr + 8pr$

$= (2p)^2 + (3q)^2 + (2r)^2 + 2(2p \times 3q + 3q \times 2r + 2r \times 2p)$

$= (2p + 3q + 2r)^2$

(iii) $4x^2 + y^2 + 9z^2 - 4xy + 6yz - 12zx$

$= (2x)^2 + (-y)^2 + (-3z)^2 + 2(2x \times -y - y \times -3z + 3z \times -2x)$

$= (2x - y - 3z)^2$

(iv) $2x^2 + 4y^2 + 3z^2 + 4\sqrt{2}xy - 4\sqrt{3}yz - 2\sqrt{6}xz$

$= (\sqrt{2}x)^2 + (2y)^2 + (-\sqrt{3}z)^2 + 2(2\sqrt{2}xy - 2\sqrt{3}yz - \sqrt{6}xz)$

$= (\sqrt{2}x + 2y - \sqrt{3}z)^2$

5. (i) $27x^3 - 8y^3 - 54x^2y + 36xy^2$

$= (3x)^3 + (-2y)^3 + 3 \times 3x \times -2y(3x - 2y)$

$= (3x - 2y)^3 \qquad [\because \quad a^3 + b^3 + 3ab(a + b) = (a + b)^3]$

(ii) $x^3 + 64y^3 + 12x^2y + 48xy^2$

$= (x)^3 + (4y)^3 + 3 \times x \times 4y(x + 4y) = (x + 4y)^3$

(iii) $8y^3 - 125z^3 - 60y^2z + 150yz^2$

$= (2y)^3 - (5z)^3 - 3 \times 2y \times 5z(2y - 5z)$

$[\because a^3 - b^3 - 3ab(a - b) = (a - b)^3]$

$= (2y - 5z)^3$

(iv) $x^3 + 125z^3 + 75xz^2 + 15x^2z$

$= (x)^3 + (5z)^3 + 3 \times x \times 5z\,(x + 5z) = (x + 5z)^3$

6. (i) $p^3 + 27$

$= (p)^3 + (3)^3 \qquad [\because\ a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)]$

$= (p + 3)\,(p^2 - 3p + 3^2) = (p + 3)\,(p^2 - 3p + 9)$

(ii) $y^3 + 125 = (y)^3 + (5)^3 = (y + 5)\,(y^2 - 5y + 25)$

(iii) $1 - 27z^3 = (1)^3 + (-3z)^3 = (1 - 3z)\,(1 + 3z + 9z^2)$

(iv) $8x^3y^3 + 27z^3 = (2xy)^3 + (3z)^3$

$= (2xy + 3z)\,(4x^2y^2 - 6xyz + 9z^2)$

(v) $64x^3 - y^3 = (4x)^3 - (y)^3 = (4x - y)\,(16x^2 + 4xy + y^2)$

(vi) $m^3 - 27n^3 = (m)^3 + (-3n)^3$

$= (m - 3n)\,(m^2 + 3nm + 9n^2)$

(vii) $\dfrac{1}{216}p^3 - 8q^3 = \left(\dfrac{p}{6}\right)^3 - (2q)^3$

$= \left(\dfrac{p}{6} - 2q\right)\left(\dfrac{p^2}{36} + \dfrac{pq}{3} + 4q^2\right)$

7. (i) $10xy^4 - 10x^4y = 10xy\,(y^3 - x^3)$

$= 10xy\,(y - x)\,(y^2 + xy + x^2)$

(ii) $54x^6y + 2x^3y^4 = 2x^3y\,(27x^3 + y^3)$

$= 2x^3y\,[(3x)^3 + (y)^3]$

$= 2x^3y\,[(3x + y)\,(9x^2 - 3xy + y^2)]$

(iii) $(p - 2q)^3 - (8q)^3$

$= (p - 2q - 8q)\,[(p - 2q)^2 + (p - 2q)\,(8q) + (8q)^2]$

$= (p - 10q)\,(p^2 + 4q^2 - 4qp + 8pq - 16q^2 + 64q^2)$

$= (p - 10q)\,(p^2 + 52q^2 - 4qp + 8pq)$

$= (p - 10q)\,(p^2 + 52q^2 + 4pq)$

(iv) $27\,(x - 1)^3 + m^3 = [3\,(x - 1)^3 + m^3]$

$= [3\,(x - 1) + m]\,[3\,(x - 1)^2 - 3m\,(x - 1) + m^2)]$

$= (3x - 3 + m)\,[9(x^2 + 1 - 2x) - 3mx + 3m + m^2]$

$= (3x - 3 + m)\,[9x^2 + 9 - 18x - 3mx + 3m + m^2]$

$= (3x - 3 + m)\,[9x^2 + m^2 - 18x - 3mx + 3m + 9]$

8. (i) $p^3 + 8q^3 + 64r^3 - 24pqr$

$= (p)^3 + (2q)^3 + (4r)^3 - 3 \times p \times 2q \times 4r$

$= (p + 2q + 4r)\,(p^2 + (2q)^2 + (4r)^2 - 2pq - 8qr - 4pr)$

$= (p + 2q + 4r)\,(p^2 + 4q^2 + 16r^2 - 2pq - 8qr - 4pr)$

(ii) $8x^3 - 27y^3 + z^3 + 18xyz$

$= (2x)^3 + (-3y)^3 + (z)^3 + 3 \times 2x \times z \times 3y$

$= (2x - 3y + z)\,[(2x)^2 + (-3y)^2 + z^2 + 6xy + 3yz - 2xz]$

$= (2x - 3y + z)\,(4x^2 + 9y^2 + z^2 + 6xy + 3yz - 2xz)$

(iii) $l^3 + m^3 - n^3 + 3lmn = l^3 + m^3 + (-n)^3 - 3lm \times (-n)$

$= (l + m - n)\,(l^2 + m^2 + n^2 - lm + mn + ln)$

(iv) $-27x^3 + y^3 - z^3 - 9xyz$

$= (-3x)^3 + (y)^3 + (-z)^3 - 3 \times -3x \times y \times -z$

$= (-3x + y - z)\,(9x^2 + y^2 + z^2 + 3xy + yz - 3xz)$

(v) $x^3 - 8y^3 - 64z^3 - 24xyz$

$= (x)^3 + (-2y)^3 + (-4z)^3 - 3 \times x \times -2y \times -4z$

$= (x - 2y - 4z)\,(x^2 + 4y^2 + 16z^2 + 2xy - 8yz + 4xz)$

(vi) $\frac{1}{27}x^3 - y^3 + 125z^3 + 5xyz$

$= \left(\frac{1}{3}x\right)^3 + (-y)^3 + (5z)^3 - 3 \times \frac{x}{3} \times -y \times 5z$

$= \left(\frac{1}{3}x - y + 5z\right)\left(\frac{x^2}{9} + y^2 + 25z^2 + \frac{xy}{3} + 5yz - \frac{5}{3}xz\right)$

9. (i) $(3x - 5y)^3 + (5y - qz)^3 + (9z - 3x)^3$

If $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

According to formula :

$3x - 5y + 5y - 9z + 9z - 3x = 0$

$(3x - 5y)^3 + (5y - 9z)^3 + (9z - 3x)^3$

$= 3\,(3x - 5y)\,(5y - 9z)\,(9z - 3x)$

(ii) $(p - 3q)^3 + (3q - 7r)^3 + (7r - p)^3$

According to formula,

$p - 3q + 3q - 7r + 7r - p = 0$

$(p - 3q)^3 + (3q - 7r)^3 + (7r - p)^3$

$= 3\,(p - 3q)\,(3q - 7r)\,(7r - p)$

(iii) $(5x-6p)^3 + (7z-5x)^3 + (6p-7z)^3$

$5x-6p+7z-5x+6p-7z=0$

$(5x-6p)^3 + (7z-5x)^3 + (6p-7z)^3$

$= 3(5x-6p)(7z-5x)(6p-7z)$

(iv) $\left(\frac{1}{2}x-3y\right)^3 + (3y-\sqrt{3}z)^3 + \left(\sqrt{3}z-\frac{1}{2}x\right)^3$

$\frac{1}{2}x-3y+3y-\sqrt{3}z+\sqrt{3}z-\frac{1}{2}x=0$

$\left(\frac{1}{2}x-3y\right)^3 + (3y-\sqrt{3}z)^3 + \left(\sqrt{3}z-\frac{1}{2}x\right)^3$

$= 3\left(\frac{1}{2}x-3y\right)(3y-\sqrt{3}z)\left(\sqrt{3}z-\frac{1}{2}x\right)$

10. (i) $(55)^3 - (25)^3 - (30)^3$

If $a+b+c=0$ then $a^3+b^3+c^3=3abc$

According to formula :

$55-25-30=0$

$(55)^3 - (25)^3 - (30)^3 = 3\times -55\times -25\times -30 = 123750$

(ii) $(47)^3 + (29)^3 - (76)^3$

$\Rightarrow \quad 47+29-76=0$

$(47)^3 + (29)^3 - (76)^3 = -3\times 47\times 29\times 76$

$= -310764$

(iii) $(9.8)^3 - (11.3)^3 + (1.5)^3$

$\Rightarrow \quad 9.8-11.3+1.5=0$

$(9.8)^3 - (11.3)^3 + (1.5)^3 = -3\times 9.8\times 11.3\times 1.5$

$= -498.33$

(iv) $(2.7)^3 - (1.6)^3 - (1.1)^3$

$\Rightarrow \quad 2.7-1.6-1.1=0$

$(2.7)^3 - (1.6)^3 - (1.1)^3 = 3\times 2.7\times 1.6\times 1.1 = 14.256$

Revision Exercise

1. (i) Addition $= ab-bc+bc-ca+ca-ab=0$

(ii) Addition $= 5p^2q^2+4pq+7+3+9pq-2p^2q^2$

$= 3p^2q^2+13pq+10$

(iii) Addition $= l^2+m^2+n^2+lm+mn+mn+nl+nl+lm$

$= l^2+m^2+n^2+2(lm+nl+mn)$

(iv) Addition $= 4x^3 - 7x^2 + 9 + 3x^2 - 5x + 4 + 7x^3 - 11x + 1 + 6x^2 - 13x$

$= 11x^3 + 2x^2 - 29x + 14$

2. (i) Subtraction $= (14a - 5ab + 7b - 5) - (8a + 3ab - 2b + 7)$

$= 14a - 5ab + 7b - 5 - 8a - 3ab + 2b - 7$

$= 6a - 8ab + 9b - 12$

(ii) Subtraction

$= (12xy - 3yz - 4zx + 5xyz) - (8xy + 4yz + 5zx)$

$= 12xy - 3yz - 4zx + 5xyz - 8xy - 4yz - 5zx$

$= 4xy - 7yz - 9zx + 5xyz$

(iii) Subtraction $= (18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q) - (4p^2q - 3pq + 5pq^2 - 8p + 7q - 10)$

$= 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q - 4p^2q + 3pq - 5pq^2 + 8p - 7q + 10$

$= p^2q - 7pq^2 + 8pq + 5p - 18q + 28$

3. (i) $(3x - 5y + 7z) \times -3yxz = -9x^2yz + 15xy^2z - 21xyz^2$

(ii) $(2p^2 - 3pq + 5q^2 + 5) \times -2pq$

$= -4p^3q + 6p^2q^2 - 10pq^3 - 10pq$

(iii) $\left(\frac{2}{3}a^2b - \frac{4}{5}ab^2 + \frac{2}{7}ab + 3\right) \times 35ab$

$= \frac{70}{3}a^3b^2 - 28a^2b^3 + 10a^2b^2 + 105ab$

(iv) $(4x^2 - 10xy + 7y^2 - 8x + 4y + 3) \times 3xy$

$= 12x^3y - 30x^2y^2 + 21xy^3 - 24x^2y + 12xy^2 + 9xy$

4. (i) $(x^2 + 3)(x - 3) + 9 = x^3 - 3x^2 + 3x - 9 + 9$

$= x^3 - 3x^2 + 3x$

(ii) $(x + 3)(x - 3)(x + 4)(x - 4) = (x^2 - 9)(x^2 - 16)$

$= x^4 - 16x^2 - 9x^2 + 144$

$= x^4 - 25x^2 + 144$

(iii) $(x + 5)(x + 6)(x + 7) = (x^2 + 5x + 6x + 30)(x + 7)$

$= (x^2 + 11x + 30)(x + 7)$

$= x^3 + 11x^2 + 30x + 7x^2 + 77x + 210$

$= x^3 + 18x^2 + 107x + 210$

(iv) $(p+q-2r)(2p-q+r)-4qr$

$=2p^2-pq+pr+2pq-q^2+qr-4pr+2qr-2r^2-4qr$

$=2p^2-q^2-2r^2+pq-qr-3pr$

(v) $(p+q)(r+s)+(p-q)(r-s)-2(pr+qs)$

$=pr+ps+qr+qs+pr-ps-qr+qs-2pr-2qs=0$

(vi) $(x+y+z)(x-y+z)+(x+y-z)(-x+y+z)-4zx$

$=x^2-xy+xz+xy-y^2+yz+zx-zy+z^2-x^2$

$+xy+xz-xy+y^2+yz+zx-zy-z^2-4zx=0$

5. Given : $x^2+\frac{1}{x^2}=23$

We know that : $\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2=23+2$

$\left(x+\frac{1}{x}\right)^2=25 \Rightarrow \left(x+\frac{1}{x}\right)=\pm\sqrt{25}$

$\therefore \quad \left(x+\frac{1}{x}\right)=\pm 5$

6. Given : $a+b=9, ab=10$

$(a+b)^2=a^2+b^2+2ab$

$a^2+b^2=(a+b)^2-2ab$

$=(9)^2-2\times 10=81-20$

$a^2+b^2=61$

7. Given : $a-b=6$ and $a^2+b^2=42$

We know that : $(a-b)^2=a^2+b^2-2ab$

$ab=\frac{(a^2+b^2)-(a-b)^2}{2}$

$=\frac{42-(6)^2}{2}=\frac{42-36}{2}=\frac{6}{2}=3$

8. Given : $a^2+b^2=41$ and $ab=4$

(i) $(a+b)^2=a^2+b^2+2ab$

$=41+2\times 4=41+8=49$

$(a+b)=\pm\sqrt{49} \Rightarrow (a+b)=\pm 7$

(ii) $(a-b)^2=a^2+b^2-2ab=41-2\times 4=41-8$

$(a-b)^2=33 \Rightarrow (a-b)=\pm\sqrt{33}$

9. (i) $x^2+3x+2=x^2+(2+1)x+2=x^2+2x+x+2$

$=x(x+2)+1(x+2)=(x+1)(x+2)$

(ii) $z^2 + 10z + 24 = z^2 + (6 + 4)z + 24$

$= z^2 + 6z + 4z + 24$

$= z(z + 6) + 4(z + 6) = (z + 4)(z + 6)$

(iii) $y^2 - 7y + 12 = y^2 - (3 + 4)y + 12$

$= y^2 - 3y - 4y + 12$

$= y(y - 3) - 4(y - 3)$

$= (y - 4)(y - 3)$

(iv) $m^2 - 23m + 42 = m^2 - (21 + 2)m + 42$

$= m^2 - 21m - 2m + 42$

$= m(m - 21) - 2(m - 21)$

$= (m - 2)(m - 21)$

(v) $y^2 - 5y - 24 = y^2 - (8 - 3)y - 24$

$= y^2 - 8y + 3y - 24$

$= y(y - 8) + 3(y - 8)$

$= (y + 3)(y - 8)$

(vi) $t^2 + 23t - 108 = t^2 + (27 - 4)t - 108$

$= t^2 + 27t - 4t - 108$

$= t(t + 27) - 4(t + 27)$

$= (t - 4)(t + 27)$

(vii) $3x^2 + 14x + 8 = 3x^2 + (12 + 2)x + 8$

$= 3x^2 + 12x + 2x + 8$

$= 3x(x + 4) + 2(x + 4)$

$= (3x + 2)(x + 4)$

(viii) $3y^2 + 10y + 8 = 3y^2 + (6 + 4)y + 8$

$= 3y^2 + 6y + 4y + 8$

$= 3y(y + 2) + 4(y + 2)$

$= (3y + 4)(y + 2)$

10. (i) $x^2 - 3xy - 40y^2 = x^2 - (8 - 5)xy - 40y^2$

$= x^2 - 8xy + 5xy - 40y^2$

$= x(x - 8y) + 5y(x - 8y)$

$= (x + 5y)(x - 8y)$

11. (i) $(a + b)^2 - 11(a + b) - 42$

$= a^2 + b^2 + 2ab - 11a - 11b - 42$

$= a^2 + ab - 14a + ab + b^2 - 14b + 3a + 3b - 42$

$= a(a + b - 14) + b(a + b - 14) + 3(a + b - 14)$

$= (a + b - 14)(a + b + 3)$

(ii) $8 + 6(p + q) - 5(p + q)^2$

$= 8 + 6p + 6q - 5(p^2 + q^2 + 2pq)$

$= 8 + 6p + 6q - 5p^2 - 5q^2 - 10pq$

$= 8 + 10p + 10q - 4p - 5p^2 - 5qp - 4q - 5qp - 5q^2$

$= 2(4 + 5p + 5q) - p(4 + 5p + 5q) - q(4 + 5p + 5q)$

$= (4 + 5p + 5q)(2 - p - q)$

12. (i) $(x - 2y)^2 - 6(x - 2y) + 5$

$= x^2 + 4y^2 - 4xy - 6x + 12y + 5$

$= x^2 - 2xy - 5x - 2xy + 4y^2 + 10y - x + 2y + 5$

$= x(x - 2y - 5) - 2y(x - 2y - 5) - 1(x - 2y - 5)$

$= (x - 2y - 5)(x - 2y - 1)$

(ii) $7 + 10(2x - 3y) - 8(2x - 3y)^2$

$= 7 + 20x - 30y - 8(4x^2 + 9y^2 - 12xy)$

$= 7 + 20x - 30y - 32x^2 - 72y^2 + 96xy$

$= 7 + 28x - 42y - 8x - 32x^2 + 48xy + 12y + 48xy - 72y^2$

$= 7(1 + 4x - 6y) - 8x(1 + 4x - 6y) + 12y(1 + 4x - 6y)$

$= (1 + 4x - 6y)(7 - 8x + 12y)$

❑

6. Division of Algebraic Expressions

Exercise 6.1

1. (i) $9x^2yz \div 3xy = \dfrac{9x^2yz \times 1}{3xy} = 3xz$

(ii) $25m^2n^3 \div 5m^2n^2 = \dfrac{25m^2n^3}{5m^2n^2} = 5n$

(iii) $x^2 - y^2 \div x + y = \dfrac{x^2 - y^2}{(x + y)} = \dfrac{(x + y)(x - y)}{(x + y)} = (x - y)$

2. (i) $\dfrac{20m^3y^2}{4m^2y} = 5my$

(ii) $\dfrac{x^2 + 4x + 4}{x + 2} = \dfrac{(x + 2)(x + 2)}{(x + 2)} = (x + 2)$

(iii) $\frac{16m^2 - 9n^2}{4m - 3n} = \frac{(4m)^2 - (3n)^2}{(4m - 3n)} = \frac{(4m + 3n)(4m - 3n)}{(4m - 3n)}$

$= (4m + 3n)$

(iv) $\frac{125x^3 + 64}{25x^2 - 20x + 16} = \frac{(5x)^3 + (4)^3}{25x^2 - 20x + 16}$

$= \frac{(5x + 4)(25x^2 - 20x + 16)}{(25x^2 - 20x + 16)} = (5x + 4)$

(v) $\frac{9x^2 - 24xy + 16y^2}{3x - 4y} = \frac{(3x)^2 - 2 \times 3x \times 4y + (4y)^2}{3x - 4y}$

$= \frac{(3x - 4y)(3x - 4y)}{(3x - 4y)} = (3x - 4y)$

(vi) $\frac{216z^3 - 343p^3}{6z - 7p} = \frac{(6z)^3 - (7p)^3}{6z - 7p}$

$= \frac{(6z - 7p)(36z^2 + 42zp + 49p^2)}{(6z - 7p)}$

$= (36z^2 + 42zp + 49p^2)$

3. (i) $-39pq^2r^5 \div -24p^3q^3r = \frac{-39pq^2r^5}{-24p^3q^3r} = \frac{13r^4}{8p^2q}$

(ii) $\frac{\frac{-3}{4}a^2b^3}{\frac{6}{7}a^3b^2} = \frac{-3}{4} \times \frac{7}{6}\frac{b}{a} = \frac{-7}{8}\frac{b}{a}$

(iii) $\frac{9x^4 - 8x^3 - 12x + 3}{3x} = 3x^3 - \frac{8}{3}x^2 - 4 + \frac{1}{x}$

(iv) $\frac{14p^2q^3 - 32p^3q^2 + 15pq^2 - 22p + 18q}{-2p^2q}$

$= -7q^2 + 16pq - \frac{15}{2}\frac{q}{p} + \frac{11}{pq} - \frac{9}{p^2}$

4. (i)

$$
\begin{array}{r|l}
2x + 1 & 3x + 5 \\
\cline{2-2}
 & 6x^2 + 13x + 5 \\
 & \underline{-\,6x^2 \pm 3x} \\
 & \quad 10x + 5 \\
 & \quad \underline{-\,10x \pm 5} \\
 & \quad\quad 0
\end{array}
$$

Quotient $= 3x + 5$ and Remainder $= 0$

(ii) $1+y \overline{)\,1 + y^3}$ $\quad y^2 - y + 1$

$y^2 + y^3$

$1 - y^2$

$-y - y^2$

$1 + y$

$1 + y$

0

Quotient $= y^2 - y + 1$ and Remainder $= 0$

(iii) $x+1 \overline{)-2x^2 + x + 5}$ $\quad -2x + 3$

$-2x^2 - 2x$

$3x + 5$

$3x + 3$

2

Quotient $= 3 - 2x$ and Remainder $= 2$

(iv) $x^3 - 6x^2 + 12x - 8 \div x - 2$

$x-2 \overline{)x^3 - 6x^2 + 12x - 8}$ $\quad x^2 - 4x + 4$

$x^3 - 2x^2$

$-4x^2 + 12x$

$-4x^2 + 8x$

$4x - 8$

$4x - 8$

0

Quotient $= x^2 - 4x + 4$ and Remainder $= 0$

5. (i)

$$
\begin{array}{r}
2x^2 + 5x + 3 \\
3x - 7 \overline{)\ 6x^3 + x^2 - 26x - 25} \\
\underline{6x^3 - 14x^2} \quad (-\ +) \\
15x^2 - 26x \\
\underline{15x^2 - 35x} \quad (-\ +) \\
9x - 25 \\
\underline{9x - 21} \quad (-\ +) \\
-4
\end{array}
$$

Quotient $= 2x^2 + 5x + 3$ and Remainder $= -4$

(ii)

$$
\begin{array}{r}
m^2 - 5m - 5 \\
m - 1 \overline{)\ m^3 - 6m^2 + 7} \\
\underline{m^3 - m^2} \quad (-\ +) \\
-5m^2 + 7 \\
\underline{-5m^2 + 5m} \quad (+\ -) \\
-5m + 7 \\
\underline{-5m + 5} \quad (+\ -) \\
2
\end{array}
$$

Quotient $= m^2 - 5m - 5$ and Remainder $= 2$

(iii)

$$
\begin{array}{r}
a + 1 \\
a^2 + a + 1 \overline{)\ a^3 + 2a^2 + 2a + 1} \\
\underline{a^3 + a^2 + a} \quad (-\ -\ -) \\
a^2 + a + 1 \\
\underline{a^2 + a + 1} \quad (+\ -\ -) \\
0
\end{array}
$$

Quotient $= a + 1$ and Remainder $= 0$

(iv)

$$
\begin{array}{r}
4x - 3 \\
3x^2 - 2x + 5 \overline{)\ 12x^3 - 17x^2 + 26x - 18} \\
\underline{12x^3 - 8x^2 + 20x} \quad (-\ +\ -) \\
-9x^2 + 6x - 18 \\
\underline{-9x^2 + 6x - 15} \quad (+\ -\ +) \\
-3
\end{array}
$$

Quotient $= 4x - 3$ and Remainder $= -3$

6. (i) $\dfrac{9m^5 + 12m^4 - 6m^2}{3m^2} = 3m^3 + 4m^2 - 2$

(ii) $\dfrac{x^2 + 7x + 12}{x + 3} = \dfrac{x^2 + 3x + 4x + 12}{x + 3}$

$= \dfrac{x(x + 3) + 4(x + 3)}{x + 3} = \dfrac{(x + 3)(x + 4)}{(x + 3)} = x + 4$

(iii) $\dfrac{4y^2 + 3y + \frac{1}{2}}{2y + 1} = \dfrac{4y^2 + 2y + y + \frac{1}{2}}{2y + 1}$

$= \dfrac{2y(2y + 1) + \frac{1}{2}(2y + 1)}{(2y + 1)} = \dfrac{\left(2y + \frac{1}{2}\right)(2y + 1)}{(2y + 1)} = \left(2y + \frac{1}{2}\right)$

(iv) $\dfrac{4z^3 + 6z^2 - z}{-z/2} = -8z^2 - 12z + 2$

(v)
$$
\begin{array}{r|l}
 & 3m^2 - 2m + 9 \\
\hline
m + 2 & 3m^3 + 4m^2 + 5m + 18 \\
 & \underline{-3m^3 \pm 6m^2} \\
 & \quad -2m^2 + 5m \\
 & \quad \underline{-2m^2 \pm 4m} \\
 & \qquad\quad 9m + 18 \\
 & \qquad\quad \underline{-9m \pm 18} \\
 & \qquad\quad\quad 0
\end{array}
$$

Quotient $= 3m^2 - 2m + 9$ and Remainder $= 0$

(vi)
$$
\begin{array}{r|l}
 & 3y^2 + 3y + 2 \\
\hline
y^2 - 2y & 3y^4 - 3y^3 - 4y^2 - 4y \\
 & \underline{-3y^4 \pm 6y^3} \\
 & \quad 3y^3 - 4y^2 \\
 & \quad \underline{-3y^3 \pm 6y^2} \\
 & \qquad\quad 2y^2 - 4y \\
 & \qquad\quad \underline{-2y^2 \pm 4y} \\
 & \qquad\quad\quad 0
\end{array}
$$

Quotient $= 3y^2 + 3y + 2$ and Remainder $= 0$

7. (i)

$$
\begin{array}{r}
2x+3 \\
7x-4\overline{)14x^2+13x-15} \\
\underline{14x^2-8x\ \ (-\ +)} \\
21x-15 \\
\underline{21x-12\ \ (-\ +)} \\
-3
\end{array}
$$

Quotient $= 2x + 3$ and Remainder $= -3$

(ii)

$$
\begin{array}{r}
5z^2+\frac{10z}{3}+11 \\
3z-6\overline{)15z^3-20z^2+13z-12} \\
\underline{15z^3-30z^2\ \ (-\ +)} \\
10z^2+13z \\
\underline{10z^2-20z\ \ (-\ +)} \\
33z-12 \\
\underline{33z-66\ \ (-\ +)} \\
54
\end{array}
$$

Quotient $= 5z^2 + \frac{10z}{3} + 11$ and Remainder $= 54$

(iii)

$$
\begin{array}{r}
3y^3-5y+\frac{3}{2} \\
2y^2-6\overline{)6y^5-28y^3+3y^2+30y-9} \\
\underline{6y^5-18y^3\ \ (-\ +)} \\
-10y^3+3y^2+30y-9 \\
\underline{-10y^3+30y\ \ (+\ -)} \\
3y^2-9 \\
\underline{3y^2-9\ \ (-\ +)} \\
0
\end{array}
$$

Quotient $= 3y^3 - 5y + \frac{3}{2}$ and Remainder $= 0$

(iv) $34x - 22x^3 - 12x^4 - 10x^2 - 75 \div 3x + 7$

$$
\begin{array}{r}
-4x^3 + 2x^2 - 8x + 30 \\
3x + 7 \overline{)\, -12x^4 - 22x^3 - 10x^2 + 34x - 75} \\
-12x^4 - 28x^3 \\
+ \quad + \\
\hline
6x^3 - 10x^2 \\
6x^3 + 14x^2 \\
- \quad - \\
\hline
-24x^2 + 34x \\
-24x^2 - 56x \\
+ \quad + \\
\hline
90x - 75 \\
90x + 210 \\
- \quad - \\
\hline
-285
\end{array}
$$

Quotient $= -4x^3 + 2x^2 - 8x + 30$ and Remainder $= -285$

8. Do it yourself.

Revision Exercise

1. (i) $\dfrac{-84a^3}{14a^2} = -6a$ (ii) $\dfrac{24x^3y^3}{-8y^2} = -3x^3y$

(iii) $\dfrac{15a^4b}{-5a^3b} = -3a$ (iv) $\dfrac{-24x^4d^3}{-2x^2d^5} = \dfrac{12x^2}{d^2}$

(v) $\dfrac{63a^4b^5c^6}{-9a^2b^4c^3} = -7a^2bc^3$

(vi) $\dfrac{8x - 10y + 6c}{2} = 4x - 5y + 3c$

(vii) $\dfrac{-14x^6y^3 - 21x^4y^5 + 7x^5y^4}{7x^2y^2} = -2x^4y - 3x^2y^3 + x^3y^2$

(viii) $\dfrac{a^2 + 7a + 12}{a + 4} = \dfrac{a^2 + (3 + 4)a + 12}{a + 4} = \dfrac{a^2 + 3a + 4a + 12}{a + 4}$

$= \dfrac{a(a + 3) + 4(a + 3)}{(a + 4)} = \dfrac{(a + 3)(a + 4)}{(a + 4)} = (a + 3)$

(ix) $\dfrac{x^2 + 3x - 54}{x - 6} = \dfrac{x^2 + (9 - 6)x - 54}{x - 6} = \dfrac{x^2 + 9x - 6x - 54}{x - 6}$

$$= \frac{x(x+9)-6(x+9)}{x-6} = \frac{(x-6)(x+9)}{(x-6)} = (x+9)$$

(x) $\dfrac{12x^2+7xy-12y^2}{3x+4y} = \dfrac{12x^2+(16-9)xy-12y^2}{3x+4y}$

$$= \frac{12x^2+16xy-9xy-12y^2}{3x+4y}$$

$$= \frac{4x(3x+4y)-3y(3x+4y)}{(3x+4y)}$$

$$= \frac{(4x-3y)(3x+4y)}{(3x+4y)} = (4x-3y)$$

(xi) $\dfrac{x^6-8}{x^2-2} = \dfrac{(x^2)^3-(2)^3}{x^2-2}$

$$= \frac{(x^2-2)(x^4+2x^2+4)}{(x^2-2)} = x^4+2x^2+4$$

(xii)
$$\begin{array}{r|l} & 3x-5 \\ \hline 2x^2-x-6 & 6x^3-13x^2-13x+30 \\ & \underline{6x^3-3x^2-18x} \\ & \quad -10x^2+5x+30 \\ & \quad \underline{-10x^2+5x+30} \\ & \quad\quad 0 \end{array}$$

Quotient $= 3x-5$ and Remainder $= 0$

2. $\dfrac{14x^2(3x^2-19x+30)}{4x(3x-10)} = \dfrac{14x^2[3x^2-(9+10)x+30]}{4x(3x-10)}$

$$= \frac{14x^2}{4x}\,\frac{[3x^2-9x-10x+30]}{(3x-10)}$$

$$= \frac{7x}{2}\,\frac{[3x(x-3)-10(x-3)]}{(3x-10)}$$

$$= \frac{7x}{2}\,\frac{(3x-10)(x-3)}{(3x-10)} = \frac{7x(x-3)}{2}$$

3. $\dfrac{(21m^2-10m^3+m^4)}{(3-m)}$

$$= \frac{m^2(21-10m+m^2)}{(3-m)} = \frac{m^2(21-7m-3m+m^2)}{(3-m)}$$

$$= \frac{m^2[7(3-m) - m(3-m)]}{(3-m)} = \frac{m^2(7-m)(3-m)}{(3-m)}$$

$$= m^2(7-m)$$

4. $$\frac{48(2y^4 - 36y^2 + 162)}{4(y-3)^2} = \frac{48 \times 2}{4}\left(\frac{y^4 - 18y + 81}{(y-3)^2}\right)$$

$$= \frac{24[(y^2)^2 - 2 \times 9 \times y + (9)^2]}{(y-3)^2}$$

$$= \frac{24(y^2 - 9)^2}{(y-3)^2} = \frac{24(y^2 - 9)(y^2 - 9)}{(y-3)^2}$$

$$= \frac{24(y+3)(y-3)(y+3)(y-3)}{(y-3)^2} = 24(y+3)^2$$

5. $$\frac{44(5x^2 - 20x - 8y + 2xy)}{(5x+2y)} = \frac{44[x(5x+2y) - 4(5x+2y)]}{(5x+2y)}$$

$$= \frac{44(x-4)(5x+2y)}{(5x+2y)} = 44(x-4)$$

6. $$\frac{(x-2)^2 + 9(x-2)}{(x+7)} = \frac{(x-2)[(x-2)+9]}{(x+7)} = \frac{(x-2)(x+7)}{(x+7)}$$

$$= (x-2)$$

7. $$\frac{(16c^2 - 4a^2 - 12ab - 9b^2)}{(2a+3b+4c)} = \frac{(4c)^2 - (2a+3b)^2}{(2a+3b+4c)}$$

$$(2a+3b+4c) = \frac{(4c - 2a - 3b)(4c + 2a + 3b)}{(2a+3b+4c)}$$

$$= (4c - 2a - 3b)$$

8. (i)

$$\begin{array}{r|l}
 & a^2 - 6a + 14 \\
\hline
a + 1 & a^3 - 5a^2 + 8a + 15 \\
 & \underline{-\,a^3 \pm a^2} \\
 & \quad -6a^2 + 8a \\
 & \quad \underline{-6a^2 - 6a} \\
 & \quad\quad 14a + 15 \\
 & \quad\quad \underline{14a + 14} \\
 & \quad\quad\quad 1
\end{array}$$

Quotient $= a^2 - 6a + 14$ and Remainder $= 1$

(ii)
$$
\begin{array}{r}
3x^3 + 15x^2 + 39x + 119 \\
x-3 \overline{)\,3x^4 + 6x^3 - 6x^2 + 2x - 7} \\
3x^4 - 9x^3 \\
\hline
15x^3 - 6x^2 \\
15x^3 - 45x^2 \\
\hline
39x^2 + 2x \\
39x^2 - 117x \\
\hline
119x - 7 \\
119x - 357 \\
\hline
350
\end{array}
$$

Quotient $= 3x^3 + 15x^2 + 39x + 119$ and Remainder $= 350$

(iii)
$$
\begin{array}{r}
2x - 3 \\
3x + 5 \overline{)\,6x^2 + x - 15} \\
6x^2 + 10x \\
\hline
-9x - 15 \\
-9x - 15 \\
\hline
0
\end{array}
$$

Quotient $= 2x - 3$ and Remainder $= 0$

9. Area of rectangle $= x^3 - 8x^2 + 7$

One side $= x - 1$

Length of adjacent side $= \dfrac{x^3 - 8x^2 + 7}{x - 1}$

$$
\begin{array}{r}
x^2 - 7x - 7 \\
x-1 \overline{)\,x^3 - 8x^2 + 7} \\
x^3 - x^2 \\
\hline
-7x^2 + 7 \\
-7x^2 + 7x \\
\hline
-7x + 7 \\
-7x + 7 \\
\hline
\times
\end{array}
$$

Therefore, length of adjacent side $= x^2 - 7x - 7$

10. Product of two number $= 16x^4 - 1$

One number $= 2x - 1 \Rightarrow$ Let other number $= A$

$$A \times (2x-1) = 16x^4 - 1 = (4x^2)^2 - 1^2$$

$$A = \frac{(2x)^4 - 1}{(2x-1)} = \frac{(4x^2-1)(4x^2+1)}{(2x-1)}$$

$$= \frac{[(2x)^2 - 1](4x^2+1)}{(2x-1)}$$

$$= \frac{(2x-1)(2x+1)(4x^2+1)}{(2x-1)}$$

$$= (2x+1)(4x^2+1)$$

$$= 8x^3 + 4x^2 + 2x + 1$$

11. $$\frac{x^6 - y^6}{(x-y)(x^2+xy+y^2)} = \frac{(x^3)^2 - (y^3)^2}{(x-y)(x^2+xy+y^2)}$$

$$= \frac{(x^3-y^3)(x^3+y^3)}{(x-y)(x^2+xy+y^2)}$$

$$= \frac{(x^3+y^3)(x-y)(x^2+xy+y^2)}{(x-y)(x^2+xy+y^2)}$$

$$= (x^3+y^3)$$

❑

7. Linear Equations in One Variable

Exercise 7.1

1. (i) $\frac{2y+6}{y+4} = 1 \Rightarrow 2y + 6 = (y+4) \times 1$

$2y + 6 = y + 4 \Rightarrow 2y - y = 4 - 6 \Rightarrow y = -2$

(ii) $\frac{3x+5}{2x+7} = 4 \Rightarrow (3x+5) = 4(2x+7)$

$3x + 5 = 8x + 28 \Rightarrow -28 + 5 = 8x - 3x$

$5x = -23 \Rightarrow x = \frac{-23}{5}$

(iii) $\frac{2x+1}{3x-2}=\frac{5}{9} \Rightarrow (2x+1)9=5(3x-2)$

$18x+9=15x-10 \Rightarrow 18x-15x=-10-9$

$3x=-19 \Rightarrow x=\frac{-19}{3}$

(iv) $\frac{5z-3}{2z}=\frac{8}{9} \Rightarrow (5z-3)9=8(2z)$

$45z-27=16z \Rightarrow 45z-16z=27$

$29z=27 \Rightarrow z=\frac{27}{29}$

(v) $\frac{1-9y}{19-3y}=\frac{5}{8} \Rightarrow 8(1-9y)=5(19-3y)$

$8-72y=95-15y \Rightarrow 8-95=-15y+72y$

$-87=57y \Rightarrow y=\frac{-29}{19}$

2. (i) $\frac{5y-3}{2y+1}=\frac{2}{5} \Rightarrow 5(5y-3)=2(2y+1)$

$25y-15=4y+2 \Rightarrow 25y-4y=15+2$

$21y=17 \Rightarrow y=\frac{17}{21}$

(ii) $\frac{0.4z-3}{1.5z+9}=\frac{-7}{5}$

$5(0.4z-3)=-7(1.5z+9)$

$2z-15=-10.5z-63$

$2z+10.5z=-63+15$

$2\times 12.5z=-48\times 2$

$25z=-96 \Rightarrow z=\frac{-96}{25}$

(iii) $\frac{3y+5}{3-2y}=\frac{5}{3}$

$3(3y+5)=5(3-2y)$

$9y+15=15-10y$

$9y+10y=15-15$

$19y=0 \Rightarrow y=0$

(iv) $\frac{2x}{3x+1}=-3$

$2x=-3(3x+1)$

$2x=-9x-3 \Rightarrow 2x+9x=-3$

$11x=-3 \Rightarrow x=\frac{-3}{11}$

(v) $$\frac{17(2-x)-5(x+12)}{1-7x}=8$$

$$17(2-x)-5(x+12)=8(1-7x)$$
$$34-17x-5x-60=8-56x$$
$$34-22x+56x=8+60$$
$$34x=68-34$$
$$34x=34 \quad \Rightarrow \quad x=1$$

(vi) $$\frac{y-(7-8y)}{9y-(3+4y)}=\frac{2}{3}$$

$$3[y-(7-8y)]=2[9y-(3+4y)]$$
$$3(y-7+8y)=2(9y-3-4y)$$
$$3(9y-7)=2(5y-3)$$
$$27y-21=10y-6$$
$$27y-10y=21-6$$
$$17y=15 \quad \Rightarrow \quad y=\frac{15}{17}$$

3. (i) $$\frac{5y-7}{3y}=2 \quad \Rightarrow \quad (5y-7)=3y\times 2$$

$$5y-7=6y \quad \Rightarrow \quad 6y-5y=-7$$
$$y=-7$$

(ii) $$\frac{2x-4}{3x+2}=\frac{-2}{3} \quad \Rightarrow \quad 3(2x-4)=-2(3x+2)$$

$$6x-12=-6x-4 \quad \Rightarrow \quad 6x+6x=-4+12$$
$$12x=8 \quad \Rightarrow \quad x=\frac{2}{3}$$

(iii) $$\frac{0.4x-3}{1.5x+9}=\frac{7}{5} \quad \Rightarrow \quad 5(0.4x-3)=7(1.5x+9)$$

$$2x-15=10.5x+63 \quad \Rightarrow \quad -15-63=10.5x-2x$$
$$2\times 8.5x=-78\times 2 \quad \Rightarrow \quad 17x=-156$$
$$x=\frac{-156}{17}$$

(iv) $$\frac{m-3}{m+4}=\frac{m+1}{m-2}$$

$$(m-3)(m-2)=(m+1)(m+4)$$
$$m^2-3m-2m+6=m^2+m+4m+4$$
$$m^2-5m+6=m^2+5m+4$$
$$6-4=5m+5m$$
$$10m=2 \quad \Rightarrow \quad m=\frac{1}{5}$$

(v) $\dfrac{x^2-9}{x^2+5}=\dfrac{-5}{9}$

$(x^2-9)\times 9=-5\times(x^2+5)$

$9x^2-81=-5x^2-25$

$9x^2+5x^2=81-25$

$14x^2=56 \quad\Rightarrow\quad 2x^2=8$

$x^2=4 \quad\Rightarrow\quad x=\sqrt{4} \quad\Rightarrow\quad x=2$

(vi) $\dfrac{x+3}{7}-\dfrac{2x-5}{3}=\dfrac{4x-2}{5}-4$

$\dfrac{3(x+3)-7(2x-5)}{21}=\dfrac{4x-2}{5}-4$

$\dfrac{3x+9-14x+35}{21}=\dfrac{4x-2}{5}-4$

$\dfrac{-11x+44}{21}=\dfrac{4x-2}{5}-4$

$5(-11x+44)=21[(4x-2)-20]$

$-55x+220=84x-42-420$

$84x+55x=220+42+420$

$139x=262+420$

$139x=682 \quad\Rightarrow\quad x=\dfrac{682}{139}$

4. (i) $4x+5=13$

$4x=13-5=8 \quad\Rightarrow\quad x=2$

(ii) $5z-3=2z+4 \quad\Rightarrow\quad 5z-2z=4+3$

$3z=7 \quad\Rightarrow\quad z=\dfrac{7}{3}$

(iii) $7x+11=\dfrac{3}{4}+2x \quad\Rightarrow\quad 7x-2x=\dfrac{3}{4}-11$

$5x=\dfrac{3-11\times 4}{4} \quad\Rightarrow\quad 5x=\dfrac{3-44}{4}$

$5x=\dfrac{-41}{4} \quad\Rightarrow\quad x=\dfrac{-41}{20}$

(iv) $\dfrac{4y}{9}=\dfrac{3y+2}{4} \quad\Rightarrow\quad 4y\times 4=9(3y+2)$

$16y=27y+18 \quad\Rightarrow\quad 27y-16y=-18$

$11y=-18 \quad\Rightarrow\quad y=\dfrac{-18}{11}$

(v) $5(x+2)=2(x-1) \Rightarrow 5x+10=2x-2$

$5x-2x=-10-2 \Rightarrow 3x=-12$

$x=-4$

(vi) $\frac{4x}{5}-\frac{2x}{15}=\frac{3}{10} \Rightarrow \frac{1}{5}\left[4x-\frac{2x}{3}\right]=\frac{3}{10}$

$\frac{4x\times 3-2x}{3}=\frac{3\times 5}{10} \Rightarrow \frac{12x-2x}{3}=\frac{3}{2}$

$\frac{10x}{3}=\frac{3}{2} \Rightarrow x=\frac{9}{20}$

Revision Exercise

1. (i) $\frac{3-2x}{2x+5}=\frac{-3}{11} \Rightarrow 11(3-2x)=-3(2x+5)$

$33-22x=-6x-15 \Rightarrow 22x-6x=33+15$

$16x=48 \Rightarrow x=3$

(ii) $\frac{5p+2}{8-2p}=\frac{7}{6} \Rightarrow 6(5p+2)=7(8-2p)$

$30p+12=56-14p \Rightarrow 30p+14p=56-12$

$44p=44 \Rightarrow p=1$

(iii) $\frac{5}{x}=\frac{7}{x-4} \Rightarrow 5(x-4)=7(x)$

$5x-20=7x \Rightarrow 7x-5x=-20$

$2x=-20 \Rightarrow x=-10$

(iv) $\frac{4}{2x+3}=\frac{5}{x+4} \Rightarrow 4(x+4)=5(2x+3)$

$4x+16=10x+15 \Rightarrow 10x-4x=16-15$

$6x=1 \Rightarrow x=\frac{1}{6}$

(v) $\frac{2x-3}{2x-1}=\frac{3x-1}{3x+1}$

$(2x-3)(3x+1)=(3x-1)(2x-1)$

$6x^2+2x-9x-3=6x^2-3x-2x+1$

$-7x-3=-5x+1 \Rightarrow 7x-5x=-4$

$2x=-4 \Rightarrow x=-2$

(vi) $\frac{2y+3}{3y+2}=\frac{4y+5}{6y+7}$

$(2y+3)(6y+7)=(4y+5)(3y+2)$

$$12y^2 + 14y + 18y + 21 = 12y^2 + 8y + 15y + 10$$
$$32y + 21 = 23y + 10$$
$$32y - 23y = 10 - 21$$
$$9y = -11 \quad \Rightarrow \quad y = \frac{-11}{9}$$

2. Given : Two number ratio $= 5 : 3$
Number $= 5x, 3x$
Difference of number $= 28$
$5x - 3x = 28 \Rightarrow 2x = 28 \Rightarrow x = 14$
Therefore, number are $5 \times 14 = 70$,
$3 \times 14 = 42$

3. Let three consecutive multiples of 4 be,
$x, x + 4, x + 8$
According to condition :
$x + x + 4 + x + 8 = 144 \quad \Rightarrow \quad 3x = 144 - 12$
$3x = 132 \quad \Rightarrow \quad x = 44$
The number are $x = 44$
$x + 4 = 44 + 4 = 48$
$x + 8 = 44 + 8 = 52$
Hence, required number are 44, 48 and 52.

4. Let age of Lipika $= x$
Age of Lipika's mother $= 5x$
According to question,
$(x + 5) = \frac{1}{3}(5x + 5) \quad \Rightarrow \quad 3(x + 5) = (5x + 5)$
$3x + 15 = 5x + 5 \quad \Rightarrow \quad 5x - 3x = 15 - 5$
$2x = 10 \quad \Rightarrow \quad x = 5$
Hence, age of Lipika $= 5$ year
Age of Lipika's mother $= 5 \times 5 = 25$ year

5. Let age of Naman $= x$
Age of Rohit $= x + 15$
According to question,
$8(x - 3) = (x + 15 + 3) \quad \Rightarrow \quad 8x - 24 = x + 18$
$8x - x = 18 + 24 \quad \Rightarrow \quad 7x = 42 \quad \Rightarrow \quad x = 6$
Present age 6, 21 year.

6. Let number x and y.
According to given question,
$x - y = 3 \qquad \ldots(i)$
Then two digit number $= 10x + y$
Reverse number $= 10y + x$
Sum of number $= 143$

$$10x + y + 10y + x = 143$$
$$11x + 11y = 143$$
$$x + y = 13 \quad \ldots(ii)$$

Taking eq. (i) and (ii),

$$x - y = 3$$
$$x + y = 13$$
$$2x = 16$$

$\Rightarrow \quad x = 8$ and $y = 5$

The number is 58 or 85.

7. Let unit place = x and tens place = y

According to question,

$$x + y = 11 \quad \ldots(i)$$
$$\text{Two digit number} = 10y + x$$
$$\text{Reverse number} = 10x + y$$
$$10x + y - (10y + x) = 63$$
$$9x - 9y = 63$$
$$x - y = 7 \quad \ldots(ii)$$

Taking eq. (i) and (ii),

$$x + y = 11$$
$$x - y = 7$$
$$2x = 18$$

$\Rightarrow \quad x = 9$ and $y = 2$

Thus, two digit number 29.

8. Given : Riya's age $= 4 \times$ Sandeep's age

Let Sandeep's age $= x$

Riya's age $= 4x$

According to question,

$$4x + 4 = 2(x + 4)$$
$$4x + 4 = 2x + 8$$
$$4x - 2x = 8 - 4 \Rightarrow 2x = 4 \Rightarrow x = 2$$

Thus, Riya's age $= 4 \times 2 = 8$ year

Sandeep's age $= 2$ year.

9. Let unit place number = x

tens place number = y

According to question,

$$y = 3x \quad \ldots(i)$$
$$10x + y + 10y + x = 88$$
$$11x + 11y = 88$$
$$x + y = 8 \quad \ldots(ii)$$

Taking eq. (i) and (ii),

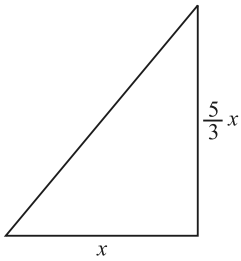
$$x = 2 \quad \text{and} \quad y = 6$$

Thus, number is 62 or 26.

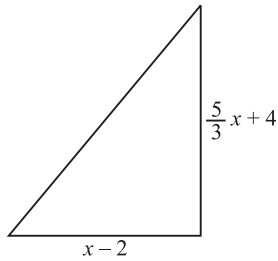
10. Let the base of triangle be x.

Altitude of triangle be $\frac{5}{3}x$

Case I



Case II



$$\text{Area of triangle} = \frac{1}{2} \text{base} \times \text{altitude}$$

$$\frac{1}{2} \times \frac{5}{3}x \times x = \frac{1}{2} \times (x-2)\left(\frac{5}{3}x + 4\right)$$

$$\frac{1}{2} \times x \times \frac{5x}{3} = \frac{1}{2}(x-2)\left(\frac{5}{3}x + 4\right)$$

$$\frac{5x^2}{3} = (x-2)\left(\frac{5x + 4 \times 3}{3}\right)$$

$$5x^2 = (x-2)(5x + 12)$$

$$5x^2 = 5x^2 + 12x - 10x - 24$$

$$2x = 24 \quad \Rightarrow \quad x = 12 \text{ cm}$$

$$\text{Thus, base of triangle} = 12 \text{ cm}$$

$$\text{Altitude of triangle} = 12 \times \frac{5}{3} = 20 \text{ cm.}$$

11. Let the breadth of rectangle $= x$

length of rectangle $= x + 4$

Case I

Case II

Now, according to question,

$$\text{Area of rectangle} = l \times b$$

$$x(x + 4) + 81 = (x + 3)(x + 7)$$

$$x^2 + 4x + 81 = x^2 + 3x + 7x + 21$$
$$10x - 4x = 81 - 21$$
$$6x = 60 \quad \Rightarrow \quad x = 10\,\text{cm}.$$

Thus, breadth of rectangle $= 10\,\text{cm}$

Length of rectangle $= 10 + 4 = 14$ cm.

12. Let the volume of iron piece be $= x$

Volume of copper piece be $= 2x$

According to question,

$$x \times 7.8 + 2x \times 8.9 = 1280$$
$$x \times 7.8 + 17.8x = 1280$$
$$25.6x = 1280$$
$$x = \frac{12800}{256} \quad \Rightarrow \quad x = 50$$

Thus, volume of iron piece $= 50\ \text{cm}^3$

Volume of copper piece $= 2 \times 50 = 100\ \text{cm}^3$.

13. Let the land which can be ploughed in one day be x.

According to question,

$$14 \times x = (x + 20) \times 10$$
$$14x = 10x + 200$$
$$14x - 10x = 200$$
$$4x = 200$$
$$x = 50\,\text{hectares}$$

Area of field $= 14 \times 50 = 700\,\text{hectares}$.

14. Let supplementary angle,

$$\angle A + \angle B = 180°$$

According to question,

$$x + x + 50° = 180°$$
$$2x + 50° = 180°$$
$$2x = 180° - 50° = 130°$$
$$x = 65°$$

Thus, angle is $65°, 65° + 50° = 115°$.

15. Let three angle $= 5x, 6x, 7x$

Sum of angle of triangle $= 180°$

$$5x + 6x + 7x = 180°$$
$$18x = 180° \quad \Rightarrow \quad x = 10°.$$

Thus, angle of triangle is

$$5 \times 10° = 50°$$
$$6 \times 10° = 60°$$
$$7 \times 10° = 70°.$$

❑

8. Ratios and Percentages

Exercise 8.1

1. (i) First convert both rupees in same unit :

So, $5 \times 100 = 500$ paise

Thus, required ratio $500 : 50 = 10 : 1$

(ii) First convert both distance in same unit :

So, $10 \times 1000 = 10{,}000$ m

Thus, required ratio $= 5 : 10{,}000 = 1 : 2{,}000$

(iii) First convert both time in same unit :

So, $30 \times 24 = 720$ hours

Thus, required ratio $= 720 : 36 = 20 : 1$

(iv) First convert weight in same unit :

So, $15 \times 1000 = 15{,}000$ g

Thus, required ratio $= 15{,}000 : 210 = 500 : 7$

2. (i) ₹ $1 = 1 \times 100 = 100$ paise

$$\text{Required percentage} = \left(\frac{30}{100} \times 100\right) = 30\%$$

(ii) $1 \text{ km} = 1 \times 1000 = 1000$ m

$$\text{Required percentage} = \frac{45}{1000} \times 100 = 4.5\%$$

(iii) $1 \text{ kg} = 1 \times 1000 = 1000$ g

$$\text{Required percentage} = \frac{1000}{800} \times 100 = 125\%$$

3. (i) $1 \text{ hour} = 1 \times 60 = 60$ minutes

$$\text{Required percentage} = \frac{45}{60} \times 100 = 75\%$$

(ii) $1 \text{ year} = 1 \times 12 = 12$ months

$$\text{Required percentage} = \frac{4}{12} \times 100 = 33\frac{1}{3}\%$$

(iii) $5 \text{ m} = 5 \times 100 = 500$ cm

$$\text{Required percentage} = \frac{335}{500} \times 100 = 67\%$$

(iv) $$\text{Required percentage} = \frac{60}{360} \times 100 = \frac{100}{6} = 16\frac{2}{3}\%$$

4. Given, Total students $= 36$, Passing students $= 30$

(i) $$\text{Fraction of students passed} = \frac{30}{36} = \frac{5}{6}$$

(ii) Percentage of students passed $= \frac{30}{36} \times 100 = \frac{500}{6} = 83\frac{1}{3}\%$

5. (i) $2\frac{1}{2}\% = \frac{5}{2}\%$

Fraction of percentage $= \frac{5}{2} \times \frac{1}{100} = \frac{1}{40}$

(ii) $16\frac{2}{3}\% = \frac{50}{3}\%$

Fraction of percentage $= \frac{50}{3} \times \frac{1}{100} = \frac{1}{6}$

6. (i) Percentage of fraction $= \frac{9}{20} \times 100 = 45\%$

(ii) Percentage of fraction $= 1\frac{1}{4} = \frac{5}{4} \times 100 = 125\%$

7. (i) Fraction as decimal $= \frac{3}{4} = 0.75$

Fraction as percentage $= \frac{3}{4} \times 100 = 75\%$

(ii) Fraction as decimal $= \frac{5}{8} = 0.625$

Fraction as percentage $= \frac{5}{8} \times 100 = 62.5\%$

8. (i) Fraction as decimal $= \frac{5}{6} = 0.8333$

Fraction as percentage $= \frac{5}{6} \times 100 = 83.33\%$

(ii) Fraction as decimal $= \frac{4}{7} = 0.5714$

Fraction as percentage $= \frac{4}{7} \times 100 = 57.14\%$

Word Problems

1. New number $= \left(1 - \frac{37\frac{1}{2}}{100}\right) \times 216 = \left(\frac{100 - 37\frac{1}{2}}{100}\right) \times 216$

$= \frac{62\frac{1}{2}}{100} \times 216 = \frac{125}{200} \times 216 = \frac{5}{8} \times 216$

$= 5 \times 27 = 135$

Result $= 135$

2. $\text{New number} = \left(1 + \frac{125}{100}\right) \times 28 = \left(\frac{100 + 125}{100}\right) \times 28$

$$= \frac{225}{100} \times 28 = \frac{9}{4} \times 28 = 9 \times 7 = 63$$

$$\text{Result} = 63$$

3. Let number $= x$

According to question,

$$161 = \left(1 + \frac{15}{100}\right) \times x \quad \Rightarrow \quad 161 = \left(\frac{100 + 15}{100}\right) x$$

$$161 = \frac{115}{100} \times x \quad \Rightarrow \quad 161 = \frac{23}{20} x$$

$$7 = \frac{1}{20} \times x \quad \Rightarrow \quad x = 140$$

Thus, number $= 140$

4. Let number $= x$

According to question,

$$192 = \left(1 - \frac{20}{100}\right) \times x \quad \Rightarrow \quad 192 = \frac{80}{100} \times x$$

$$24 = \frac{1}{10} \times x \quad \Rightarrow \quad x = 24 \times 10 \quad \Rightarrow \quad x = 240$$

5. Let the price today $= ₹\ x$

According to question,

$$x = \left(1 + \frac{36}{100}\right) \times 19{,}00{,}000 = \frac{136}{100} \times 19{,}00{,}000$$

$$= 136 \times 19 \times 1000 = ₹\ 25{,}84{,}000$$

6. Let the rent money $= ₹\ x$

$$x = \frac{26}{100} \times 880 = 26 \times 8.8 = ₹\ 228.80$$

7. Given : $\text{Total candidates} = 8000$

$$\text{Boys} = \frac{8000 \times 60}{100} = 4800$$

$$\text{Girls} = \frac{8000 \times 40}{100} = 3200$$

$$\text{Fail Boys} = \frac{4800 \times 20}{100} = 960$$

$$\text{Fail Girls} = \frac{3200 \times 10}{100} = 320$$

$$\text{Total failed candidates} = 960 + 320 = 1280$$

8. Let original price = ₹ x

16% Increasing price = ₹ 1479

$$\frac{116}{100} \times x = 1479 \quad \Rightarrow \quad x = 12.75 \times 100$$

$$x = ₹\ 1275$$

9. Let earlier weight = x

15% reduce weight = 59.5

$$x \times \left(1 - \frac{15}{100}\right) = 59.5 \quad \Rightarrow \quad \frac{x \times 85}{100} = 59.5$$

$$x = 0.7 \times 100 \quad \Rightarrow \quad x = 70\,\text{kg}$$

10. Given houses sold in 2005 = 4260

Let 20% more houses sold in 2006 are x

So, $$x = \left(1 + \frac{20}{100}\right) \times 4260 = \frac{120}{100} \times 4260 = 5112$$

11. Let value of car after two year = x

$$x = 4,20,000\left(1 - \frac{20}{100}\right)\left(1 - \frac{10}{100}\right)$$

$$= 4,20,000\left(\frac{80}{100}\right)\left(\frac{90}{100}\right) = 42 \times 80 \times 90 = ₹\ 3,02,400$$

12. In 2005, a train carried 8% more passengers than in 2004.
In 2006, a train carried 8% more passenger than in 2005.
Let total percentage = x

We know that $$x\% = a + b + \frac{ab}{100} = 8 + 8 + \frac{8 \times 8}{100}$$

$$= 16 + \frac{64}{100} = 16 + 0.64 = 16.64\%$$

Revision Exercise

1. (i) $$x = 15\frac{1}{2}\% \times 640 = \frac{31}{2} \times \frac{640}{100} = \frac{9920}{100} = ₹\ 99.20$$

(ii) $$x = 6.5\% \times 5000 = \frac{65}{100 \times 10} \times 5000 = 325\ \text{persons}$$

(iii) $$x = 80\% \times 4.5 = \frac{80}{100} \times 4.5 = \frac{8}{10} \times \frac{45}{10} = \frac{360}{100} = 3.60\ \text{kg}$$

(iv) $$x = 125\% \times 50 = \frac{125}{100} \times 50 = \frac{125}{2} = 62.5\,\text{m}$$

(v) $$x = 30.6\% \times 300\,\text{kg} = \frac{306}{10} \times \frac{1}{100} \times 300 = \frac{918}{10} = 91.8\,\text{kg}$$

(vi) $$x = 60.5\% \times 8\,\text{hours} = \frac{605}{10} \times \frac{1}{100} \times 8 = \frac{4840}{1000} = 4.84\ \text{hours}$$

2. (i) $17 : 20 \quad \Rightarrow \quad x = \frac{17}{20} \times 100 \quad \Rightarrow \quad x = 17 \times 5 = 85\%$

(ii) $13 : 18 \quad \Rightarrow \quad x = \frac{13}{18} \times 100 = \frac{1300}{18} = \frac{650}{9} = 72\frac{2}{9}\%$

3. (i) $x = \frac{2}{100} = 0.02$ (ii) $x = \frac{3\frac{1}{4}}{100} = \frac{13}{400} = 0.0325$

4. (i) $x = \frac{27}{100} \times ₹\ 50 = \frac{27}{2} = ₹\ 13.50$

(ii) $x = 6\frac{1}{4}\% \times 25\,\text{kg} = \frac{25}{4} \times \frac{1}{100} \times 25 = \frac{25}{16} = 1\frac{9}{16}\,\text{kg}$

(iii) $x = \frac{300\,\text{g}}{2\,\text{kg}} \times 100 = \frac{300}{2 \times 1000} \times 100 = \frac{30}{2} = 15\%$

(iv) $x = \frac{7.5}{6} \times 100 = \frac{750}{6} = 125\%$

5. (i) $x = \frac{65}{50} \times 100 = 65 \times 2 = 130\%$ (ii) $x = \frac{4}{9} \times 100 = \frac{400}{9} = 44\frac{4}{9}\%$

6. (i) Let number $= x$

$16\frac{2}{3}\% \times x = 25 \quad \Rightarrow \quad \frac{50}{3} \times \frac{1}{100} \times x = 25$

$\therefore \quad x = 25 \times 6 = 150$

(ii) Let number $= x \quad \Rightarrow \quad 13.25\% \times x = 159$

$\frac{1325}{100} \times \frac{1}{100} x = 159$

$x = \frac{159}{1325} \times 100 \times 100$

$= 0.12 \times 100 \times 100 = 1200$

(iii) Let result $= x$

$x = \left(1 + \frac{30}{100}\right) \times 60 \quad \Rightarrow \quad x = \frac{130}{100} \times 60 = 13 \times 6 = 78$

(iv) Let result $= x$

$x = \left(1 - \frac{10}{100}\right) \times 750 \quad \Rightarrow \quad x = \frac{90}{100} \times 750 = 9 \times 75 = 675$

(v) Let number $= x$

$\left(1 + \frac{15}{100}\right) \times x = 299 \quad \Rightarrow \quad \frac{115}{100} \times x = 299$

$x = \frac{299 \times 20}{23} \quad \Rightarrow \quad x = 13 \times 20 = 260$

(vi) Let number = x

$$\left(1-\frac{18}{100}\right)\times x = 697 \quad \Rightarrow \quad \frac{82}{100}\times x = 697$$

$$x = \frac{697}{82}\times 100 = 8.5\times 100 = 850$$

7. Let income = ₹ x

After spending 88% of income = ₹ 2160

$$\left(1-\frac{88}{100}\right)\times x = ₹\ 2160 \quad \Rightarrow \quad \frac{12}{100}\times x = 2160$$

$$x = 180\times 100 = ₹\ 18{,}000$$

8. Let the bill = ₹ x

10% reduced bill ₹ x = ₹ 58.50

$$\left(1-\frac{10}{100}\right)\times x = 58.50 \quad \Rightarrow \quad \frac{90}{100}\times x = 58.50$$

$$x = 6.5\times 10 = ₹\ 65$$

9. Let new height = x m

$$x = \left(1+\frac{12.5}{100}\right)\times 4.8 = \frac{112.5}{100}\times 4.8 = \frac{540}{100} = 5.4\text{ m}$$

10. Let decrease percentage = x

$$x = \frac{500}{10000}\times 100 = 5\%$$

11. Let new salary = ₹ x

$$x = \left(1+\frac{10}{100}\right)\times 50000 = \frac{110}{100}\times 50000 = ₹\ 55{,}000$$

12. Let total strength = x $\Rightarrow$ 60% boys = 480

$$\frac{60}{100}x = 480 \quad \Rightarrow \quad x = 80\times 10 = 800$$

13. Let money = ₹ x

After spending 60% of money = ₹ 8000

$$\left(1-\frac{60}{100}\right)\times x = ₹\ 8000 \quad \Rightarrow \quad \frac{40}{100}\times x = 8000$$

$$x = 200\times 100 = ₹\ 20{,}000$$

14. Let new prize = ₹ x

$$x = \left(1+\frac{12}{100}\right)\times 60000 = \frac{112}{100}\times 60000 = ₹\ 67{,}200$$

15. Let week students = x

Given : 30% of 50 students are goods in maths

$$x = 70\%\times 50$$

70% week,

$$x = \frac{70}{100}\times 50 = 35$$

16. Let weight of tin $= x$

$$x = 90 \times \frac{45}{55} = 73.63 \text{ kg}$$

17. Let percentage $= x$

$$x = \left(\frac{1080 - 960}{960}\right) \times 100 = \frac{120}{960} \times 100 = 12.5\%.$$

18. Let total number of voters $= x$

Loser polled $= 42\%$; Winner polled $= 58\%$

According to question,

$$58\%\ x - 42\% \text{ of } x = 14400$$

$$16\% \text{ of } x = 14400$$

$$\frac{x}{100} \times 16 = 14400 \quad \Rightarrow \quad x = 900 \times 100 = 90{,}000.$$

Percentage of voters did not vote $= x$

$$x = \left(\frac{100{,}000 - 90{,}000}{100{,}000}\right) \times 100 = \frac{10{,}000}{100{,}000} \times 100 = 10\%$$

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9. Profit and Loss

Exercise 9.1

1. Given : SP $= ₹\ 3240$; Profit $\% = 8$; CP $= ?$

$$\text{SP} = \text{CP} \times \left(\frac{100 + P}{100}\right)$$

We know that, Let CP $= x$

$$3240 = \text{CP} \times \left(\frac{100 + 8}{100}\right)$$

$$3240 = \text{CP} \times \frac{108}{100}$$

$$\text{CP} = \frac{3240}{108} \times 100 = 30 \times 100 = ₹\ 3000$$

2. Given : Cost of pens $= ₹\ 200$

Cost of pencils $= ₹\ 50$

Total amount $= 200 + 50 = 250$

$$10\% \text{ gain in pens} = \frac{200 \times 110}{100} = 220$$

$$20\% \text{ loss in pencils} = 50 \times \left(1 - \frac{20}{100}\right) = \frac{50 \times 80}{100} = 40$$

Total amount = 220 + 40 = 260

$$P\% = \left(\frac{260 - 250}{250}\right) \times 100 = \frac{10}{250} \times 100 = 4\%$$

3. Given, SP = ₹ 1320; L% = 12%; CP = ?

$$SP = CP \times \left(\frac{100 - L}{100}\right) \quad \Rightarrow \quad 1320 = CP \times \frac{(100 - 12)}{100}$$

$$1320 = \frac{CP \times 88}{100} \quad \Rightarrow \quad CP = \frac{1320}{88} \times 100 = ₹\ 1500.$$

4. Let $CP = x$

$$x = 1200 \times \left(\frac{110}{100}\right) \times \left(\frac{112}{100}\right)$$

$$= \frac{1200 \times 110 \times 112}{100 \times 100} = ₹\ 1478.40.$$

5. Cost of 20 quires = 250,

$$\text{Cost of 1 quires} = \frac{250}{20} = ₹\ 12.5$$

Gain = 20%, CP = ₹ 12.5

$$SP = 12.5 \times \frac{(100 + 20)}{100} = \frac{120 \times 12.5}{100} = ₹\ 15$$

6. Cost of book = ₹ 450; Spent = ₹ 20

Total CP = 450 + 20 = ₹ 470; P = 15%

$$SP = 470\left(1 + \frac{15}{100}\right) = 470 \times \frac{115}{100} = ₹\ 540.5$$

$$\text{SP of one book} = \frac{540.5}{10} = ₹\ 54.05.$$

7. Selling price of Sofa = ₹ 1750; L = 30%; CP = ?

$$SP = CP \times \left(1 - \frac{30}{100}\right)$$

$$1750 = CP \times \frac{70}{100} \quad \Rightarrow \quad CP = ₹\ 2500$$

Then, $$SP = 2500 \times \left(1 + \frac{20}{100}\right) = 2500 \times \frac{120}{100} = ₹\ 3,000$$

8. Selling price of fan = ₹ 360; L = 10%

If selling price of fan = ₹ 460

Loss % or profit % = ?

We know that,

$$SP = CP\,\frac{(100 - L)}{100}$$

$$CP = \frac{SP \times 100}{(100 - 10)} = \frac{360 \times 100}{90} \quad \ldots(i)$$

$$CP = \frac{460 \times 100}{(100 \pm x)} \quad \ldots(ii)$$

Eq. (i) and (ii),

$$\frac{460 \times 100}{(100 \pm x)} = \frac{360 \times 100}{90}$$

$$46 \times 90 = 36(100 \pm x)$$

$$4140 = 3600 + 36x$$

$$36x = 540 \quad \Rightarrow \quad x\% = 15\%$$

Thus, 15% profit.

9. By selling 180 oranges, the man lost the SP of 20 oranges.

$$\text{Loss percentage} = \frac{20}{(180 + 20)} \times 100 = \frac{20 \times 100}{200} = 10\%$$

$$\text{SP of one orange} = ₹\ \frac{270}{200} = ₹\ 1.35$$

10. SP of 4 oranges = CP of 3 oranges

$$SP \times 4 = CP \times 3$$

$$\frac{SP}{CP} = \frac{3}{4}$$

$$\text{We know that } L\% = \left(\frac{CP - SP}{CP}\right) \times 100$$

$$= \left(\frac{4 - 3}{4}\right) \times 100 = \frac{1}{4} \times 100 = 25\%$$

11. CP of article = ₹ 2000; SP of article = ₹ 2500

Let Profit percentage = ?

$$P\% = \left(\frac{SP - CP}{CP}\right) \times 100$$

$$= \left(\frac{2500 - 2000}{2000}\right) \times 100 = \frac{500}{2000} \times 100 = 25\%$$

12. CP of scooter = ₹ 3000; Repairing amounts = ₹ 2000

Total CP = ₹ 5000; SP = ₹ 8000

$$\text{Profit } \% = \left(\frac{8000 - 5000}{5000}\right) \times 100 = \frac{3000}{5000} \times 100 = 60\%$$

13. CP of 8 articles = SP of 10 articles

$$CP \times 8 = SP \times 10 \quad \Rightarrow \quad \frac{CP}{SP} = \frac{10}{8}$$

$$L\% = \left(\frac{CP - SP}{CP}\right) \times 100 = \left(\frac{10 - 8}{10}\right) \times 100$$

$$= \frac{2}{10} \times 100 = 20\%$$

14. CP of 20 articles = SP of 12 articles

$$CP \times 20 = SP \times 12 \quad \Rightarrow \quad \frac{CP}{SP} = \frac{12}{20}$$

$$Profit\% = \left(\frac{SP - CP}{CP}\right) \times 100$$

$$= \left(\frac{20 - 12}{12}\right) \times 100 = \frac{8}{12} \times 100 = 66.67\%$$

15. SP of LED = ₹ 18000; L% = 10%

let SP of LED = x $\Rightarrow$ P% = 30%

We know that

$$\frac{SP \times 100}{(100 - L)} = \frac{SP \times 100}{(100 + P)}$$

$$\frac{18{,}000 \times 100}{(100 - 10)} = \frac{x \times 100}{(100 + 30)}$$

$$\frac{18000}{90} = \frac{x}{130}$$

$$200 \times 130 = x \quad \Rightarrow \quad x = 26{,}000$$

$\therefore$ SP = ₹ 26,000

16. CP_1 of tea = 40×150 = ₹ 6000

CP_2 of tea = 60×100 = ₹ 6000

Total CP = 6000 + 6000 = ₹ 12000

SP of tea = $(40 + 60) \times 130$ = ₹ 13000

$$Profit\% = \frac{(13000 - 12000)}{12000} \times 100$$

$$= \frac{1000}{12000} \times 100 = \frac{100}{12} = 8.33\%$$

17. SP of laptop = x; Profit % = 20%

SP of laptop = x + 1000; Profit% = 25%

We know that

$$SP = CP\,\frac{(100 + 20)}{100} \qquad \ldots(i)$$

$$SP + 1000 = \frac{CP\,(100 + 25)}{100} \qquad \ldots(ii)$$

Taking eq. (i) and (ii),

$$\frac{SP}{SP+1000} = \frac{CP\,(100+20)}{100} \times \frac{100}{CP\,(100+25)}$$

$$\frac{SP}{SP+1000} = \frac{120}{125} \quad \Rightarrow \quad \frac{SP}{SP+1000} = \frac{24}{25}$$

$$25\,SP = 24\,SP + 24000$$

$$SP = 24000$$

Put in eq. (i)

$$24000 = \frac{CP\,(120)}{100}$$

$$CP = 20 \times 1000 = 20{,}000$$

18. $\text{SP of fan} = ₹\,644;\ P\% = \frac{1}{6}\,CP$

We know that $SP = CP + P$

$$644 = CP + \frac{CP}{6} \quad \Rightarrow \quad 644 = \frac{6\,CP + CP}{6}$$

$$CP = \frac{6 \times 644}{7} = 92 \times 6 = ₹\,252$$

$$\text{Profit\%} = \left(\frac{SP - CP}{CP}\right) \times 100 = \left(\frac{644 - 552}{552}\right) \times 100$$

$$= \frac{92}{552} \times 100 = \frac{1}{6} \times 100 = 16.67\%$$

19. Case I. $CP = 8000 \times \frac{3}{4} = 600, \quad \text{Loss} = \frac{600 \times 10}{100} = 60$

$$\text{Loss} = CP - SP$$

$$60 = 600 - SP \quad \Rightarrow \quad SP = ₹\,540$$

Case II. $CP = 800 \times \frac{1}{4} = 200, \quad \text{Profit} = \frac{200 \times 10}{100} = 20$

$$SP = 200 + 20 = 220$$

$$\text{Total SP} = 540 + 220 = ₹\,760$$

$$\text{Total CP} = 600 + 200 = ₹\,800$$

$$\text{Loss} = CP - SP = 800 - 760 = ₹\,40$$

20. SP_1 of bed-sheet $= ₹\,48;\ \text{Loss} = 4\%$

$SP_2 = ?;\ \text{Profit} = 20\%$

We know that

$$\frac{48 \times 100}{96} = \frac{SP_2 \times 100}{120}$$

$$SP_2 = ₹\,60$$

Exercise 9.2

1. (i) MP = ₹ 85; Discount = 20%

We know that $SP = MP \times \left(\frac{100 - D}{100}\right) = 85 \times \frac{(100 - 20)}{100}$

$= \frac{85 \times 80}{100} = \frac{85 \times 4}{5} = 17 \times 4 = ₹\ 68$

(ii) MP = ₹ 990 ; Discount = 15%

$SP = \frac{990 \times (100 - 15)}{100}$

$= \frac{99}{10} \times 85 = \frac{99 \times 17}{2} = ₹\ 841.50$

2. (i) SP = ₹ 1860; Discount = 7%

We know that $SP = \frac{MP\,(100 - D)}{100}$

$1860 = \frac{MP\,(100 - 7)}{100}$

$1860 = \frac{MP\,(93)}{100} \quad \Rightarrow \quad MP = 2{,}000$

(ii) SP = ₹ 1056, Discount = 4%

$SP = MP \times \frac{(100 - D)}{100} = \frac{MP \times (100 - 4)}{100}$

$1056 = \frac{MP \times 96}{100} \quad \Rightarrow \quad MP = 11 \times 100 = ₹\ 1100$

3. (i) MP = ₹ 40, SP = ₹ 34

Discount = MP – SP = 40 – 34 = 6

$D\% = \frac{6}{40} \times 100 = 15\%$

(ii) MP = ₹ 12.5, SP = ₹ 10.5

Discount = MP – SP = 12.5 – 10.5 = ₹ 2

$D\% = \frac{2}{12.5} \times 100 = 2 \times 8 = 16\%$

Word Problems

1. MP of desert cooler = ₹ 8,000; Discount = 24%

$SP = \frac{MP \times (100 - D)}{100} = \frac{8{,}000 \times (100 - 24)}{100}$

$= 80 \times 76 = ₹\ 6080$

2. SP of almirah = ₹ 13,120; Discount = 18%

$SP = \frac{MP \times (100 - D)}{100} = \frac{MP\ (100 - 18)}{100}$

$$13120 = \frac{MP \times 82}{100}$$

$$MP = 160 \times 100 = ₹\ 16{,}000$$

3. Let CP of goods = ₹ 100; MP of goods = 100 + 30 = 130

Discount = 10%

$$SP = \frac{130 \times (100 - 10)}{100} = \frac{130 \times 90}{100} = 117$$

$$\text{Profit\%} = \left(\frac{SP - CP}{CP}\right) \times 100 = \frac{117 - 100}{100} \times 100 = 17\%$$

4. CP of suit = ₹ 16,000; Discount = 20%; Profit% = 30%

$$SP = \frac{16000 \times (130)}{100} \quad \ldots(i)$$

$$SP = \frac{MP \times 80}{100} \quad \ldots(ii)$$

Taking eqs. (i) and (ii),

$$16000 \times 130 = MP \times 80$$

$$MP = ₹\ 26{,}000$$

5. MP of bicycle = ₹ 5000; Discount = 10%; Profit = 40%

We know that,

$$SP = \frac{CP \times (100 + P)}{100} = \frac{CP \times (100 + 40)}{100} \quad \ldots(i)$$

$$SP = \frac{MP \times (100 - D)}{100} = \frac{5000 \times (100 - 10)}{100} \quad \ldots(ii)$$

Taking eqs. (i) and (ii),

$$\frac{CP \times 140}{100} = \frac{5000 \times 90}{100}$$

$$CP = \frac{5000 \times 9}{14} = \frac{45000}{14} = ₹\ 3214.29$$

6. Let price of an article = ₹ 100

$$\text{So, single discount} = 100\left(1 - \frac{40}{100}\right) = ₹\ 60$$

$$\text{Two successive discounts} = 100 \times \left(1 - \frac{20}{100}\right)\left(1 - \frac{20}{100}\right)$$

$$= 100 \times \frac{80 \times 80}{100 \times 100} = ₹\ 64$$

So, single discount is better.

7. SP = ₹ 54; Discount = 10%

$$SP = \frac{MP \times (100 - 10)}{100} = \frac{MP \times 90}{100}$$

$$54 = \frac{MP \times 90}{100} \quad \Rightarrow \quad MP = 6 \times 10 = ₹\ 60$$

8. SP of table $= 405 - 30 = 375$; Profit $= 25\%$

$$SP = \frac{CP \times (100 + 25)}{100} \quad \Rightarrow \quad 375 = \frac{CP \times 125}{100}$$

$$CP = \frac{375 \times 4}{5} = 75 \times 4 = ₹\ 300$$

9. Let cost price $= ₹\ 100$; MP $= 100 + 25 = ₹\ 125$

$$SP = \frac{MP \times (100 - 20)}{100} = \frac{125 \times 80}{100} = \frac{5}{4} \times 80 = ₹\ 100$$

Thus, no loss no gain.

10. Discount $= 12\%$; MP $= ₹\ 5400$; Profit $= 8\%$

$$SP = \frac{CP \times (100 + 8)}{100} \qquad \ldots(i)$$

$$SP = \frac{MP \times (100 - 12)}{100} \qquad \ldots(ii)$$

Taking eqs. (i) and (ii)

$$CP \times \frac{108}{100} = \frac{5400 \times 88}{100}$$

$$CP = \frac{5400}{108} \times 88 = 50 \times 88 = ₹\ 4400$$

11. $D_1 = 12\%, D_2 = 4\%$; MP $= ₹\ 800$

$$SP = \frac{800 \times (100 - 12)}{100} \times \frac{(100 - 4)}{100}$$

$$= \frac{800 \times 88}{100} \times \frac{96}{100} = \frac{8 \times 88 \times 96}{100}$$

$$= \frac{704 \times 96}{100} = \frac{67584}{100} = ₹\ 675.84$$

12. Discount $= 10\%$; Profit $= 8\%$

Let CP $= ₹\ 100$

$$\frac{100 \times (100 + 8)}{100} = ₹\ 108 = \frac{MP \times 90}{100}$$

$\because$ $SP = MP$

$$SP = \frac{108}{90} \times 100 = ₹\ 120$$

$$\text{Profit}\% = \left(\frac{SP - CP}{CP}\right) \times 100$$

$$= \left(\frac{120 - 100}{100}\right) \times 100 = \frac{20}{100} \times 100 = 20\%$$

Exercise 9.3

1. (i) The towels price = ₹ 50

5% ST added on purchase price = ₹ $50 + \frac{50 \times 5}{100} = 50 + 2.5 = 52.5$

$\therefore$ Buying price = ₹ 52.50

(ii) Price of 5 kg flour = ₹ $15 \times 5 =$ ₹ 75

5% ST added on purchase price = $75 + \frac{75 \times 5}{100} = 75 + 3.75 = 78.75$

$\therefore$ Buying price = ₹ 78.75

2. (i) Price of a TV = ₹ 12,000

5% GST added on purchase price = $12000 + \frac{5 \times 12000}{100}$

$\therefore$ Buying price = $12000 + 600 =$ ₹ 12600

(ii) Price of leather coat = ₹ 1800

5% GST added on purchase price = $1800 + \frac{1800 \times 5}{100}$

$\therefore$ Buying price = $1800 + 90 =$ ₹ 1890

(iii) Price of two bars of soup = $25 \times 2 =$ ₹ 50

5% GST added on purchase price = $50 + \frac{50 \times 5}{100}$

$= 50 + 2.5 =$ ₹ 52.5

$\therefore$ Buying price = ₹ 52.50

(iv) Price of an air cooler = ₹ 3300

5% GST added on the purchase price = $3300 + \frac{3300 \times 5}{100}$

$= 3300 + 165 =$ ₹ 3465

3. (i) Price of a shamppo bottle = ₹ 208

4% GST added on the purchase price = buying price

$$\left(\frac{104}{100}\right)x = 208$$

$$x = \frac{208 \times 100}{104} = 2 \times 100 = ₹\ 200$$

(ii) Price of bicycle = ₹ 3120

4% GST added on the purchase price = buying price

$$\left(\frac{104}{100}\right)x = 3120$$

$$x = 30 \times 100 = 3000$$

Original price x = ₹ 3,000

Word Problems

1. Price of a commodity $= ₹\ 2500$

Paid amount $= ₹\ 2700$

Sales Tax $= 2700 - 2500 = ₹\ 200$

$$\text{Rate of sales tax} = \frac{200}{2500} \times 100 = 8\%$$

5. Price of air-conditioner $= ₹\ 33{,}000$

Rate of tax $= 10\%$, Let original price $= ₹\ x$

According to question,

$$\left(\frac{100+10}{100}\right)x = 33000$$

$$\frac{110}{100} \times x = 33000$$

$$x = \frac{33000 \times 100}{110} = ₹\ 30{,}000$$

Thus, original price is 30,000

Revision Exercise

1. (i) $CP = ₹\ 400, SP = ₹\ 468$

$$\text{Profit\%} = \frac{SP - CP}{CP} \times 100 = \left(\frac{468-400}{400}\right) \times 100$$

$$= \frac{68}{400} \times 100 = \frac{68}{4} = 17\%$$

(ii) $CP = ₹\ 13600, SP = ₹\ 12104$

$$\text{Loss\%} = \left(\frac{CP - SP}{CP}\right) \times 100 = \left(\frac{13600-12104}{13600}\right) \times 100$$

$$= \frac{1496}{13600} \times 100 = 0.11 \times 100 = 11\%$$

2. (i) $MP = ₹\ 780, SP = ₹\ 721.50$

$\text{Discount} = 780 - 721.5 = 58.5$

$$D\% = \frac{58.5}{780} \times 100 = 0.075 \times 100 = 7.5\%$$

(ii) Advertised price $= ₹\ 28500, SP = ₹\ 24510$

Discount $= 28500 - 24510 = ₹\ 3990$

$$D\% = \frac{D}{MP} \times 100 = \frac{3990}{28500} \times 100$$

$$= 0.14 \times 100 = 14\%$$

3. $SP = ₹\ 1636.25$; Profit $= ₹\ 96.25$

$CP = SP - P = 1636.25 - 96.25 = ₹\ 1540$

$$\text{Profit\%} = \frac{96.25}{1540} \times 100 = 0.0625 \times 100 = 6.25\%$$

4. $\text{SP} = ₹\ 770, \text{Loss} = ₹\ 110$

$\text{CP} = \text{SP} + \text{Loss} = 770 + 110 = ₹\ 880$

$$\text{Loss\%} = \frac{\text{Loss}}{\text{CP}} \times 100$$

Thus, $$\text{Loss\%} = \frac{110}{880} \times 100 = \frac{100}{8} = 12.5\%$$

5. CP of 25 dozen eggs $= 9.6 \times 25 = ₹\ 240$

Total eggs $= 12 \times 25 = 300$

30 eggs were broken, them

Left eggs $= 300 - 30 = 270$

SP of eggs $= 270 \times ₹\ 1 = ₹\ 270$

$$\text{Profit\%} = \left(\frac{270 - 240}{240}\right) \times 100$$

$$= \frac{30}{240} \times 100 = \frac{1}{8} \times 100 = 12.5\%$$

6. Cost price of an article $= ₹\ 20{,}000$

₹ 1400 are the repairing charges.

Total CP $= 20{,}000 + 1400 = ₹\ 21400$

Profit % $= 20\%$

$$\text{SP} = \frac{\text{CP} \times (100 + 20)}{100}$$

$$= \frac{21400 \times 120}{100} = ₹\ 25{,}680$$

Thus, SP of the article is ₹ 25,680.

7. CP of 200 bicycles $= 1200 \times 200 = ₹\ 2{,}40{,}000$

₹ 30 per bicycles transportation charges $= 200 \times 30 = 6000$

Advertising $= 4{,}000$

Total CP of bicycles $= 2{,}40{,}000 + 6{,}000 + 4000 = 2{,}50{,}000$

SP of bicycles $= 200 \times 1350 = 2{,}70{,}000$

Profit $= \text{SP} - \text{CP}$

$= 2{,}70{,}000 - 2{,}50{,}000 = 20{,}000$

$$\text{Profit \%} = \frac{20{,}000}{2{,}50{,}000} \times 100 = 0.08 \times 100 = 8\%$$

8. MP of note book $= ₹\ 30$

Price of 1 dozen note books $= ₹\ 30 \times 12 = ₹\ 360$

Discount $= 15\%$

$$\text{SP} = \frac{\text{MP} \times (100 - \text{D})}{100} = \frac{360 \times (100 - 15)}{100}$$

$$= \frac{360 \times 85}{100} = ₹\ 306$$

Thus, SP of books is ₹ 306.

9. SP of fan $= ₹\ 728$; Discount $= 9\%$

$$SP = MP \times \left(\frac{100 - D}{100}\right)$$

$$728 = MP \times \left(\frac{100 - 9}{100}\right) = \frac{MP \times 91}{100}$$

$$= 8 \times 100 = ₹\ 800$$

Thus, MP of the fan is ₹ 800.

10. (i) CP of Calculator ₹ $= 650$, Discount $= 20\%$, Profit $= 20\%$

$$SP = 650 \times \frac{(100 + 20)}{100} = \frac{650 \times 120}{100} = ₹\ 780$$

(ii) $$SP = \frac{MP \times 80}{100} \quad \Rightarrow \quad 780 = \frac{MP \times 80}{100}$$

$$MP = 9.75 \times 100 = ₹\ 975$$

Thus, SP of calculator is ₹ 780 and MP is ₹ 975.

11. Rate of 12 for ₹ 10, Profit $= 20\%$, Rate of $1 = ₹\ \frac{10}{12}$

$$SP = CP \times \left(\frac{100 + 20}{100}\right) = \frac{10}{12} \times \frac{120}{100} = ₹\ \frac{1}{1}$$

Thus, SP is ₹ 1 each.

12. SP of radio $= ₹\ 360$, Loss $\% = 25\%$, Profit $\% = 25\%$

$$SP = CP \times \frac{(100 - 25)}{100} \quad \Rightarrow \quad 360 = CP \times \frac{75}{100}$$

$$CP = ₹\ 480$$

$$SP = \frac{480 \times 125}{100} = \frac{480 \times 5}{4} = 120 \times 5 = ₹\ 600$$

13. Let SP of 1 chair $= x$

$$SP = 17 \text{ chair}$$

$$\text{Loss} = 3 \text{ chairs}$$

$$CP = SP + \text{Loss}$$

$$CP = 17 + 3 = 20 \text{ chair}$$

$$\text{Loss } \% = \frac{\text{Loss}}{CP} \times 100 = \frac{3}{20} \times 100 = 15\%$$

$\therefore$ SP of one chair $= \frac{3400}{20} = ₹\ 170$

14. SP of 4 fans $=$ CP of 5 fans

$$SP \times 4 = CP \times 5 \quad \Rightarrow \quad \frac{SP}{CP} = \frac{5}{4}$$

$$\text{Profit } \% = \left(\frac{SP - CP}{CP}\right) \times 100 = \left(\frac{5 - 4}{4}\right) \times 100 = \frac{1}{4} \times 100$$

$$\text{Profit}\% = 25\%$$

15. SP of 3 oranges = CP of 4 oranges

$$SP \times 3 = CP \times 4$$

$$\frac{SP}{CP} = \frac{4}{3}$$

$$\text{Profit\%} = \left(\frac{SP - CP}{CP}\right) \times 100 = \left(\frac{4-3}{3}\right) \times 100$$

$$= \frac{1}{3} \times 100 = 33\frac{1}{3}\%$$

16.
$$SP = \frac{CP \times (100 - 5)}{100} \quad \ldots(i)$$

$$SP + 375 = \frac{CP \times (100 + 10)}{100} \quad \ldots(ii)$$

Taking eqs. (i) and (ii),

$$\frac{SP}{SP + 375} = \frac{(100 - 5)}{100} \times \frac{100}{(110)}$$

$$110SP = 95\,SP + 95 \times 375$$

$$15\,SP = 95 \times 375$$

$$SP = ₹2375$$

Put in eqs. (i)

$$CP = \frac{2375 \times 100}{95} = 25 \times 100 = ₹\,2500$$

Thus, CP of TV is ₹ 2500.

17. Case I SP = ₹ 67.50, Loss = 10%, CP = ?

$$CP = \frac{100}{(100 - 10)} \times 67.50 = \frac{100}{90} \times 67.50 = ₹\,75$$

Case II SP = 82.50 and CP = ₹ 75

$$\text{Profit} = 82.50 - 75 = 7.5$$

$$\text{Profit\%} = \frac{\text{Profit} \times 100}{CP} = \frac{7.5 \times 100}{75} = \frac{750}{75} = 10\%$$

Thus, profit of stool is 10%.

18.
$$\text{Profit \%} = \left(\frac{\frac{CP}{5} \times 100}{CP}\right) = \frac{1}{5} \times 100 = 20\%$$

19. SP of two buffaloes = ₹ 60,000 (each), Profit% = 30%,
Loss% = 10%

$$SP = CP_1 \times \frac{(100 + 30)}{100}$$

$$60{,}000 = \frac{CP_1 \times (130)}{100}$$

$$CP_1 = 46153.8 \quad \ldots(i)$$

$$60000 = \frac{CP_2 \times (100 - 10)}{100} = \frac{CP_2 \times 90}{100}$$

$$CP_2 = 66666.66$$

$$\text{Total CP} = 46153.8 + 66666.66 = ₹\ 112820.44$$

$$\text{Total SP} = 2 \times 60{,}000 = ₹\ 1{,}20{,}000$$

$$\text{Profit} = 1{,}20{,}000 - 1{,}12{,}820.44 = 7179.56$$

$$\text{Profit\%} = \frac{7179.56}{1{,}12{,}820} \times 100 = 0.0636 \times 100 = 6.36\%$$

20. $SP = ?$; Loss % = 8%

$$SP = \frac{CP \times (100 - 8)}{100} \quad \ldots(i)$$

When she sell it ₹ 150 more, then

$$SP + 150 = CP \times \left(\frac{100 + 12}{100}\right) \quad \ldots(ii)$$

Taking eq. (i) and (ii),

$$\frac{SP}{SP + 150} = \frac{(100 - 8)}{100} \times \frac{100}{(100 + 12)}$$

$$\frac{SP}{SP + 150} = \frac{92}{112} \Rightarrow \frac{SP}{SP + 150} = \frac{23}{28}$$

$$28\ SP = 23\ SP + 23 \times 150$$

$$28\ SP - 23\ SP = 23 \times 150 \Rightarrow 5\ SP = 23 \times 150$$

$$SP = 23 \times 30 = ₹\ 690$$

Thus, SP of the leather purse is ₹ 690.

❑

10. Compound Interest

Exercise 10.1

1. Principal for the first year = ₹ 5000

Interest for the first year $= \frac{5000 \times 1 \times 10}{100} = 500$

Amount at the end of the first year = ₹ (5000 + 500) = ₹ 5500

Principal for the second year = ₹ 5500

Interest for the second year $= \frac{5500 \times 1 \times 10}{100} =$ ₹ 5500

Amount at the end of the second year = 5500 + 550 = ₹ 6050

Compound Interest = ₹ 6050 – ₹ 5000 = ₹ 1050

2. Do yourself.

3. Principal for the first year = ₹ 625

Interest for the first year $= \frac{625 \times 4 \times 1}{100} =$ ₹ 25

Amount at the end of the first year = 625 + 25 = ₹ 650

Principal for the second year = ₹ 650

Interest for the second year $= \frac{650 \times 4 \times 1}{100} = 26$

Amount at the end of the second year = 650 + 26 = ₹ 676

Compound Interest = ₹ 676 – ₹ 625 = ₹ 51

4. to **8.** Do yourself.

9. Principal for the first year = ₹ 46, 875

Interest for the first year $= \frac{46875 \times 4 \times 1}{100} =$ ₹ 1875

(i) The interest for the first year = ₹ 1875

Amount at the end of the first year = 46875 + 1875 = ₹ 48750

(ii) Principal for the second year = ₹ 48,750

Interest for the second year $= \frac{48750 \times 4 \times 1}{100} =$ ₹ 1950

Amount at the end of the second year = 48,750 + 1950 = ₹ 50,700

(iii) Principal for the third year = ₹ 50,700

Interest for the third year $= \frac{50700 \times 4 \times 10}{100} =$ ₹ 2028

10. P = ₹ 6000, R = 10%, T = 3 years

$$\text{CI for first year} = \frac{6000 \times 10 \times 1}{100} = ₹\ 600$$

$$\text{CI of second year} = \frac{(6000 + 600) \times 10 \times 1}{100}$$

$$= \frac{6600 \times 10}{100} = ₹\ 660$$

$$\text{CI of third year} = \frac{(6600 + 660) \times 10}{100}$$

$$= 7260 \times \frac{1}{10} = ₹\ 726$$

$$\text{CI for 3 years} = 600 + 660 + 726 = ₹\ 1986$$

$$\text{Amount} = 6000 + 1986 = ₹\ 7986$$

11. Principal for the first year = ₹ 5000

Interest for the first year $= \frac{5000 \times 6 \times 1}{100} =$ ₹ 300

Amount at the end of the first year = 5000 + 300 = 5300

Principal for the second year = ₹ 5300

Interest for the second year $= \frac{5300 \times 8 \times 1}{100} = 424$

Amount at the end of the second year $= 5300 + 424 = 5724$

Compound Interest $= ₹\,5724 - 5000 = ₹\,724$

12. Principal $= 20{,}000$, $R = 8\%$, $T = 2$ year

Case I Simple interest $= \frac{20{,}000 \times 8 \times 2}{100} = 3{,}200$

Case II Interest for the first year $= \frac{20{,}000 \times 8 \times 1}{100} = 1600$

Amount at the end of the first year $= 20000 + 1600 = ₹\,21600$

Principal for the second year $= \frac{21600 \times 8 \times 1}{100} = ₹\,1728$

Compound interest for the two years $= 1600 + 1728 = ₹\,3328$

Difference between CI and SI $= ₹\,3328 - ₹\,3200 = ₹\,128$

Exercise 10.2

1. Principal $= ₹\,625$, Rate $= 4\%$, Time $= 2$ years

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 625\left(1 + \frac{4}{100}\right)^2 = 625 \times \left(\frac{104}{100}\right)^2$$

$$= 625 \times \frac{26}{25} \times \frac{26}{25} = ₹\,676$$

2. $P = ₹\,2000$, $R = 4\%$, $T = 3$ years

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 2000\left(1 + \frac{4}{100}\right)^3$$

$$= 2000\left(\frac{104}{100}\right)^3 = 2000 \times \frac{26}{25} \times \frac{26}{25} \times \frac{26}{25}$$

$$= \frac{3.2 \times 26 \times 26 \times 26}{25} = \frac{56243.2}{25}$$

$$\text{Amount} = ₹\,2249.728 = ₹\,2249.73$$

3. $P = ₹\,3000$, $R = 5\%$, $T = 2$ years

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 3000\left(1 + \frac{5}{100}\right)^2$$

$$= 3000\left(\frac{105}{100}\right)^2 = 3000 \times \frac{21}{20} \times \frac{21}{20}$$

$$= 7.5 \times 21 \times 21 = ₹\,3307.5$$

4. $P = ₹5000$, $R = 10\%$, $T = 2$ years

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 5000\left(1 + \frac{10}{100}\right)^2$$

$$= 5000\left(\frac{110}{100}\right)^2 = 5000 \times \left(\frac{11}{10}\right)^2$$

$$= 5000 \times \frac{11}{10} \times \frac{11}{10} = 50 \times 121 = ₹\ 6050$$

5. $P = ₹\ 8000, R = 15\%, T = 3$ years

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 8000\left(1 + \frac{15}{100}\right)^3$$

$$= 8000 \times \left(\frac{115}{100}\right)^3 = 8000 \times \left(\frac{23}{20}\right)^3$$

$$= 8000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{23}{20} = 23 \times 23 \times 23 = ₹\ 12167$$

Word Problems

1. $P = ₹\ 1000, R = 10\%, T = 3$ years

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 1000\left(1 + \frac{10}{100}\right)^3$$

$$= 1000 \times \left(\frac{110}{100}\right)^3 = 1000 \times \left(\frac{11}{10}\right)^3$$

$$= \frac{1000 \times 11 \times 11 \times 11}{10 \times 10 \times 10} = 11 \times 11 \times 11 = ₹\ 1331$$

Thus, Palak will pay amount ₹ 1331.

2. $P = ₹\ 4000, R = 2.5\%, T = 2$ years

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 4000\left(1 + \frac{2.5}{100}\right)^2$$

$$= 4000 \times \left(\frac{1025}{1000}\right)^2 = 4000 \times \left(\frac{41}{40}\right)^2 = 4000 \times \frac{41}{40} \times \frac{41}{40}$$

$$= \frac{10 \times 41 \times 41}{4} = \frac{16810}{4} = ₹\ 4202.50$$

3. $P = ₹\ 1{,}80{,}000, R = 11\frac{1}{4}\%, T = 2\frac{1}{2}$ years

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T \left(1 + \frac{R}{100}\right)^{1/2}$$

$$= 1{,}80{,}000\left(1 + \frac{45}{400}\right)^2 \left(1 + \frac{45}{4 \times 2 \times 100}\right)$$

$$= 1{,}80{,}000\left(1 + \frac{9}{80}\right)^2 \left(1 + \frac{9}{160}\right)$$

$$= 1{,}80{,}000\left(\frac{89}{80}\times\frac{89}{80}\right)\left(\frac{169}{160}\right)$$

$$= \frac{180}{8\times 8\times 16}\times 89\times 89\times 169$$

$$= \frac{240956820}{64\times 16} = \frac{3764950.31}{16} = ₹\ 235309.39$$

4. $P = ₹\ 1{,}20{,}000, R = 12\%, T = 4$ years

$$\text{Simple Interest} = \frac{P\times R\times T}{100} = \frac{1{,}20{,}000\times 12\times 4}{100}$$

$$= 1200\times 12\times 4$$

$$\text{SI} = ₹\ 57600 \qquad \ldots(\text{i})$$

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^{T} = 1{,}20{,}000\left(1+\frac{12}{100}\right)^{4}$$

$$= 1{,}20{,}000\left(\frac{112}{100}\right)^{4} = 1{,}20{,}000\times\left(\frac{28}{25}\right)^{4}$$

$$= 1{,}20{,}000\times\frac{28}{25}\times\frac{28}{25}\times\frac{28}{25}\times\frac{28}{25}$$

$$\text{Amount} = ₹\ 188822.32$$

$$\text{CI} = A - P = 188822.32 - 1{,}20{,}000$$

$$\text{CI} = ₹\ 68822.32 \qquad \ldots(\text{ii})$$

Difference between CI and SI

$$= 68822.32 - 57600 = ₹\ 11222.32$$

5. $P = ₹\ 1{,}00{,}000, R_1 = 6\frac{1}{2}\%, R_2 = 8\frac{1}{4}\%, R_3 = 10\%, T = 3$ years

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^{T}$$

$$= 100000\left(1+\frac{13}{200}\right)\left(1+\frac{33}{400}\right)\left(1+\frac{10}{100}\right)$$

$$= 100{,}000\left(\frac{213}{200}\right)\times\left(\frac{433}{400}\right)\left(\frac{110}{100}\right)$$

$$\text{Amount} = \frac{213\times 433\times 110}{80} = ₹\ 126814.88$$

$$\text{CI} = 126814.88 - 100000 = ₹\ 26814.88$$

6. $P = ₹\ 60{,}000, R = 10\%, T = 3\frac{1}{2}$ years

$$\text{SI} = \frac{P\times R\times T}{100} = \frac{60{,}000\times 10\times 7}{2\times 100}$$

$$\text{SI} = 3000\times 7 = ₹\ 21000$$

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^{T}\left(1+\frac{R}{200}\right)^{1/2}$$

$$= 60{,}000\left(1+\frac{10}{100}\right)^3\left(1+\frac{1}{2}\times\frac{10}{100}\right)$$

$$= 60{,}000\left(\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}\right)\left(\frac{21}{20}\right)$$

$$= \frac{6\times 11\times 11\times 11\times 21}{2} = ₹\ 83853$$

$$\text{CI} = A - P = 83853 - 60{,}000 = ₹\ 23853$$

Difference between CI and SI $= 23853 - 21000 = ₹\ 2853$

7. $P = ₹\ 18{,}000$, $A = ₹\ 20{,}480$, $T = 2$ years, $R = ?$

$$A = P\left(1+\frac{R}{100}\right)^T$$

$$20480 = 18000\left(1+\frac{R}{100}\right)^2$$

$$\frac{20480}{18000} = \left(1+\frac{R}{100}\right)^2 \Rightarrow \frac{1024}{900} = \left(1+\frac{R}{100}\right)^2$$

$$\left(\frac{32}{30}\right)^2 = \left(1+\frac{R}{100}\right)^2$$

Comparing both side :

$$\frac{32}{30} = 1+\frac{R}{100} \Rightarrow \frac{32}{30} - 1 = \frac{R}{100}$$

$$\frac{2}{30} = \frac{R}{100} \Rightarrow R = \frac{2\times 100}{30} \Rightarrow R = \frac{20}{3}\%$$

Exercise 10.3

1. $P = ₹\ 4096$, $R = 12\frac{1}{2}\%$, $T = 18$ months for half yearly

$$R = \frac{25}{2}\times\frac{1}{2}, T = 18\times 2 \text{ half} = 3 \text{ half years}$$

$$A = P\left(1+\frac{R}{100}\right)^T = 4096\left(1+\frac{25}{4\times 100}\right)^3$$

$$= 4096\left(1+\frac{1}{4\times 4}\right)^3 = 4096\left(1+\frac{1}{16}\right)^3$$

$$= 4096\times\frac{17}{16}\times\frac{17}{16}\times\frac{17}{16}$$

$$= 17\times 17\times 17 = ₹\ 4913$$

2. $P = ₹\ 700, R = 20\%, T = 1 \text{ year and } \frac{1}{2} \text{ year}$

$$R = \frac{20}{2} = 10\% \quad \Rightarrow \quad T = \frac{3}{2} \text{ year} = 3 \text{ half year}$$

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 700\left(1 + \frac{10}{100}\right)^3$$

$$= 700 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} = \frac{7 \times 1331}{10} = ₹\ 931.70$$

$$\Rightarrow \quad \text{CI} = 931.70 - 700 = ₹\ 231.70$$

3. $P = ₹\ 1000, R = 2\%, T = 1 \text{ year}$

$$R = \frac{2}{2} = 1\%, \ T = 2 \times 1 = 2 \text{ half years}$$

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 1000\left(1 + \frac{1}{100}\right)^2$$

$$= 1000 \times \frac{101}{100} \times \frac{101}{100}$$

$$= \frac{101 \times 101}{10} = ₹\ 1020.1 \approx ₹\ 1020$$

4. $P = ₹\ 256, R = 12\frac{1}{2}\% = \frac{25}{2}\%, T = 1 \text{ year (Take half yearly)}$

$$R = \frac{\frac{25}{2}}{2} = \frac{25}{4}\%, T = 2 \times 1 = 2 \text{ half years}$$

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 256\left(1 + \frac{25}{4 \times 100}\right)^2$$

$$= 256\left(1 + \frac{1}{4 \times 4}\right)^2 = 256\left(\frac{17}{16} \times \frac{17}{16}\right)$$

$$= 17 \times 17 = ₹\ 289$$

5. $P = ₹\ 8000, R = 10\%, T = 1\frac{1}{2} = \frac{3}{2} \text{ years}$

$$R = \frac{10}{2} = 5\%, T = \frac{3}{2} \times 2 = 3 \text{ half years}$$

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 8000\left(1 + \frac{5}{100}\right)^3$$

$$= 8000\left(\frac{105}{100}\right)^3 = 8000 \times \left(\frac{21}{20}\right)^3$$

$$= 8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20}$$

$$\text{Amount} = 21 \times 21 \times 21 = ₹\ 9261$$

6. $P = ₹\ 8192, R = 12.5\%, T = 1\frac{1}{2}$ years

$$R = \frac{12.5}{2} = \frac{25}{4}\%, T = 3 \text{ half years}$$

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 8192\left(1 + \frac{25}{4 \times 100}\right)^3$$

$$= 8192\left(1 + \frac{1}{4 \times 4}\right)^3 = 8192 \times \left(\frac{17}{16}\right)^3$$

$$= \frac{8192 \times 17 \times 17 \times 17}{16 \times 16 \times 16} = ₹\ 9826$$

7. $P = ₹\ 64{,}000, R = 5\%, T = \frac{3}{2}$ years

$$R = \frac{5}{2}\%, T = \frac{2 \times 3}{2} = 3 \text{ half years}$$

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 64{,}000\left(1 + \frac{5}{2 \times 100}\right)^3$$

$$= 64{,}000\left(\frac{41}{40}\right)^3 = 64{,}000 \times \frac{41}{40} \times \frac{41}{40} \times \frac{41}{40}$$

$$= ₹\ 68921$$

$$\text{CI} = A - P = 68921 - 64000 = ₹\ 4921$$

8. $P = ₹\ 40{,}960, R = 12.5\%, T = \left(\frac{3}{2} \text{ years}\right)$

$$R = \frac{12.5}{2} = \frac{25}{4}\%, T = 2 \times \frac{3}{2} = 3 \text{ half years}$$

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 40960\left(1 + \frac{25}{4 \times 100}\right)^3$$

$$= 40960\left(1 + \frac{1}{4 \times 4}\right)^3 = 40960\left(\frac{17}{16}\right)^3$$

$$= \frac{40960 \times 17 \times 17 \times 17}{16 \times 16 \times 16} = ₹\ 49130$$

$$\text{CI} = A - P = 49130 - 40960 = ₹\ 8170$$

Exercise 10.4

1. (i) $P = ₹\ 1{,}56{,}250, R = 8\%, T = 1\frac{1}{2} = \frac{3}{2}$ years

$$R = \frac{8}{2} = 4\%, T = 2 \times \frac{3}{2} = 3 \text{ half years}$$

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^T = 156250\left(1+\frac{4}{100}\right)^3$$

$$= 1{,}56{,}250\left(1+\frac{1}{25}\right)^3 = 156250\times\left(\frac{26}{25}\right)^3$$

$$= 10\times 26\times 26\times 26 = ₹\ 175760$$

$$\text{CI} = A - P = 175760 - 156250 = ₹\ 19510$$

(ii) $P = 1{,}00{,}000, R = 4\%, T = 9$ months (quarter)

$R = \frac{4}{4} = 1\%, T = 3$ quarter years

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^T = 100000\left(1+\frac{1}{100}\right)^3$$

$$= 100000\times\left(\frac{101}{100}\right)^3$$

$$= \frac{101\times 101\times 101}{10} = ₹\ 103030.10$$

$$\text{CI} = A - P = 103030.10 - 100000 = ₹\ 3030.10$$

2. $P = ₹\ 2500, R = 4\%, T = 2$ years

$$R = \frac{4}{2} = 2\%, T = 2\times 2 = 4 \text{ half years}$$

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^T$$

$$= 2500\left(1+\frac{2}{100}\right)^4 = 2500\times\left(\frac{51}{50}\right)^4$$

$$= 2500\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}\times\frac{51}{50}$$

$$= \frac{6765201}{2500} = ₹\ 2706.08$$

$$\text{CI} = A - P = 2706.08 - 2500 = ₹\ 206.08$$

$$\text{SI} = \frac{P\times R\times T}{100} = \frac{2500\times 2\times 4}{100} = ₹\ 200$$

Difference between CI and SI $= 206.08 - 200 = ₹\ 6.08$

3. $P = ₹\ 3125, R_1 = 4\%, R_2 = 5\%, R_3 = 6\%, T = 3$ years

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^T$$

$$= 3125\left(1+\frac{4}{100}\right)\left(1+\frac{5}{100}\right)\left(1+\frac{6}{100}\right)$$

$$= 3125\times\frac{26}{25}\times\frac{21}{20}\times\frac{106}{100}$$

$$= \frac{125 \times 26 \times 21 \times 106}{2000}$$

$$= \frac{3617250 \times 2}{2000} = ₹\,3617.25$$

$$\text{CI} = A - P = 3617.25 - 3125 = ₹\,492.25$$

4. $A = ₹\,4913, R = 12\frac{1}{2}\%, T = \frac{3}{2}$ years

$$R = \frac{25}{4}\%, T = 2 \times \frac{3}{2} = 3 \text{ half years}$$

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T$$

$$4913 = P\left(1 + \frac{25}{4 \times 100}\right)^3 = P\left(1 + \frac{1}{4 \times 4}\right)^3$$

$$4913 = P \times \frac{17}{16} \times \frac{17}{16} \times \frac{17}{16}$$

$$P = 16 \times 16 \times 16 = ₹\,4096$$

5. $P = ₹\,2000$, CI $= ₹\,163.2, R = 4\%, T = ?$

$$\text{CI} = A - P$$

$$A = \text{CI} + P = 2000 + 163.2 = ₹\,2163.2$$

$$A = P\left(1 + \frac{R}{100}\right)^T$$

$$2163.2 = 2000\left(1 + \frac{4}{100}\right)^T$$

$$\Rightarrow \quad \frac{2163.2}{2000} = \left(\frac{104}{100}\right)^T \quad \Rightarrow \quad = \frac{21632}{20000} = \left(\frac{104}{100}\right)^T$$

$$\Rightarrow \quad \frac{10816}{10000} = \left(\frac{104}{100}\right)^T \quad \Rightarrow \quad \left(\frac{104}{100}\right)^2 = \left(\frac{104}{100}\right)^T$$

$\Rightarrow$ Comparing both side, $T = 2$ years.

6. $P = ?$, CI $= ₹\,331, R = 10\%, T = 3$ years

$$A = P\left(1 + \frac{R}{100}\right)^T \quad \Rightarrow \quad A = P\left(1 + \frac{10}{100}\right)^3 \qquad \ldots(\text{i})$$

$$\text{CI} = A - P \quad \Rightarrow \quad A = \text{CI} + P = 331 + P$$

Put in eq. (i),

$$331 + P = P\left(1 + \frac{1}{10}\right)^3 \quad \Rightarrow \quad 331 + P = P\left(\frac{11}{10}\right)^3$$

$$331 + P = P\left(\frac{1331}{1000}\right)$$

$$331 \times 1000 + 1000P = 1331P$$
$$1331P - 1000P = 331 \times 1000$$
$$331P = 331 \times 1000 \quad \Rightarrow \quad P = ₹\ 1000$$

Thus, principal is ₹ 1000.

7. CI = ₹ 1290, $P = ?$, $R = 15\%$, $T = 2$ years.

$$A = P\left(1 + \frac{15}{100}\right)^2 \qquad \ldots(i)$$
$$\text{CI} = A - P \quad \Rightarrow \quad A = \text{CI} + P = 1290 + P$$

Put in eq. (i),

$$1290 + P = P\left(1 + \frac{3}{20}\right)^2 \quad \Rightarrow \quad 1290 + P = P\left(\frac{23}{20}\right)^2$$
$$1290 \times 400 + 400P = 529P \quad \Rightarrow \quad 129P = 1290 \times 400$$
$$P = 10 \times 400 = ₹\ 4000$$

8. $P = ₹\ 8000$, $A = ₹\ 9261$, $R = 10\%$ (half-yearly), $T = ?$

$$R = \frac{10}{2} = 5\%$$
$$A = P\left(1 + \frac{R}{100}\right)^T \quad \Rightarrow \quad \frac{9261}{8000} = \left(1 + \frac{5}{100}\right)^T$$
$$\frac{9261}{8000} = \left(1 + \frac{1}{20}\right)^T \quad \Rightarrow \quad \frac{9261}{8000} = \left(\frac{21}{20}\right)^T$$
$$\left(\frac{21}{20}\right)^3 = \left(\frac{21}{20}\right)^T$$

Comparing both side and taking half-yearly

$$T = \frac{3}{2}\text{ years} = 1\frac{1}{2}\text{ years}$$

9. $P = ₹\ 1000$, $A = ₹\ 1331$, $T = 3$ years, $R = ?$

$$A = P\left(1 + \frac{R}{100}\right)^T$$
$$1331 = 1000\left(1 + \frac{R}{100}\right)^3 \quad \Rightarrow \quad \frac{1331}{1000} = \left(1 + \frac{R}{100}\right)^3$$
$$\left(\frac{11}{10}\right)^3 = \left(1 + \frac{R}{100}\right)^3$$

Comparing both side,

$$\frac{11}{10} = 1 + \frac{R}{100} \quad \Rightarrow \quad \frac{11}{10} - 1 = \frac{R}{100}$$
$$\frac{1}{10} = \frac{R}{100} \quad \Rightarrow \quad R = 10\%$$

10. $P = ₹\ 26400, R = 15\%, T =$ 2 years and 4 months

$$= 2 \text{ year } \frac{4}{12} = 2 + \frac{1}{3} = 2\frac{1}{3} \text{ years}$$

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^2 \left(1 + \frac{R}{100}\right)^{1/3}$$

$$= 26400\left(1 + \frac{15}{100}\right)^2 \left(1 + \frac{15}{3 \times 100}\right)$$

$$= 26400\left(1 + \frac{3}{20}\right)^2 \left(1 + \frac{3}{3 \times 20}\right)$$

$$= 26400\left(\frac{23}{20}\right)^2 \times \left(\frac{21}{20}\right)$$

$$= \frac{26400 \times 23 \times 23}{20 \times 20} \times \frac{21}{20}$$

$$= \frac{33 \times 23 \times 23 \times 21}{10} = \frac{366597}{10}$$

$$= ₹\ 36659.7$$

11. $P = ₹\ 18000, R = 8\%, T = 2$ years

$$\text{SI} = \frac{P \times R \times T}{100} = \frac{18000 \times 8 \times 2}{100} = ₹\ 2880$$

$$\text{Amount} = 18000\left(1 + \frac{8}{100}\right)^2 = 18000\left(1 + \frac{2}{25}\right)^2$$

$$= 18000 \times \frac{27}{25} \times \frac{27}{25} = 28.8 \times 27 \times 27$$

$$= ₹\ 20995.2$$

$$\text{CI} = A - P = 20995.2 - 18000 = ₹\ 2995.2$$

Difference between CI and SI $= 2995.2 - 2880 = ₹\ 115.2$

12. $P = ₹\ 86000, R = 5\%, T = 2$ years

$$\text{SI} = \frac{P \times R \times T}{100} = \frac{86000 \times 5 \times 2}{100} = ₹\ 8600$$

$$\text{Amount} = 86000\left(1 + \frac{5}{100}\right)^2 = 86000 \times \frac{21}{20} \times \frac{21}{20}$$

$$= 215 \times 21 \times 21 = ₹\ 94815$$

$$\text{CI} = A - P = 94815 - 86000 = ₹\ 8815$$

Difference between CI and SI $= 8815 - 8600 = ₹\ 215$

Exercise 10.5

1. $P = ₹\ 16{,}000,\ R = 5\%,\ T = 2$ years

$$\text{Value of boat} = P\left(1 - \frac{R}{100}\right)^T = 16{,}000\left(1 - \frac{5}{200}\right)^2$$

$$= 16{,}000\left(1 - \frac{1}{20}\right)^2 = 16{,}000\left(\frac{19}{20} \times \frac{19}{20}\right)$$

$$= 40 \times 19 \times 19 = ₹\ 14440$$

2. $P = ₹\ 1{,}00{,}000,\ R = 10\%,\ T = 3$ years

$$\text{Value of residential flat} = P\left(1 - \frac{R}{100}\right)^3 = 100000\left(1 - \frac{1}{10}\right)^3$$

$$= 100000\left(\frac{9}{10} \times \frac{9}{10} \times \frac{9}{10}\right) = 100 \times 729$$

$$= ₹\ 72900$$

3. $P = 1250,\ R = 20\%,\ T = 3$ months

$$\text{Number of stray dogs in city} = P\left(1 - \frac{R}{100}\right)^T = 1250\left(1 - \frac{20}{100}\right)^3$$

$$= 1250\left(1 - \frac{1}{5}\right)^3 = 1250\left(\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}\right)$$

$$= 10 \times 4 \times 4 \times 4 = ₹\ 640$$

4. $P = 8000,\ R = 10\%,\ T = 3$ half years

$$\text{Blood donors} = P\left(1 + \frac{R}{100}\right)^T = 8000\left(1 + \frac{10}{100}\right)^3$$

$$= 8000\left(\frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}\right)$$

$$= 8 \times 1331 = 10648$$

$$\text{New registrations} = 10648 - 8000 = 2648$$

5. $P = ₹\ 9{,}000,\ R = 5\%,\ T = 2$ years

$$\text{Cost of refrigerator after two year} = P\left(1 - \frac{R}{100}\right)^2$$

$$= 9000\left(1 - \frac{5}{100}\right)^2 = 9000\left(1 - \frac{1}{20}\right)^2$$

$$= 9000\left(\frac{19}{20} \times \frac{19}{20}\right) = \frac{90}{4} \times 19 \times 19$$

$$= 22.5 \times 19 \times 19 = 8122.5$$

$$\text{Total depreciation value} = 9000 - 8122.5 = ₹\ 877.5$$

6. Wheat produced = 2187 quintals, $R = 8\%, T = 2$ years

$$2187 = P\left(1 + \frac{8}{100}\right)^2 = P\left(1 + \frac{2}{25}\right)^2$$

$$2187 = P\left(\frac{27}{25} \times \frac{27}{25}\right)$$

$$P = \frac{2187}{27 \times 27} \times 25 \times 25 = 3 \times 25 \times 25 = 1875$$

Thus, two years ago produced wheat was 1875 quintal.

7. Present value of a property = ₹ 411540, $R = 5\%, T = 3$ years

$$411540 = P\left(1 - \frac{5}{100}\right)^3 \quad \Rightarrow \quad 411540 = P\left(1 - \frac{1}{20}\right)^3$$

$$411540 = P \times \frac{19}{20} \times \frac{19}{20} \times \frac{19}{20}$$

$$60 = \frac{P}{8000} \quad \Rightarrow \quad P = ₹\,4,80,000$$

8. P = ₹ 16,000, after depreciation cost of the scooter = ₹ 14440, $R = ?, T = 2$

$$14440 = 16000\left(1 - \frac{R}{100}\right)^2$$

$$\frac{14440}{16000} = \left(1 - \frac{R}{100}\right)^2 \quad \Rightarrow \quad \left(\frac{38}{40}\right)^2 = \left(1 - \frac{R}{100}\right)^2$$

Comparing both side :

$$\frac{38}{40} = 1 - \frac{R}{100} \quad \Rightarrow \quad \frac{R}{100} = 1 - \frac{38}{40}$$

$$\frac{R}{100} = \frac{2}{40} \quad \Rightarrow \quad R = 5\%$$

9. Production of car in 2006 – 7007 = 80,000

Production of car in 2009 – 2010 = 92,160

$$92610 = 80000\left(1 + \frac{R}{100}\right)^3$$

$$\frac{9261}{8000} = \left(1 + \frac{R}{100}\right)^3 \quad \Rightarrow \quad \left(\frac{21}{20}\right)^3 = \left(1 + \frac{R}{100}\right)^3$$

Comparing both side :

$$\frac{21}{20} = 1 + \frac{R}{100} \quad \Rightarrow \quad \frac{21}{20} - 1 = \frac{R}{100}$$

$$\frac{1}{20} = \frac{R}{100} \quad \Rightarrow \quad R = 5\%$$

10. $P = 4{,}000$, $R_1 = 5\%$ (increase), $R_2 = 5\%$ (decrease), $R_3 = 10\%$ (decrease)

Total number of ticketless traveller

$$= 4000\left(1 + \frac{5}{100}\right)\left(1 - \frac{5}{100}\right)\left(1 - \frac{10}{100}\right)$$

$$= 4000\left(\frac{21}{20} \times \frac{19}{20} \times \frac{9}{10}\right) = \frac{4000}{4000} \times 21 \times 19 \times 9$$

$$= 21 \times 19 \times 9 = 3591$$

Revision Exercise

1. Do it yourself.

2. $P = ₹\ 1000$, $R = 10\%$, $T = 3$ years

$$\text{Amount} = 1000\left(1 + \frac{10}{100}\right)^3 = 1000\left(1 + \frac{1}{10}\right)^3$$

$$= 1000\left(\frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}\right)$$

$$= 1000 \times \frac{11 \times 11 \times 11}{1000} = 11 \times 11 \times 11 = ₹\ 1331$$

$$\text{CI} = A - P = 1331 - 1000 = ₹\ 331$$

3. $P = ₹\ 16{,}000$, $R = 12\frac{1}{2}\%$, $T = 3$ years.

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 16{,}000\left(1 + \frac{25}{200}\right)^3$$

$$= 16000\left(1 + \frac{1}{8}\right)^3 = 16000 \times \frac{9}{8} \times \frac{9}{8} \times \frac{9}{8}$$

$$= 31.25 \times 9 \times 81 = ₹\ 22781.25$$

$$\text{CI} = A - P = 22781.25 - 16000 = ₹\ 6781.25$$

4. $P = ₹\ 2400$, $R = 20\%$, $T = 3$ years

$$A = P\left(1 + \frac{R}{100}\right)^T = 2400\left(1 + \frac{20}{100}\right)^3$$

$$= 2400\left(1 + \frac{1}{5}\right)^3 = 2400 \times \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5}$$

$$A = 19.2 \times 36 \times 6 = ₹\ 4147.2$$

$$\text{CI} = A - P = 4147.2 - 2400 = 1747.2$$

5. $P = ₹\ 50{,}000$, $R = 12\%$, $T = 2\frac{1}{2}$ years

$$A = P\left(1 + \frac{R}{100}\right)^T \times \left(1 + \frac{R}{100}\right)^{1/2}$$

$$= 50{,}000\left(1+\frac{12}{100}\right)^2\left(1+\frac{12}{100}\right)^{1/2}$$

$$= 50{,}000\times\frac{28}{25}\times\frac{28}{25}\times\left(1+\frac{1}{2}\times\frac{12}{100}\right)$$

$$= 80\times 28\times 28\times\left(\frac{53}{50}\right) = ₹\ 66483.2$$

$$\text{CI} = A - P = 66483.2 - 50{,}000 = ₹\ 16483.2$$

6. $P = 1{,}30{,}000$, Rate $= 10\%$, $T = 15$ months (quarterly)

$$R = \frac{10}{4} = 2.5\%,\ T = 4\times 15 = \frac{60}{12} = 5 \text{ quarter years}$$

$$A = P\left(1+\frac{R}{100}\right)^T = 1{,}30{,}000\left(1+\frac{2.5}{100}\right)^5$$

$$= 1{,}30{,}000\left(1+\frac{1}{40}\right)^5 = 130{,}000\left(\frac{41}{40}\right)^5$$

$$= \frac{1{,}30{,}000\times 41\times 41\times 41\times 41\times 41}{40\times 40\times 40\times 40\times 40}$$

$$= \frac{0.0126\times 41\times 41\times 41\times 41\times 41}{10} = ₹\ 147083.07$$

7. $P = ₹\ 2{,}20{,}000$, $R = 12\%$, $T = 2\frac{1}{2}$ years, $R = \frac{12}{2} = 6\%$,

$T = 2\times\frac{5}{2} = 5$ half years

For annually, $A = 2{,}20{,}000\left(1+\frac{12}{100}\right)^2\left(1+\frac{12}{100}\right)^{1/2}$

$$= 2{,}20{,}000\left(\frac{28}{25}\times\frac{28}{25}\right)\left(1+\frac{1}{2}\times\frac{12}{100}\right)$$

$$= 352\times 28\times 28\times\frac{53}{50}$$

$$= 7.04\times 28\times 28\times 53 = 292526.08$$

$$\text{CI} = A - P = 292526.08 - 2{,}20{,}000 = ₹\ 72526.08$$

For half-yearly,

$$A = 2{,}20{,}000\left(1+\frac{6}{100}\right)^5$$

$$= 2{,}20{,}000\times\left(\frac{53}{50}\right)^5 = ₹\ 294409.62$$

$$\text{CI} = A - P = 74409.62$$

$$\text{Difference} = 74409.62 - 72526.08 = ₹\ 1883.54$$

8. $A = ₹\ 197018.63, R = 12\%, T = \frac{3}{2}$ years

$$R = \frac{12}{4} = 3\%, T = \frac{4 \times 3}{2} = 6 \text{ quarter years}$$

$$197018.63 = P\left(1 + \frac{3}{100}\right)^6 = P\left(1 + \frac{3}{100}\right)^6$$

$$197018.63 = P \times \left(\frac{103}{100}\right)^6$$

$$P = \frac{197018.63 \times (100)^6}{(103)^6} = ₹\ 165000$$

Thus, money required is ₹ 165000.

9. $P = ₹\ 1600, A = 1852.2, T = \frac{3}{2}$ years $= 2 \times \frac{3}{2} = 3$ half-yearly), $R = 9\%$

$$1852.2 = 1600\left(1 + \frac{R}{100}\right)^3$$

$$\frac{1852.2}{16000} = \left(1 + \frac{R}{100}\right)^3 \Rightarrow \frac{9261}{8000} = \left(1 + \frac{R}{100}\right)^3$$

$$\left(\frac{21}{20}\right)^3 = \left(1 + \frac{R}{100}\right)^3$$

Comparing both side :

$$\frac{21}{20} - 1 = \frac{R}{100} \Rightarrow \frac{1}{20} = \frac{R}{100}$$

$R = 5\%$, per half-yearly, $R = 5 \times 2 = 10\%$ per annum

10. $P = ₹\ 2400, R = 20\%, T = 3$ years

$$A = P\left(1 + \frac{R}{100}\right)^T = 2400\left(1 + \frac{20}{100}\right)^3$$

$$= 2400\left(1 + \frac{1}{5}\right)^3 = 2400 \times \frac{6}{5} \times \frac{6}{5} \times \frac{6}{5}$$

$$= 19.2 \times 216 = ₹\ 4147.2$$

11. $P = ₹\ 16{,}000, R = \frac{25}{2}\%, T = 3$ years

$$A = P\left(1 + \frac{R}{100}\right)^T = 16000\left(1 + \frac{25}{2 \times 100}\right)^3$$

$$= 16000\left(1 + \frac{1}{2 \times 4}\right)^3 = 16000\left(\frac{9}{8} \times \frac{9}{8} \times \frac{9}{8}\right)$$

$$= 31.25 \times 729 = ₹\ 22781.25$$

12. $P = ₹\ 8000, R = 5\%, T = 3$ years

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 8000\left(1 + \frac{5}{100}\right)^3$$

$$= 8000\left(1 + \frac{1}{20}\right)^3 = 8000\left(\frac{21}{20}\right)^3$$

$$= 8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = 21 \times 21 \times 21 = ₹\ 9261$$

❑

11. Direct and Inverse Variation

Exercise 11.1

1. Do it yourself. **2.** Do it yourself.

3. Let the charge for a journey = ₹ x

It is a direct variation.

$$\frac{100}{1800} = \frac{120}{x} \quad \Rightarrow \quad x = 120 \times 18 = ₹\ 2160$$

Thus, charge for a journey of 120 km is ₹ 2160.

4. Let the number of the articles = x

It is a direct variation.

$$\frac{27}{1890} = \frac{x}{1750} \quad \Rightarrow \quad x = \frac{27 \times 175}{189} = 25$$

5. Let weight of the rice = x kg

It is a direct variation.

$$\frac{7}{1120} = \frac{x}{3680} \quad \Rightarrow \quad x = \frac{7 \times 368}{112} = 23 \text{ kg}$$

6. Let cost of 54 notebooks = ₹ x

$$\frac{6}{156} = \frac{54}{x} \quad \Rightarrow \quad x = \frac{54 \times 156}{6} = ₹\ 1404$$

Thus, cost of 54 note-books is ₹ 1404.

7. Let the number of men = x

We know that

$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_1 D_1 H_1}{W_2} \quad \Rightarrow \quad \frac{22 \times 1}{27} = \frac{x \times 1}{135}$$

$$x = \frac{22 \times 135}{27} = 110$$

8. Let the agent gets commission = ₹ x

It is a direct variation. $\frac{73}{1000} = \frac{x}{100}$

$$x = \frac{73 \times 100}{1000} = ₹\ 7.3$$

9. Let the number of bottles $= x$

It is a direct variation.

$$\frac{8}{5} = \frac{x}{40} \quad \Rightarrow \quad x = \frac{8 \times 40}{5} = 64 \text{ bottles}$$

10. Let the number of stamps $= x$

It is a direct variation.

$$\frac{5}{18} = \frac{x}{36} \quad \Rightarrow \quad x = \frac{5 \times 36}{18} = 10 \text{ stamps}$$

11. Let the number of tools $= x$

It is a direct variation.

$$\frac{5}{120} = \frac{20}{x} \quad \Rightarrow \quad x = \frac{20 \times 120}{5} = 480 \text{ tools}$$

12. Let the thickness of 275 sheets of paper $= x$ cm.

It is a direct variation.

$$\frac{500}{3.5} = \frac{275}{x} \quad \Rightarrow \quad x = \frac{275 \times 3.5}{500} = 1.925 \text{ cm}$$

13. Let the number of words $= x$

It is a direct variation.

$$\frac{540}{\frac{60}{2}} = \frac{x}{6} \quad \Rightarrow \quad x = \frac{540 \times 6}{30} = 108$$

14. Let Gitika covered a distance in 315steps $= x$ m

It is a direct variation.

$$\frac{125}{100} = \frac{315}{x} \quad \Rightarrow \quad x = \frac{315 \times 4}{5} = 252 \text{ m}$$

15. Let the cost of 105 m plastic sheet = ₹ x

It is a direct variation.

$$\frac{93}{1395} = \frac{105}{x} \quad \Rightarrow \quad x = \frac{105 \times 1395}{93} = ₹\ 1575$$

Exercise 11.2

1. Do it yourself. **2.** Do it yourself.

3. Let the number of days $= x$

Ration of the days = Inverse ratio of no. of cows.

$$15 \times 50 = 60 \times x$$

$$x = \frac{15 \times 50}{60} = 12.5 \text{ days or } 12\frac{1}{2} \text{ days}$$

4. Let the required time = x hours

Ratio of the number of pumps = Inverse ratio of time taken

$$20 \times 12 = 45 \times x \quad \Rightarrow \quad x = \frac{240}{45} = 5\frac{1}{3} \text{ hours}$$

5. Let the required average speed = x km/h

Ratio of average speed = Inverse ratio of the time taken by the car

$$12 \times 20 = x \times 15 \quad \Rightarrow \quad x = \frac{240}{15} = 16 \text{ km/h}$$

6. Let the number of days = x

Ratio of the number of men = Inverse ratio of time taken by the men.

$$72 \times 25 = x \times 30 \quad \Rightarrow \quad x = \frac{72 \times 25}{30} = 60 \text{ days}$$

7. Let the required workers = x

Ratio of the number of workers = Inverse ratio of time taken by the worker

$$56 \times 180 = x \times 70 \quad \Rightarrow \quad x = \frac{56 \times 180}{70} = 144$$

8. Let the required time = x hours

Ratio of the time taken = Inverse ratio of speed

$$6 \times 50 = x \times 75 \quad \Rightarrow \quad x = \frac{300}{75} = 4 \text{ hours}$$

9. Let the required person = x

Ratio of the persons = Inverse ratio of number of days

$$1800 \times 40 = x \times 24$$

$$x = \frac{72000}{24} = 3000 \text{ persons}$$

10. Let the required time = x weeks

Ratio of persons = Inverse ratio of time taken.

$$500 \times 8 = x \times 400 \quad \Rightarrow \quad x = 10 \text{ weeks.}$$

11. Let the number of the bats = x

$$39 \times 58 = (58 + 20) \times x$$

$$39 \times 58 = 78 \times x$$

$$x = \frac{39 \times 58}{78} \quad \Rightarrow \quad x = 29 \text{ bats.}$$

12. Given : Number of persons = 567

Let the number of extra persons = x

We know that,

$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$$

$$567 \times 9 = (567 + x) \times 7$$

$$\frac{567 \times 9}{7} = (567 + x)$$

$$81 \times 9 = 567 + x \quad \Rightarrow \quad 729 = 567 + x$$

$$x = 729 - 567 \quad \Rightarrow \quad x = 162 \text{ persons}$$

Exercise 11.3

1. Neeraj can do a work in 10 days.
Aman can do a work in 12 days
Neeraj, Aman and Virat together finish the work in 4 days
Let virat can do this work in x days.
We know that

$$\frac{1}{10} + \frac{1}{12} + \frac{1}{x} = \frac{1}{4}$$

$$\frac{1}{x} = \frac{1}{4} - \left(\frac{1}{10} + \frac{1}{12}\right) = \frac{1}{4} - \left(\frac{6+5}{60}\right) = \frac{1}{4} - \frac{11}{60}$$

$$\frac{1}{x} = \frac{15-11}{60} = \frac{4}{60} \quad \Rightarrow \quad x = \frac{60}{4} = 15 \text{ days}$$

2. One day's work of Rajesh and Rahul $= \frac{1}{20}$ parts

$$\text{After 8 days} = \frac{8}{20} \text{ parts}$$

$$\text{Remaining work} = 1 - \frac{8}{20} = \frac{12}{20}$$

Rahul can do the remaining work in $= \frac{12}{20} \times 30 = 18$ days

3. A's one day work $= \frac{1}{15}$; B's one day work $= \frac{1}{20}$

We know that

$$\text{A and B's one day work} = \frac{1}{15} + \frac{1}{20} = \frac{4+3}{60} = \frac{7}{60}$$

$$\text{A and B's 6 days work} = \frac{7}{60} \times 6 = \frac{42}{60}$$

$$\text{Remaining work} = 1 - \frac{42}{60} = \frac{60-42}{60} = \frac{18}{60}$$

$$\text{Let } A \text{ completed work} = x \text{ days}$$

$$\frac{18}{60} = \frac{x}{15} \quad \Rightarrow \quad x = \frac{18}{4} = 4\frac{1}{2} \text{ days.}$$

4. $A + B = \frac{1}{10} \quad \Rightarrow \quad B + C = \frac{1}{15} \quad \Rightarrow \quad C + A = \frac{1}{20}$

$$2(A + B + C) = \frac{1}{10} + \frac{1}{15} + \frac{1}{20}$$

$$2\,(A+B+C) = \frac{6+4+3}{60}$$

$$A+B+C = \frac{13}{60}\times\frac{1}{2} = \frac{13}{120}$$

A, B and C will complete the work in $9\frac{3}{13}$ days.

$A = 24$ days, $B = 17\frac{1}{7}$ days, $C = 120$ days

5. (Riya + Megha's) 1 day's work $= \frac{1}{6}$

(Megha + Ekta's) 1 day's work $= \frac{1}{8}$

(Ekta + Riya's) 1 day's work $= \frac{1}{4}$

2 (Riya + Megha + Ekta's) 1 day's work $= \frac{1}{6}+\frac{1}{8}+\frac{1}{4}$

$$= \frac{4+3+6}{24} = \frac{13}{24}$$

(Riya + Megha + Ekta's) 1 day's work $= \frac{13}{24\times 2} = \frac{13}{48}$

Thus, Riya + Megha + Ekta can weave a basket in

$$= \frac{48}{13} = 3\frac{9}{13} \text{ days}$$

Megha's 1 day's work $= \frac{13}{48}-\frac{1}{4} = \frac{13-12}{48}$

Megha can weave a basket in $= \frac{1}{48}$ parts $= 48$ days

Ekta's 1 days's work $= \frac{13}{48}-\frac{1}{6} = \frac{13-8}{48} = \frac{5}{48}$

Ekta's can weave a basket in $= \frac{48}{5} = 9\frac{3}{5}$ days

Riya's work's a day's work $= \frac{13}{48}-\frac{1}{8} = \frac{13-6}{48} = \frac{7}{48}$

Riya can weave a basket in $= \frac{48}{7} = 6\frac{6}{7}$ days.

6. A's 1 day's work $= 40$ days $= \frac{1}{40}$ parts

A's 8 days work $= 8\times\frac{1}{40} = \frac{8}{40}$

Remaining work $= 1-\frac{8}{40} = \frac{40-8}{40} = \frac{32}{40} = \frac{4}{5}$

$$B \text{ Completed the work in } = \frac{4}{5}x = 16 \quad \Rightarrow \quad 16 \times \frac{5}{4} = 20 \text{ days}$$

$$B \text{ finish } = x = 20 \text{ days}$$

If A and B work together, then,

$$1 \text{ day's work of } A \text{ and } B = \frac{1}{40} + \frac{1}{20} = \frac{1+2}{40} = \frac{3}{40}$$

$$A + B = \frac{40}{3} = 13\frac{1}{3} \text{ days}$$

Thus, A and B will be complete the work in $13\frac{1}{3}$ days.

7. $A + B = \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{5}{30} = \frac{1}{6}$

$$A \text{ work } = \frac{5}{10} = \frac{1}{2}$$

$$\text{Remaining work } = \frac{\frac{1}{6}}{\frac{1}{2}} = \frac{1}{6} \times \frac{2}{1} = \frac{1}{3}$$

Thus, A and B worked together for 3 days.

8. Given : $3W \times 17 = 5G \times 17$

Let days $= x$

$$(7W + 11G) \times x = 3W \times 17$$

$$\left(7W + \frac{11 \times 3}{5}W\right) \times x = 3W \times 17$$

$$\left(\frac{35W + 33W}{5}\right)x = 51W$$

$$68W \times x = 255W$$

$$x = \frac{255}{68} = \frac{15}{4} = 3\frac{3}{4} \text{ days}$$

Thus, $(7W + 11G)$ complete the work in $3\frac{3}{4}$ days.

9. $A + B + C = \frac{1}{2} + \frac{1}{6} + \frac{1}{3} = \frac{3+1+2}{6} = \frac{6}{6} = 1$

$A + B + C = 1$ day

$6 \longrightarrow$ ₹ 960

$1 \longrightarrow$ ₹ 160

A get $= 3 \times 160 =$ ₹ 480

B get $= 1 \times 160 =$ ₹ 160

C get $= 2 \times 160 =$ ₹ 320

10. $A + B + C = \frac{1}{15}$

$$A \text{ days} = \frac{1}{15} - \left(\frac{1}{30} + \frac{1}{40}\right) = \frac{1}{15} - \left(\frac{4+3}{120}\right) = \frac{1}{15} - \frac{7}{120}$$

$$= \frac{8-7}{120} = \frac{1}{120}$$

Thus, A alone do the work in 120 days.

Revision Exercise

1. Do it yourself. **2.** Do it yourself.

3. Do it yourself.

4. Cost of 12 pencils $= ₹\ 21$

Cost of 1 pencil $= \frac{21}{12}$

1 score pencils $= 20$

Cost of one score pencils $= \frac{21}{12} \times 20$

Cost of one score pencils $= ₹\ 35$

5. Labourer earns in one week $= ₹\ 672$

Labourer earns in 7 days $= ₹\ 672$

Labourer earns in 1 day $= ₹\ \frac{672}{7}$

Labourer earns in 18 days $= ₹\ \frac{672}{7} \times 18 = ₹\ 1728$

6. Let cost of ₹ 24024 $= x$ dollars

It is a direct variation.

$$\frac{175}{7350} = \frac{x}{24024} \Rightarrow \frac{1}{42} = \frac{x}{24024}$$

$$x = \frac{24024}{42} = 572$$

7. Let the car travels in 26.4 L of petrol $= x$ km

It is a direct variation.

$$\frac{67.5}{4.5} = \frac{x}{26.4} \Rightarrow 15 = \frac{x}{26.4}$$

$$x = 15 \times 26.4 = 396 \text{ km}$$

8. Let speed of car $= x$ km/h

$$60 \times 6 = x \times 5 \Rightarrow x = \frac{60 \times 6}{5}$$

$$x = 12 \times 6 \Rightarrow x = 72 \text{ km/h}$$

9. Let the remaining machines can make the same number of shirts in x days.

$$28 \times 20 = (28 - 4) \times x \quad \Rightarrow \quad 28 \times 20 = 24 \times x$$

$$7 \times 10 = 3 \times x \quad \Rightarrow \quad x = \frac{70}{3} = 23\frac{1}{3} \text{ days.}$$

10. Let time of each period $= x$ min

$$8 \times 30 = 6 \times x \quad \Rightarrow \quad 8 \times 5 = x \quad \Rightarrow \quad x = 40 \text{ min.}$$

Thus, time of 6 period is 40 minutes.

11. Let the stock of food is enought for 25 persons for x days.

$$40 \times 30 = 25 \times x$$

$$8 \times 6 = x \quad \Rightarrow \quad x = 48 \text{ days.}$$

12. A can do $\frac{1}{5}$ th of work $= 2$ days

A can do complete work in 10 days

B can do $\frac{2}{3}$ rd of work $= 8$ days

B can do complete work in 12 days

$$A + B = \frac{1}{10} + \frac{1}{12} = \frac{6 + 5}{60} = \frac{11}{60}$$

$(A + B)$ can do complete work in $\frac{60}{11} = 5\frac{5}{11}$ days.

13. One tap fills a tank $= 20$ minutes

Another tap fills $= 12$ minutes

Both taps are opened together, then $= \frac{1}{20} + \frac{1}{12} = \frac{3 + 5}{60} = \frac{8}{60}$

The tank will be full in $= \frac{60}{8} = \frac{15}{2} = 7\frac{1}{2}$ minutes

14. A can do a work $= 6$ days

B can do a work $= 8$ days

$$A + B = \frac{1}{6} + \frac{1}{8} = \frac{4 + 3}{24} = \frac{7}{24}$$

Work for 2 days done by both $= \frac{2 \times 7}{24} = \frac{14}{24} = \frac{7}{12}$

Remaining work $= 1 - \frac{7}{12} = \frac{5}{12}$

Number of days that A requires to finish the work

$$= \frac{\frac{5}{12}}{\frac{1}{6}} = \frac{5}{12} \times \frac{6}{1} = \frac{5}{2} = 2\frac{1}{2} \text{ days}$$

15. A farmer can reap a field in = 10 days

His wife can reap a field in = 8 days

Field reaped by them in 1 day $= \frac{1}{8} + \frac{1}{10} = \frac{5+4}{40} = \frac{9}{40}$

Both can reap a field in $= \frac{40}{9} = 4\frac{4}{9}$ days.

❑

12. Understanding Quadrilaterals

Exercise 12.1

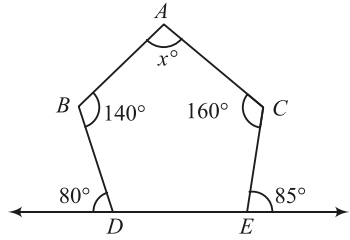
1. Do it yourself.

2. (i) 8 $\Rightarrow$ Each exterior angle of polygons $= \frac{360}{n} = \frac{360}{8} = 45°$

(ii) 12 $\Rightarrow$ Each exterior angle of polygons $= \frac{360}{n} = \frac{360}{12} = 30°$

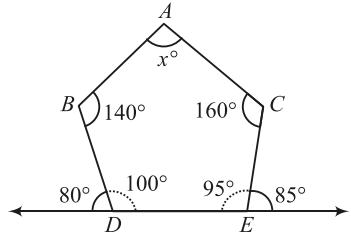
(iii) 24 $\Rightarrow$ Each exterior angle of polygons $= \frac{360}{n} = \frac{360}{24} = 15°$

3. $\square BCDE + \Delta ABC = 180 + 360$



$100° + 95° + 160° + 140° + x = 540°$

$x = 540° - 495° = 45°$



4 Two angles $= 140° + 160° = 300°$

Remaining four angles $= x$

Sum of hexagon angles $= 720°$

$$4x + 300 = 720°$$
$$4x = 720° - 300°$$
$$4x = 420°$$
$$x = 105°$$

Each angles of hexagon is 105°.

5. Each exterior angle $= 40°$

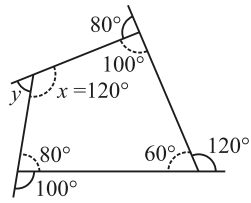
We know that,

$$\text{Number of sides of regular polygon} = \frac{360°}{\text{Exterior angle}}$$
$$= \frac{360°}{40°} = 9$$

Thus, number of sides is 9.

6. (i) Sum of angles of figure $= 360°$

$$x + 100° + 60° + 80° = 360°$$
$$x = 360° - 240° = 120°$$

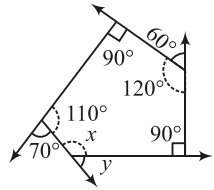


Then, $x + y = 180°$

$$120 + y = 180°$$
$$y = 180 - 120 = 60°$$

(ii) Sum of angles of figure $= 540°$

$$90° + 120° + 90° + x + 110° = 540°$$
$$x = 540° - 410° = 130°$$



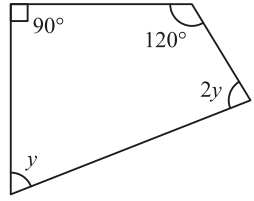
Then, $x + y = 180°$

$$130 + y = 180°$$
$$y = 180° - 130° = 50°$$

Exercise 12.2

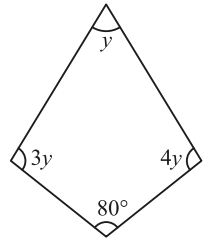
1. (i) Sum of angles of figure $= 360°$

$$90° + 120° + 2y + y = 360°$$
$$3y = 360° - 210°$$
$$3y = 150°$$
$$y = 50°$$



(ii) Sum of angles of figure $= 360°$

$$y + 3y + 80° + 4y = 360°$$
$$8y = 360° - 80°$$
$$8y = 280°$$
$$y = 35°$$



2. Four angles of quadrilateral $= 3x, 7x, 5x, 9x$

Sum of angles of quadrilateral $= 360°$

$$3x + 7x + 5x + 9x = 360° \Rightarrow 24x = 360° = 15°$$

Thus, angles $3 \times 15 = 45° \Rightarrow 7 \times 15 = 105°$

$\Rightarrow$ $5 \times 15 = 75° \Rightarrow 9 \times 15 = 135°$

3. Do it yourself.

4. Two angles of quadrilateral is 75° and 105°

Let one angle $= x$ and other angle $= 2x$

Sum of angles of quadrilateral $= 360°$

$$75 + 105 + x + 2x = 360° \Rightarrow 3x = 360 - 180$$

$$3x = 180 \Rightarrow x = 60°$$

Thus, remaining angles are 60° and 120°

5. One angle of quadrilateral is 150°.

Let other three angle $= x, x$ and x.

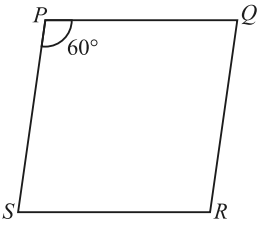
Sum of angles of quadrilateral $= 360°$

$$x + x + x + 150° = 360° \Rightarrow 150 + 3x = 360°$$

$$3x = 360° - 150° = 210°$$

$$x = 70°$$

6. Let $\angle R = x, \angle Q = 2x, \angle S = 2x$



Sum of angles of figure $= 360°$

$$x + 2x + 2x + 60° = 360°$$

$$5x = 360 - 60$$

$$5x = 300$$

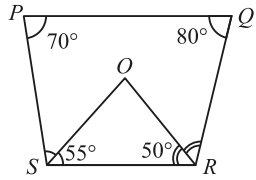
$$x = 60°$$

Thus, angles are $\angle R = 60° \Rightarrow \angle Q = 2 \times 60° = 120°$

$$\angle S = 2 \times 60° = 120°$$

7. $\angle P = 70°, \angle Q = 80°$

$\angle ROS = ?$



We know that

$$80° + \angle R = 180°$$

$$\angle R = 100°$$

$$70° + \angle S = 180°$$

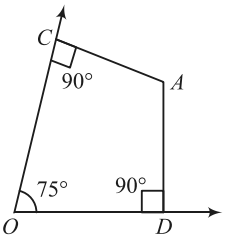
$$\angle S = 180 - 70 = 110°$$

$$\angle ROS = 180 - 55 - 50 = 75°$$

8. Sum of angles of quadrilateral = 360°

$$75° + 90° + 90° + \angle A = 360°$$
$$\angle A = 360° - 255°$$
$$\angle A = 105°$$

Thus, $\angle CAD$ is 105°.

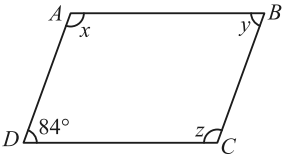


Exercise 12.3

1. Do it yourself.

2. (i) We know that

$$x + 84° = 180°$$
$$x = 180° - 84° = 96°$$
$$96° + y = 180°$$
$$y = 180° - 96° = 84°$$
$$84° + z = 180°$$
$$z = 180° - 84 = 96°$$

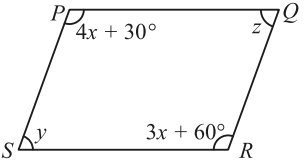


(ii) We know that

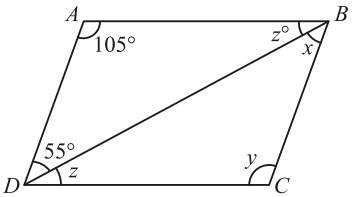
$$z + 3x + 60° = 180° \ldots (i)$$
$$z + 4x + 30° = 180° \ldots (ii)$$

Taking eq. (i) and (ii),

$$-x + 30 = 0$$
$$x = 30°, y = 30°, z = 30°$$



3. We know that

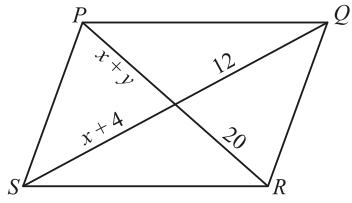


In ΔABD,

$$105 + 55 + z = 180°$$
$$z = 180 - 160 = 20°$$
$$55° + 20° + y = 180°$$
$$y = 180° - 75° = 105°$$
$$20 + 105 + x = 180°$$
$$x = 180 - 125 \Rightarrow x = 55°$$

Thus, angles are $x = 55° \Rightarrow y = 105° \Rightarrow z = 20°$

4. $x + 4 = 12 \Rightarrow x = 8$



$\Rightarrow \quad x + y = 20 \Rightarrow y = 20 - 8 = 12$

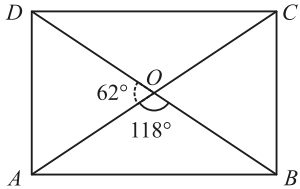
5. (i) We know that,

$\angle ABO + \angle BAO = -118° + 180°$

$\therefore \quad \angle ABO = \angle BAO$

$\angle ABO + \angle ABO = -118° + 180°$

$2\angle ABO = -118° + 180° = 62 \Rightarrow \angle ABO = 31°$



Thus, angle of $\angle ABO$ is 31°.

(ii) $\angle ADO + \angle DAO = 180° - 62°$

$\angle ADO + \angle DAO = 118°$

$\therefore \quad \angle ADO = \angle DAO \Rightarrow \angle ADO + \angle ADO = 118°$

$2\angle ADO = 118° \Rightarrow \angle ADO = 59°$

Thus, angle of $\angle ADO = 59°$.

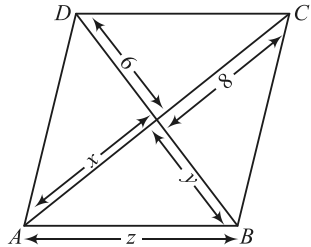
6. Given : $x = 8, y = 6$

$z = \sqrt{x^2 + y^2}$

$= \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36} = \sqrt{100}$

$= 10$



7. Given : $\angle A : \angle D = 5 : 7$

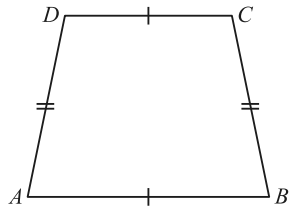
$5x + 7x = 180°$

$12x = 180°$

$x = 15°$

$\angle A = 5 \times 15 = 75°$

$\angle D = 7 \times 15 = 105°$



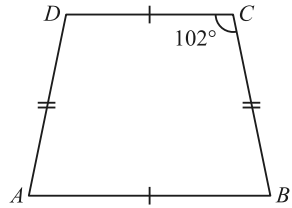
$$\angle B + \angle C = 180°$$
$$3x + 11 + 5x - 31 = 180°$$
$$8x - 20° = 180°$$
$$8x = 180° + 20° = 200° \quad \Rightarrow \quad x = 25°$$
$$\angle B = 3 \times 25 + 11 = 86°$$
$$\angle C = 5 \times 25 - 31 = 94°$$

Thus, angles are: $\angle A = 75°$, $\angle B = 86°$, $\angle C = 94°$ and $\angle D = 105°$

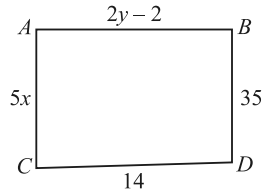
8.
$$\angle C = 102°$$
$$\angle C + \angle B = 180°$$
$$\angle B = 180° - 102° = 78°$$

We given, $\angle A = \angle B = 78°$

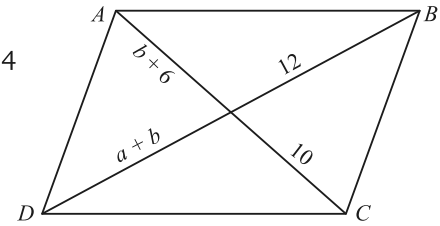
$\angle C = \angle D = 102°$



9.
$$5x = 35, x = 7$$
$$2y - 2 = 14$$
$$2y = 14 + 2 = 16$$
$$y = 8$$

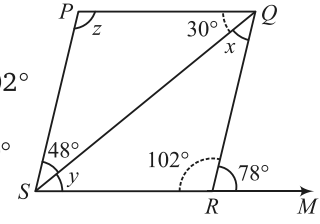


10. Given :
$$b + 6 = 10$$
$$b = 10 - 6 = 4$$
$$a + b = 12$$
$$a + 4 = 12$$
$$a = 12 - 4$$
$$a = 8$$



11. We know that
$$\angle P = \angle R$$
$$102° = \angle P \quad \Rightarrow \quad z = 102°$$
$$102° + 48° + y = 180°$$
$$y = 180° - 150° = 30°$$
$$30° + x + 102° = 180°$$
$$x = 48°$$

Thus, angles $x = 48°$, $y = 30°$,
$$z = 102°$$



Revision Exercise

1. Do it yourself.

2. Do it yourself.

3. $$\frac{\text{Exterior angle}}{\text{Interor angle}} = \frac{360/n}{\frac{(n-2)\times 180}{n}} = \frac{360}{(n-2)\times 180^\circ}$$

$$\frac{360}{(n-2)\times 180} = \frac{2}{7}$$

$$7 = (n-2) \quad \Rightarrow \quad n = 7 + 2 = 9$$

Thus, number of sides in the polygon is 9.

4. One angle of parallelogram $(A) = 80^\circ$

Other angle $(B) = 180^\circ - 80^\circ = 100^\circ$

Similarly $C = 80^\circ, D = 100^\circ$

5. One angle of quadrilateral $= 65^\circ$

Measure of two angles $= 180^\circ - 65^\circ = 115^\circ$

6. Angle of quadrilateral $= 2x, 3x, 5x, 8x$

We know that,

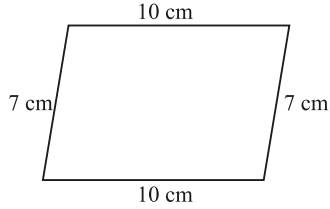
$$2x + 3x + 5x + 8x = 360^\circ$$

$$18x = 360^\circ \quad \Rightarrow \quad x = 20^\circ$$

Thus, angles are $2 \times 20 = 40^\circ, 3 \times 20 = 60^\circ$

$5 \times 20 = 100^\circ$ and $8 \times 20^\circ = 160^\circ$.

7. Two sides of parallelogram is 7 cm and 10 cm



Perimeter $= 7 + 10 + 7 + 10 = 34$ cm

8. Let the two adjacent angles are x and y.

Then, $x - y = 30^\circ$ ...(i)

$x + y = 180^\circ$ ...(ii)

Taking eq. (i) and (ii),

$2x = 210 \quad \Rightarrow \quad x = 105^\circ \quad \Rightarrow \quad y = 105^\circ - 30^\circ = 75^\circ$

Thus, angles are 105°, 75°, 105°, 75°.

9. $\angle A = 50^\circ$

$\angle B = 180^\circ - 50^\circ = 130^\circ$

$\angle C = 180 - 50^\circ = 130^\circ$

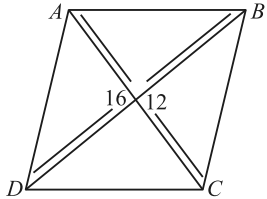
$\angle D = 50^\circ$

Thus, angles are 50°, 130°, 50°, 130°.

10. Diagonal 16 and 12 cm

$$\text{Sides} = \sqrt{\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2}$$

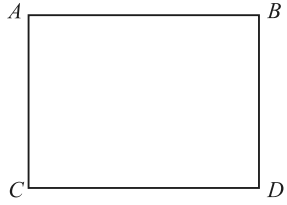
$$= \sqrt{\left(\frac{16}{2}\right)^2 + \left(\frac{12}{2}\right)^2} = \sqrt{100} = 10$$



Each sides $= 10\,\text{cm}$

11. Given :

Ratio of the sides of a rectangle $= \frac{2}{3}$



$$\text{Perimeter} = 20\,\text{cm}$$

$$2(2x + 3x) = 20$$

$$5x = 10$$

$$x = 2\,\text{cm}$$

Thus, sides of rectangles are

$$2 \times 2 = 4\,\text{cm} \quad \Rightarrow \quad 2 \times 3 = 6\,\text{cm}.$$

❑

13. Areas of Rectilinear Figures

Exercise 13.1

1. Given :

(i) Diagonal of a rectangle $= 34$ cm

Breadth of rectangle $= 16$ cm

We know that $d = \sqrt{l^2 + b^2}$

$$34 = \sqrt{l^2 + (16)^2}$$

Squaring both sides

$$(34)^2 = l^2 + 256 = l^2 = 1156 - 256 = 900$$

$$l = \sqrt{900} = 30\,\text{cm}.$$

Thus, length is 30 cm.

(ii) Area of rectangle $= l \times b = 30 \times 16 = 480\ \text{cm}^2$

2. Given :

(i) Perimeter of rectangle $= 46$ m, Length $= 15$ m

$$\text{Perimeter} = 2(l + b) \quad \Rightarrow \quad 46 = 2(15 + b)$$

$$15 + b = 23 \quad \Rightarrow \quad b = 23 - 15 = 8$$

$$\text{Breadth } b = 8\ \text{m}$$

(ii) Diagonal $d = \sqrt{l^2 + b^2} = \sqrt{(15)^2 + (8)^2}$

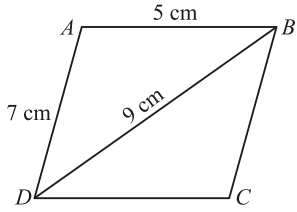
$= \sqrt{225 + 64} \quad \Rightarrow \quad d = \sqrt{289} = 17$ m

3. Given : Area of square $= 729\ \text{m}^2$

We know that Area of square $= (\text{side})^2$

$a^2 = 729 \quad \Rightarrow \quad a = \sqrt{729} \quad \Rightarrow \quad a = 27$ m

4. $l = \dfrac{5 + 7 + 9}{2} = \dfrac{21}{2}$ cm.



Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\dfrac{21}{2} \times \dfrac{11}{2} \times \dfrac{3}{2} \times \dfrac{7}{2}} = \dfrac{21}{4}\sqrt{11}\ \text{cm}^2$

Area of parallelogram $ABCD = 2 \times \dfrac{21}{4}\sqrt{11} = \dfrac{21}{2} \times \sqrt{11}$

$= 34.83\ \text{cm}^2$

5. Given : $A_1 = 5.76\text{m} \times 3.1\text{m} = 17.856\ \text{m}^2$

$A_2 = \dfrac{24 \times 10}{100 \times 100} = \dfrac{24}{1000}\ \text{m}^2$

Number of tiles $= \dfrac{17.856}{\frac{24}{1000}} = \dfrac{0.744}{\frac{1}{1000}} = 0.744 \times 1000 = 744$

Cost of tiles $= ₹1.5 \times 744 = ₹\ 1116$

6. Area of rhombus $= 10.2\ \text{cm}^2$, Sides $= 6$ cm

Let altitude $= x$

$x \times 6 = 10.2 \quad \Rightarrow \quad x = \dfrac{10.2}{6} = 1.7$ cm

7. Area of rhombus $= 28\,\text{cm}^2$, Perimeter $= 28$ cm

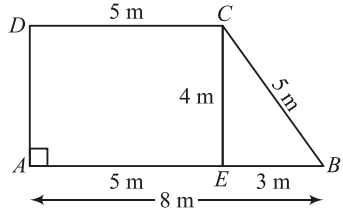
$4a = 28 \quad \Rightarrow \quad a = 7$ cm

We know that $a \times h = 28 \quad \Rightarrow \quad h = \dfrac{28}{7} = 4$ cm

Thus altitude is 4 cm.

8. Area of quadrilateral = Area of rectangle $AECD$ + Area of ΔBEC

$= 5 \times 4 + \dfrac{1}{2} \times 4 \times 3 = 26\ \text{m}^2$



9. Area of a trapezium $= \frac{1}{2}$ (sum of parallel sides) $\times$ height

$$91 = \frac{1}{2} \times (a + b)h = \frac{1}{2}(a + a + 8) \times 7$$

$$13 = \frac{1}{2}(2a + 8)$$

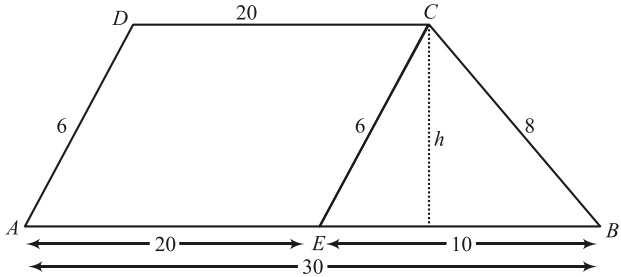
$$(2a + 8) = 26$$

$$2a = 26 - 8 = 18$$

$$a = 9\,\text{cm}, b = 9 + 8 + 17\,\text{cm}$$

10. Do it yourself.

11.



$$\text{In } \Delta EBC, s = \frac{a+b+c}{2} = \frac{6+8+10}{2} = \frac{24}{2} = 12\,\text{cm}$$

$$\text{Area of } \Delta EBC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{12(12-6)(12-8)(12-10)}$$

$$= \sqrt{12 \times 6 \times 4 \times 2}$$

$$= \sqrt{6 \times 2 \times 6 \times 4 \times 2} = 6 \times 4 = 24\ \text{m}^2$$

$$\text{Area of } \Delta EBC = \frac{1}{2} \times \text{base} \times \text{height}$$

$$24 = \frac{1}{2} \times 10 \times h \quad \Rightarrow \quad h = \frac{24 \times 2}{10} = 4.8\ \text{m}$$

$$\text{Area of } DABC = \frac{1}{2}(a + b) \times h = \frac{1}{2}(20 + 30) \times 4.8$$

$$= \frac{1}{2}(50) \times 4.8 = 25 \times 4.8 = 120\ \text{m}^2$$

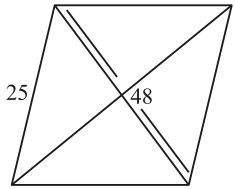
12. One diagonal $= 48$ m

One side $= 25$ m

Other diagonal $= 2\sqrt{(25)^2 - (24)^2}$

$= 2\sqrt{625 - 576} = 2\sqrt{(49)}$

$= 2 \times 7 = 14$ m



Area of rhombus $= \frac{1}{2} \times 14 \times 48 = 336 \text{ m}^2$

13. Area of quadrilateral $= \frac{1}{2}(a + b) \times h = \frac{1}{2}(7.2 + 8.8) \times 28.4$

$= \frac{1}{2} \times 16 \times 28.4 = 227.2 \text{ m}^2$

14. Area of quadrilateral $= \frac{1}{2}(a + b) \times h = \frac{1}{2}(8.5 + 7.5) \times 24$

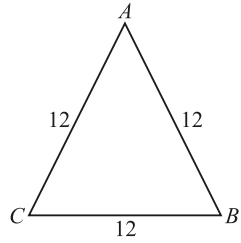
$= \frac{1}{2} \times 16 \times 24 = 192 \text{ cm}^2$

15. $s = \frac{a + b + c}{2} = \frac{12 + 12 + 12}{2} = \frac{36}{2} = 18$

Area of triangle $= \sqrt{s(s - a)(s - b)(s - c)}$

$= \sqrt{18 \times 6 \times 6 \times 6} = 36\sqrt{3}$

$= 36 \times 1.732 = 62.35 \text{ cm}^2$



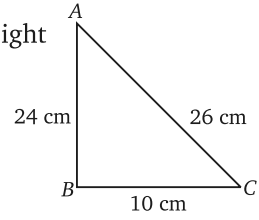
16. $s = \frac{12 + 10 + 10}{2} = \frac{32}{2} = 16$

Area of triangle $= \sqrt{s(s - a)(s - b)(s - c)}$

$= \sqrt{16(4) \times 6 \times 6} = 4 \times 4 \times 3 = 48 \text{ cm}^2$

17. Right angle of triangle $ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 10 \times 24 = 120 \text{ cm}^2$



18. $s = \frac{25 + 39 + 56}{2} = \frac{120}{2} = 60$

Area of triangle $= \sqrt{s(s - a)(s - b)(s - c)}$

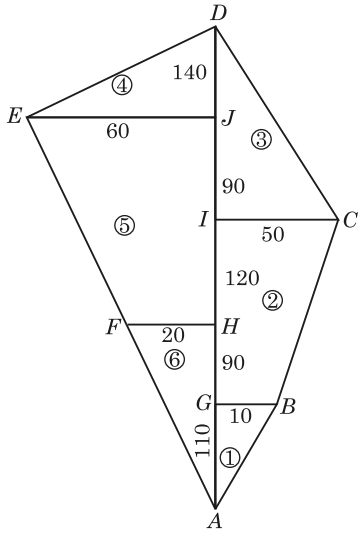
$= \sqrt{60 \times 4 \times 21 \times 35}$

$= \sqrt{15 \times 4 \times 4 \times 7 \times 3 \times 7 \times 5} = 15 \times 4 \times 7$

$\frac{1}{2} \times 56 \times \text{height} = 15 \times 4 \times 7 \quad \Rightarrow \quad \text{height} = 15 \text{ cm}$

Exercise 13.2

1. Area of the field = Δ Area (ΔAGB) + Area (trap $GBCI$)
+ Area (ΔDIC) + Area (ΔDJE) + Area (trap $EJHF$) + Area (ΔAHF)



1. Area (ΔAGB) $= \frac{1}{2} \times 110 \times 10 = 550\,\text{m}^2$
2. Area (trap $GBCI$) $= \frac{1}{2}(50 + 10) \times 210 = 210 \times 30 = 6300\,\text{m}^2$
3. Area (ΔDIC) $= \frac{1}{2} \times 230 \times 50 = 5750\,\text{m}^2$
4. Area (ΔDJE) $= \frac{1}{2} \times 140 \times 60 = 4200\,\text{m}^2$
5. Area (trap $EJHF$) $= \frac{1}{2}(60 + 20) \times 210 = 40 \times 210 = 8400\,\text{m}^2$
6. Area (ΔAHF) $= \frac{1}{2} \times 200 \times 20 = 2000\,\text{m}^2$

Area of the field $= 550\,\text{m}^2 + 6300\,\text{m}^2 + 5750\,\text{m}^2 + 4200\,\text{m}^2$
$+ 8400\,\text{m}^2 + 2000\,\text{m}^2 = 27200\,\text{m}^2$

3. Do yourself. **4.** Do yourself. **5.** Do yourself.

Revision Exercise

1. Given :

Area of rectangle $= 120\ \text{cm}^2$, Breadth $= 8\,\text{cm}$

We know that

Area of rectangle $= l \times b \quad \Rightarrow \quad 120 = l \times 8$

$l = 15\,\text{cm}$

Perimeter $= 2(l + b) = 2(15 + 8)$

$= 2(23) = 46$ cm

2. $l \times b = 84\ \text{cm}^2 \quad \Rightarrow \quad l - b = 5\ \text{m}$

$(l + b)^2 = (l - b)^2 + 4lb = (5)^2 + 4 \times 84$

$(l + b)^2 = 361 \quad \Rightarrow \quad l + b = \sqrt{361} = 19$

Perimeter $= 2(l + b) = 2 \times 19 = 38$ cm

3. Perimeter $= 36$ cm

$4a = 36 \quad \Rightarrow \quad a = 9$ cm

Area of square $= a^2 = (9)^2 = 81\ \text{cm}^2$

4. Area of square $= a^2 \quad \Rightarrow \quad a^2 = 1.69\ \text{m}^2 \quad \Rightarrow \quad a = 1.3$ m

Perimeter of square $= 4a = 4 \times 1.3 = 5.2$ m

5. Diagonal of square $= a\sqrt{2}$

$a\sqrt{2} = 12 \quad \Rightarrow \quad a = 6\sqrt{2}$ cm $\quad \Rightarrow \quad a = 8.48$ cm

Area of square $= a^2 = (6\sqrt{2})^2 = 72\ \text{cm}^2$

6. Diagonal of square $= a\sqrt{2}$

$$a\sqrt{2} = 15 \quad \Rightarrow \quad a = \frac{15}{\sqrt{2}} = 10.6\ \text{m}$$

Perimeter $= 4a = 4 \times 10.6 = 42.4$ m.

7. Area of square $= a^2 \quad \Rightarrow \quad a^2 = 169 \quad \Rightarrow \quad a = 13$ cm

(i) $a = 13$ cm

(ii) Perimeter $= 4a = 4 \times 13 = 52$ cm

8. Length of rectangle $= 16$ cm

$2(l + b) = 4a \quad \Rightarrow \quad 2(16 + b) = 4 \times 12.5$

$2(16 + b) = 50 \quad \Rightarrow \quad 16 + b = 25$

$b = 25 - 16 \quad \Rightarrow \quad b = 9$ cm

Area of rectangle $= l \times b = 16 \times 9 = 144\ \text{cm}^2$

9. Perimeter of square $=$ Area of square

$4a = a^2 \quad \Rightarrow \quad a = 4$

Area of square $= a^2 = (4)^2 = 16$ sq. units

10. (i) Area of triangle $= \frac{1}{2} \times 10 \times 24 = 120\ \text{cm}^2$

(ii) Area of triangle $= \frac{1}{2} \times 18 \times 24 = 216\ \text{m}^2$

11. (i) Area of triangle $= \frac{1}{2} \times 6 \times 4 = 12\ \text{cm}^2$

(ii) $\frac{1}{2} \times 8 \times h = \frac{1}{2} \times 6 \times 4 \quad \Rightarrow \quad h = 3$ cm

12. (i) Area of triangle $= \frac{1}{2} \times 16 \times 12 = 96 \text{ cm}^2$

(ii) $\frac{1}{2} \times 16 \times 12 = \frac{1}{2} \times 20 \times h$

$h = \frac{192}{20} = 9.6 \text{ cm.}$

Thus, height is 9.6 cm.

13. (i) Area of triangle $= \frac{1}{2} \times 4.8 \times 6 = 14.4 \text{ m}^2$

(ii) $\frac{1}{2} \times 6.4 \times h = \frac{1}{2} \times 4.8 \times 6 \quad \Rightarrow \quad h = \frac{4.8 \times 6}{6.4} = 4.5 \text{ m}$

14. $(4x + 5x + 3x) = 96$

$12x = 96 \quad \Rightarrow \quad x = 8 \text{ cm}$

Lengths $= 4 \times 8, 5 \times 8, 3 \times 8 = 32 \text{ cm}, 40 \text{ cm}, 24 \text{ cm}$

Area of triangle $= \frac{1}{2} \times 32 \times 24 = 384 \text{ cm}^2$

15. Perimeter $= 50 \text{ cm}$

$\therefore \quad s = \frac{a + b + c}{2} = \frac{13 + 13 + 24}{2} = \frac{50}{2} = 25 \text{ cm}$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{25 \times 12 \times 12 \times 1} = 5 \times 12 = 60 \text{ cm}^2$

16. Perimeter of rhombus $= 40 \text{ cm.}$

One diagonal $= 16 \text{ cm}$

(i) Other diagonal $= \frac{40 - 16}{2} = \frac{24}{2} = 12 \text{ cm}$

(ii) Area of rhombus $= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 16 \times 12 = 96 \text{ cm}^2$

17. Area of rhombus $= x \times h = 18 \times 12 = 216 \text{ cm}^2$

18. $d_1 = 4x, \; d_2 = 3x$

$\frac{1}{2} \times 4x \times 3x = 384$

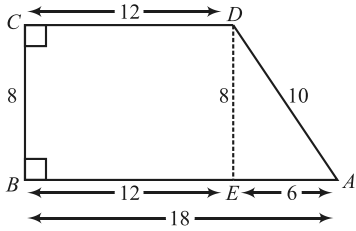
$x^2 = 64 \quad \Rightarrow \quad x = 8 \text{ cm} \quad \Rightarrow \quad d_1 = 4 \times 8 = 32 \text{ cm}$

and $d_2 = 3 \times 8 = 24 \text{ cm}$

Sides $= \sqrt{\left(\frac{32}{2}\right)^2 + \left(\frac{24}{2}\right)^2}$

$= \sqrt{256 + 144} = \sqrt{400} \quad \Rightarrow \quad \text{Sides} = 20 \text{ cm}$

19.



$$\text{Area of Trapezium, } ABCD = \frac{1}{2}(a+b)\times h = \frac{1}{2}(12+18)\times 8$$
$$= \frac{1}{2}\times 30\times 8 = 30\times 4 = 120 \text{ cm}^2$$

20.
$$\text{Area of trapezium} = \frac{1}{2}(a+b)\times h$$
$$279 = \frac{1}{2}(a+a+5)\times 18$$
$$\frac{31}{2} = \frac{1}{2}(a+a+5) \quad \Rightarrow \quad 31 = 2a+5$$
$$2a = 31-5 = 26 \quad \Rightarrow \quad a = 13 \text{ cm}$$
$$\text{Other sides} = a+5 = 13+5 = 18 \text{ cm}$$

❑

14. Circumference and Area of a Circle

Exercise 14.1

1. (i) $c = 7.7$ m, Circumference $= 2\pi r$

$$2\times\frac{22}{7}\times r = 7.7$$
$$\frac{77}{10} = \frac{44r}{7} \quad \Rightarrow \quad r = 1.225 \text{ m}$$
$$\text{diameter} = 2\times r$$
$$d = 2\times 1.225 = 2.45 \text{ m}$$

(ii) $\text{Circumference} = 2\pi r = 2\times\frac{22}{7}\times 2.8 = 17.6 \text{ m}$

$$d = 2r = 2\times 2.8 = 5.6 \text{ m}$$

(iii) $d = 2r$

$$r = \frac{d}{2} = \frac{4.2}{2} = 2.1 \text{ m}$$
$$c = 2\pi r = 2\times\frac{22}{7}\times 2.1 = 13.2 \text{ m}$$

2. (i) $c = 2\pi r = 2 \times \frac{22}{7} \times 7 = 44 \text{ cm}$

(ii) $c = 2\pi r = 2 \times \frac{22}{7} \times 5 = \frac{220}{7} \text{ m}$

(iii $c = 2\pi r = 2 \times \frac{22}{7} \times 2 = \frac{88}{7} \text{ km}$

3. (i) $c = 2\pi r = 2 \times \frac{22}{7} \times 3.5 = 22 \text{ cm}$

(ii) $c = 2\pi r = 2 \times \frac{22}{7} \times 2.1 = 13.2 \text{ m}$

(iii) $c = 2\pi r = 2 \times \frac{22}{7} \times 5.6 = 35.2 \text{ km}$

4. (i) $c = 2\pi r = 6.28$

$$2 \times \frac{22}{7} \times r = 6.28$$

$$r = 1 \text{ cm}$$

$$d = 2r = 2 \times 1 = 2 \text{ cm}$$

(ii) $2\pi r = 44$

$$2 \times \frac{22}{7} \times r = 44 \quad \Rightarrow \quad r = 7$$

$$d = 2r = 2 \times 7 = 14 \text{ m}$$

(iii) $2\pi r = 5.5 \text{ km}$

$$2 \times \frac{22}{7} \times r = 5.5$$

$$r = 0.875 \text{ km}$$

$$d = 2r = 2 \times 0.875 = 1.75 \text{ km}.$$

5. (i) $2\pi r = 26.4$

$$2 \times \frac{22}{7} \times r = 26.4 \quad \Rightarrow \quad r = 4.2 \text{ cm}.$$

(ii) $2\pi r = 35$

$$2 \times \frac{22}{7} \times r = 35 \quad \Rightarrow \quad r = 5.57 \text{ m}$$

(iii) $2\pi r = 6.6 \text{ km}$

$$2 \times \frac{22}{7} \times r = 6.6 \quad \Rightarrow \quad r = 1.05 \text{ km}$$

6. (i) Area of circle $= \pi r^2 = \frac{22}{7} \times 3.5 \times 3.5 = 38.5 \text{ cm}^2$

(ii) Area of circle $= \pi r^2 = \frac{22}{7} \times 4.2 \times 4.2 = 55.44 \text{ m}^2$

(iii) Area of circle $= \pi r^2 = \frac{22}{7} \times 7 \times 7 = 154 \text{ km}^2$

7. (i) $\text{Area of circle} = \pi r^2 = \frac{22}{7} \times 2.1 \times 2.1 = 13.86\ \text{cm}^2$

(ii) $\text{Area of circle} = \pi r^2 = \frac{22}{7} \times 5.6 \times 5.6 = 98.56\ \text{m}^2$

(iii) $\text{Area of circle} = \pi r^2 = \frac{22}{7} \times \frac{3.5}{2} \times \frac{3.5}{2} = 9.625\ \text{km}^2$

8. (i) $\pi r^2 = \pi\ \text{cm}^2 \Rightarrow r = 1\ \text{cm}$

(ii) $\pi r^2 = 55.44$

$\frac{22}{7} \times r^2 = 55.44 \Rightarrow r = 4.2\ \text{m}$

(iii) $\pi r^2 = 1.54\ \text{km}^2$

$\frac{22}{7} \times r^2 = 1.54 \Rightarrow r^2 = 0.49 \Rightarrow r = 0.7\ \text{km}$

Word Problems

1. $\text{Area of circle} = \pi r^2 = 3.14 \times \frac{5}{2} \times \frac{5}{2}$

$A = \frac{3.14 \times 25}{4} = 19.625\ \text{cm}^2$

2. $\pi r^2 = 154$

$\frac{22}{7} \times r^2 = 154 \Rightarrow r^2 = 49 \Rightarrow r = 7\ \text{m}$

Circumference of circle $c = 2\pi r = 2 \times \frac{22}{7} \times 7 = 44\ \text{m}$

Perimeter of square = 44 cm

3. $4a = 44\ \text{cm} \Rightarrow a = 11$

$\text{Area of square} = a^2 = (11)^2 = 121\ \text{cm}^2$

Circumference of circle = 44 cm

$2\pi r = 44$

$2 \times \frac{22}{7} \times r = 44 \Rightarrow r = 7$

$\text{Area of circle} = \pi r^2 = \frac{22}{7} \times 49 = 154\ \text{cm}^2$

$\Rightarrow 154\ \text{cm}^2 > 121\ \text{cm}^2$

Circle has greater area = $154 - 121 = 33\ \text{cm}^2$

4. $\frac{r_1}{r_2} = \frac{3}{2} \Rightarrow \frac{c_1}{c_2} = \frac{2\pi r_1}{2\pi r_2} = \frac{3}{2} \Rightarrow c_1 : c_2 = 3 : 2$

5. Perimeter of equilateral triangle = Circumference of circle

$3 \times \text{side} = 2\pi r$

$$3 \times 6.6 = 2 \times \frac{22}{7} \times r \quad \Rightarrow \quad r = 3.15 \text{ cm}$$

$$d = 2r = 2 \times 3.15 = 6.3 \text{ cm}$$

6. Radius of wheel = 35 cm

One complete round of wheel = $2\pi r$

24 complete round of wheel = $24 \times 2\pi r$

$$= 24 \times 2 \times \frac{22}{7} \times 35 = 5280 \text{ cm}$$

7.
$$2\pi r_1 = 2 \times \frac{22}{7} \times 105 = 660 \text{ m}$$

$$2\pi r_2 = 2 \times \frac{22}{7} \times 112 = 704 \text{ m}$$

Circumference of second circle longer by = 704 – 660 = 44 m

8.
$$c = 2\pi r$$

$$66 \times 400 = 2 \times \frac{22}{7} \times r$$

$$r = \frac{66 \times 400 \times 7}{44} = 4200 \text{ cm}$$

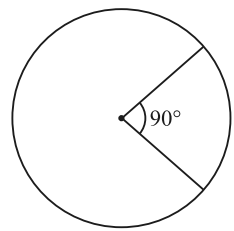
$$d = 2r = 2 \times 4200 = 8400 \text{ cm}$$

Exercise 14.2

1. (i) Area of sector $= \pi r^2 \frac{\theta}{360^\circ}$

$$= \frac{22}{7} \times \frac{14 \times 14 \times 90}{360}$$

$$= 154 \text{ cm}^2$$



Area of right angle $= \frac{1}{2} \times 14 \times 14$

$$= 98 \text{ cm}^2$$

Area of segment $= 154 - 98 = 56 \text{ cm}^2$

(ii) Do it yourself. (iii) Do it yourself.

2. (i) Length of arc $= \frac{2\pi r\ \theta}{360^\circ} = \frac{2 \times 22 \times 2.8}{7} \times \frac{90^\circ}{360^\circ} = 4.4 \text{ cm}$

(ii) Do it yourself. (iii) Do it yourself.

3. (i) Area of sector $= \frac{\pi r^2 \times \theta}{360^\circ} = \frac{22}{7} \times \frac{3.5 \times 3.5 \times 60^\circ}{360^\circ} = 6\frac{5}{12} \text{ cm}$

(ii) Do it yourself. (iii) Do it yourself.

Word Problems

1. Area of sector $= \frac{22}{7} \times \frac{2 \times 2 \times 45^\circ}{360^\circ} = \frac{22 \times 4}{7 \times 8} = 1.571 \text{ cm}^2$

Area of disc $= \pi r^2 = \frac{22}{7} \times 2 \times 2 = \frac{88}{7} = 12.57 \text{ cm}^2$

Area of remaining part of the disc $= 12.57 - 1.57 = 11 \text{ cm}^2$

2. $\frac{\pi r^2 \theta}{360^\circ} = \frac{1}{10} \pi r^2 \quad \Rightarrow \quad \theta = \frac{360^\circ}{10} = 36^\circ$

Thus, the angle of the sector is 36°.

3. $\frac{2\pi r\theta}{360^\circ} = 22 \quad \Rightarrow \quad 2 \times \frac{22}{7} \times r \times \frac{18^\circ}{360^\circ} = 22$

$r = \frac{20 \times 7}{2} = 70 \text{ m} \quad \Rightarrow \quad c = 2\pi r = 2 \times \frac{22}{7} \times 70 = 440 \text{ m}$

4. $\frac{\pi r^2 \theta}{360^\circ} = 3.85 \quad \Rightarrow \quad \frac{22}{7} \times \frac{r^2 \times 36^\circ}{360^\circ} = 3.85$

$r^2 = 12.25 \quad \Rightarrow \quad r = \sqrt{12.25} \quad \Rightarrow \quad r = 3.5 \text{ cm}$

Length of the arc of the sector $= \frac{2\pi r\theta}{360^\circ} = 2 \times \frac{22}{7} \times \frac{3.5}{360} \times 36$

Length of the arc of the sector = 2.2 cm

5. Do it yourself.

6. One area of the sector $= \frac{22}{7} \times \frac{7 \times 7 \times 90^\circ}{360^\circ}$

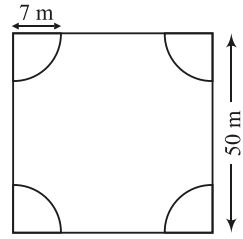
$= \frac{22}{7} \times \frac{7 \times 7}{4} = \frac{154}{4}$

Area of the 4 sector $= 4 \times \frac{154}{4} = 154 \text{ m}^2$

Area of square $= (a)^2 = (50)^2 = 2500 \text{ m}^2$

Area of remaining part of park

$= 2500 - 154 = 2346 \text{ m}^2$



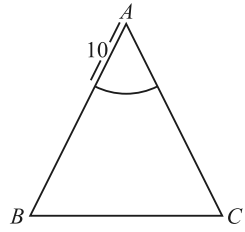
7. In equilateral triangle every angle be 60°.

Area of sector $= \frac{\pi r^2 \times \theta}{360^\circ}$

$= \frac{22}{7} \times \frac{10 \times 10 \times 60}{360}$

$= \frac{22}{7} \times \frac{10 \times 10}{6}$

Area of sector $= 52.38 \text{ m}^2$



So, the horse can graze the area is 52.38 m^2.

Revision Exercise

1. (i) $c = 2\pi r \Rightarrow 31.4 = 2 \times \frac{22}{7} \times r$

$2 \times 3.14 \times r = 31.4 \Rightarrow r = 5 \text{ m}$

Area of circle $= \pi r^2 = 3.14 \times 5 \times 5 = 78.5 \text{ m}^2$

(ii) Area of the circle $= \pi r^2 \Rightarrow \frac{22}{7} \times r^2 = 50.24$

$r^2 = 16 \Rightarrow r = 4 \text{ m}$

Circumference of circle $= 2\pi r = 2 \times 3.14 \times 4 = 25.12 \text{ m}$

(iii) $c = 2\pi r = 2 \times 3.14 \times 2.5 = 15.7 \text{ m}$

Area of circle $= \pi r^2 = 3.14 \times 2.5 \times 2.5 = 19.625 \text{ m}^2$

2. (i) $\pi r^2 = 154 \Rightarrow \frac{22}{7} \times r^2 = 154$

$r^2 = 49 \Rightarrow r = 7 \text{ cm}$

Circumference of circle $= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44 \text{ cm}$

(ii) $\pi r^2 = 6.16 \Rightarrow \frac{22}{7} \times r^2 = 6.16$

$r^2 = 1.96 \Rightarrow r = 1.4 \text{ m}$

$c = 2\pi r = 2 \times \frac{22}{7} \times 1.4 = 8.8 \text{ m}$

3. (i) $2\pi r = 132 \Rightarrow 2 \times \frac{22}{7} \times r = 132$

$r = 21 \text{ cm}$

Area of the circle $= \pi r^2 = \frac{22}{7} \times 21 \times 21 = 1386 \text{ cm}^2$

(ii) $2\pi r = 22 \Rightarrow 2 \times \frac{22}{7} \times r = 22 \Rightarrow r = \frac{7}{2} \text{ m}$

Area of the circle $= \pi r^2 = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 38.5 \text{ m}^2$

4. $\pi r^2 = 1386 \Rightarrow \frac{22}{7} \times r^2 = 1386$

$r^2 = 441 \Rightarrow r = 21$

Circumference of circle $= 2\pi r = 2 \times \frac{22}{7} \times 21 = 132 \text{ cm}$

5. Circumference of circle $= 2\pi r = 88 \Rightarrow 2 \times \frac{22}{7} \times r = 88$

$\Rightarrow r = 14 \text{ m}$

Area of circle $= \pi r^2 = \frac{22}{7} \times 14 \times 14 = 616 \text{ m}^2$

6. (i) Area of the track $= \pi r_1^2 - \pi r_2^2 = \frac{22}{7}[(70)^2 - (63)^2]$

$= \frac{22}{7}(4900 - 3969)$

$= \frac{22}{7} \times 931 = 22 \times 133 = 2926 \text{ m}^2$

(ii) Difference between the lengths of two circumference of the track $= 2\pi r_1 - 2\pi r_2 = 2 \times \frac{22}{7}(70 - 63) = 2 \times \frac{22}{7} \times 7 = 44 \text{ m}$

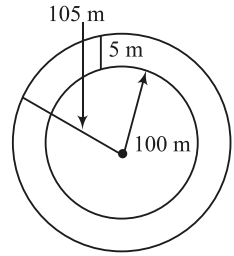
7. Area of path $= \pi r_1^2 - \pi r_2^2$

$= \frac{22}{7}[(105)^2 - (100)^2]$

$= \frac{22}{7}[11025 - 10000]$

$= \frac{22}{7} \times 1025 = \frac{22550}{7}$

Area of path $= 3221\frac{3}{7} \text{ m}^2$



8. Area of a face of the washer $= \pi r_1^2 - \pi r_2^2$

$= \pi[(4)^2 - (2)^2] = \frac{22}{7}[16 - 4]$

$= \frac{22}{7} \times 12 = \frac{264}{7} = 37.71 \text{ cm}^2$

9. Area of rectangular sheet $= 36 \times 24 = 864 \text{ cm}^2$

Area of each buttons $= \pi r^2 = \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2}$

Area of 64 buttons $= \frac{9\pi}{4} \times 64 = 16 \times 9\pi = 452.16 \text{ cm}^2$

Area of the remaining sheet $= 864 - 452.16 = 411.84 \text{ cm}^2$

10. $\pi r_1^2 = 100\,\pi r_2^2 \Rightarrow r_1 = 10r_2$

$\frac{c_1}{c_2} = \frac{2\pi r_1}{2\pi r_2} = \frac{10r_2}{r_2} = \frac{10}{1}$

Ratio of the circumferences is 10 :1.

11. Area of two plates $= \pi r_1^2 + \pi r_2^2 = \pi[r_1^2 + r_2^2]$

$= \pi[(5)^2 + (12)^2]$

$= \pi[25 + 144] = 169\pi \text{ cm}^2$

$$\text{New plate area} = \pi R^2 = 169\pi$$

$$\Rightarrow \quad R^2 = 169 \quad \Rightarrow \quad R = 13 \text{ cm}$$

Diameter of new plate is $= 2 \times 13 = 26$ cm

12. $$\text{Area of square} = a^2$$

$$a^2 = 196 \text{ cm}^2 \quad \Rightarrow \quad a = 14 \text{ cm}$$

$$\text{Perimeter of square} = \text{Circumference of circle}$$

$$4a = 2\pi r$$

$$4 \times 14 = 2 \times \frac{22}{7} \times r \quad \Rightarrow \quad r = \frac{14 \times 7}{11} \text{ cm}$$

$$\text{Area of circle} = \pi r^2 = \frac{22}{7} \times \frac{14 \times 7}{11} \times \frac{14 \times 7}{11} = 249.45 \text{ cm}^2.$$

❑

15. Volumes and Surface Areas of Solids

Exercise 15.1

1. Do it yourself.

2. (i) $\text{Volume of cuboid} = l \times b \times h = 3.6 \times 5.5 \times 3.5 = 69.3 \text{ m}^3$

$\text{Capacity of volume} = 69.3 \times 1000 = 69300$ litres.

(ii) $\text{Volume of cuboid} = l \times b \times h = 2.7 \times 4.75 \times 2.6$

$= 33.345 \text{ m}^3$

$\text{Capacity of volume} = 33.345 \times 1000 = 33345$ litres

(iii) Do it yourself. (iv) Do it yourself.

Word Problems

1. $\text{Volume of cuboid} = l \times b \times h = 30 \times 24 \times 18$

$= 12960 \text{ cm}^3$

2. Volume of rectangular tank $= l \times b \times h$

$= 65 \times 40 \times 54 = 140400 \text{ cm}^3$

$= 140400$ mL $[\because 1 \text{ cm}^3 = 1 \text{ mL}]$

$$\text{Number of glasses of sugarcane} = \frac{140400}{200} = 702$$

3. $\text{Capacity of water in water tank} = 4.8 \text{ L} = 4.8 \times 1000 = 4800 \text{ cm}^3$

$\text{Volume of water tank} = l \times b \times h$

$4800 = 20 \times 15 \times h$

$h = 16$ cm

Total surface area of cuboid $= 2(lb + bh + hl)$

$= 2(20 \times 15 + 15 \times 16 + 16 \times 20)$

$= 2(860) = 1720\ \text{cm}^2$

4. The volume of solid cube $= 64\ \text{cm}^3$

$a^3 = 64 \Rightarrow a = 4\ \text{cm}$

Total surface area of solid $= 6(a)^2 = 6(4)^2 = 6 \times 16 = 96\text{cm}^2$

5. Total surface area of solid $= 6a^2$

$6a^2 = 600 \Rightarrow a^2 = 100 \Rightarrow a = 10\ \text{cm}$

Volume of cube $= a^3 = (10)^3 = 1000\ \text{cm}^3$

6. Volume of cube $= 729\ \text{cm}^3$

$a^3 = 729 \Rightarrow a = 9\ \text{cm}$

Total surface $= 6a^2 = 6 \times (a)^2 = 6 \times 81 = 486\ \text{cm}^2$

Lateral surface area $= 2(l + b) \times h = 2(9 + 9) \times 9$

$= 18 \times 18 = 324\ \text{cm}^2$

7. Volume of cube $= a^3 = (9)^3 = 729\ \text{cm}^3$

$V_1 + V_2 + V_3 = 729 \Rightarrow a_1^3 + a_2^3 + a_3^3 = 729$

$(6)^3 + (8)^3 + (a_3)^3 = 729$

$(a_3)^3 = 729 - 216 - 512 = 729 - 728$

$(a_3^3) = 1 \Rightarrow a_3 = 1\ \text{cm}$

Edge of third smaller cube is 1 cm.

8. Number of cubical box $= \dfrac{(4.5)^3}{(0.15)^3} = \dfrac{91.125}{0.003375} = 27000$

9. Area of open tank $= 2\ h\ (l + b) + lb$

$= 2 \times 3\ (15 + 12) + 15 \times 12$

$= 162 + 180 = 342\ \text{m}^2$

Length of iron sheet $= \dfrac{342}{4} = 85.5\ \text{m}^2$

Cost of iron sheet $= 22.50 \times 85.5 = ₹\ 1923.75$

12. Total surface area or cubical $= 6a^2 = 6(14)^2$

$= 6 \times 196 = 1176\ \text{cm}^2$

Exercise 15.2

1. (i) Volume of cylinder $= \pi r^2 h = \dfrac{22}{7} \times 7 \times 7 \times 15 = 2310\ \text{cm}^3$

(ii) Volume of cylinder $= \pi r^2 h = \dfrac{22}{7} \times 2.8 \times 2.8 \times 15$

$= 369.6\ \text{m}^3$

2. (i) Volume of cylinder $= \pi r^2 h = \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 20$

$= 6930 \text{ cm}^3$

(ii) Volume of cylinder $= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12 = 462 \text{ m}^3$

Word Problems

1. Area of base of a cylinder $= \pi r^2 \Rightarrow \pi r^2 = 154$

$\frac{22}{7} \times r^2 = 154 \Rightarrow r^2 = 49 \Rightarrow r = 7 \text{ cm}$

Volume of cylinder $= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 15 = 2310 \text{ cm}^3$

Circumference of the base of a cylinder = 132 cm

2. $2\pi r = 132 \Rightarrow 2 \times \frac{22}{7} \times r = 132$

$r = 21 \text{ cm}$

Volume of cylinder $= \pi r^2 h = \frac{22}{7} \times 21 \times 21 \times 25 = 34650 \text{ cm}^3$

3. Volume of first pack whese base is square

$=$ Area of base $\times$ Height $= 5 \times 5 \times 12 = 300 \text{ cm}^3$

Volume of second pack whose base is circular

$=$ Area of base $\times$ Height $= \frac{22}{7} \times \frac{35 \times 35}{100} \times 10 = 385 \text{ cm}^3$

So, tin with a circular base has greater capacity.

Difference $= 385 - 300 = 85 \text{ cm}^3$

4. Volume of cylinder $= \pi r^2 h$

$\pi r^2 h = l \times b \times h$

$\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20 = 22 \times 14 \times h$

$h = \frac{770}{14 \times 22} = 2.5 \text{ m}$

Thus, height of platform is 2.5 m.

5. Volume of roof $= \frac{18 \times 16.5 \times 10}{100} = 29.7 \text{ m}^3$

Volume of roof $=$ Volume of cylinder tank

$29.7 = \pi r^2 h = \frac{22}{7} \times 4 \times 4 \times h$

$h = \frac{29.7 \times 7}{22 \times 16} = 0.5906 \text{ m} = 59.06 \text{ cm}$

6. Thickness = 2 cm, Height = 35 cm, $R = 12 + 2 = 14$ cm, $r = 12$ cm
Volume of wood required cylinder form

$$= \pi R^2 h - \pi r^2 h = \pi h (R^2 - r^2)$$

$$= \frac{22}{7} \times 35 (14^2 - 12^2)$$

$$= \frac{22}{7} \times 35 (196 - 144) = 5720 \text{ cm}^3$$

7. Total surface area of cylinder $= 2\pi r (h + r)$

$$= 2 \times \frac{22}{7} \times 5(5 + 15)$$

$$= 2 \times \frac{22}{7} \times 5 \times 20 = \frac{4400}{7} = 628.57 \text{ cm}^2$$

8. Circumference of cylinder $= 2\pi r$

$$2 \times \frac{22}{7} \times r = 176 \quad \Rightarrow \quad r = \frac{16}{4} \times 7 = 28 \text{ cm}$$

$$\text{Lateral surface area} = 2\pi rh = \frac{176 \times 1}{100} = 1.76 \text{ m}^2$$

9. $$\text{Lateral surface area} = 2\pi rh = 2 \times \frac{22}{7} \times \frac{3 \times 80}{10}$$

$$= 150.857 = 150.86 \text{ cm}^2$$

10. Area of roller $= 2\pi rh = 2 \times \frac{22}{7} \times 42 \times 120 = 31680 \text{ cm}^2$

$$\text{Area of playground} = 500 \times 31680$$

$$= \frac{15840000}{100 \times 100} \text{ m}^2 = 1584 \text{ m}^2$$

11. Curved surface area $= 2\pi rh = 2 \times \frac{22}{7} \times \frac{3.5}{2} \times 10$

$$= \frac{22}{7} \times 35 = 110 \text{ m}^2$$

$$\text{The cost of plastaring} = 110 \times 4 = ₹ 440$$

12. Total surface area of cylinder $= 2\pi r (h + r)$

$$= 2 \times \frac{22}{7} \times 21(21 + 100)$$

$$= 2 \times \frac{22}{7} \times 21 \times 121$$

$$= 132 \times 121 = 15972 \text{ cm}^2$$

13. $$\text{Surface area} = 2\pi rh = 2 \times \frac{22}{7} \times \frac{1}{2} \times 21 = 66 \text{ m}^2$$

$$\text{Cost of painting} = 5 \times 66 = ₹ 330$$

14. Total Surface Area $= 2\pi rh + \pi r^2$

$$= 2 \times \frac{22}{7} \times 10 \times 14 + \frac{22}{7} \times 10 \times 10$$

$$= 880 + \frac{2200}{7} = 1194.28 \text{ cm}^2$$

Cost of tin-plating $= 1194.28 \times \frac{0.5}{100} = ₹\ 5.97$

15. Curved surface $= 2\pi rh = 2 \times \frac{22}{7} \times \frac{25}{100} \times 3.5 = 5.5 \text{ m}^2$

Cost of washing $= 1.25 \times 5.5 = ₹\ 6.88$

Revision Exercise

1. Do it yourself.

2. Volume of cuboid $= l \times b \times h = 3 \times 2.2 \times 1.6 = 10.56 \text{ m}^3$

Capacity of cuboid $= 10.56 \times 1000 = 10560$ litres

3. Surface area of cuboid $= 2(lb + bh + hl)$

$= 2(896 + 504 + 576)$

$= 2(1976) = 3952 \text{ m}^2$

Area of 15 tin sheet $= 15 \times 3952 = 59280 \text{ m}^2$

4. Volume of cuboid $= l \times b \times h$

$$\frac{160 \times 1000}{1000} = 10 \times 4 \times h \quad \Rightarrow \quad h = \frac{160}{40} = 4 \text{ m}$$

Thus, depth of tank is 4 m.

5. Diagonal of cube $= a\sqrt{3} \quad \Rightarrow \quad a\sqrt{3} = 8\sqrt{3}$

(i) $a = 8$ cm, (ii) Surface area $= 6a^2 = 6(8)^2 = 384 \text{ cm}^2$

(iii) Volume of cube $= a^3 = (8)^3 = 512 \text{ cm}^3$

6. Volume of cube $= a^3$

$(6)^3 + (10)^3 + (x)^3 = (12)^3$

$216 + 1000 + x^3 = 1728$

$x^3 = 1728 - 1216 = 512 \quad \Rightarrow \quad x = 8$ cm

7. Volume of cube $= a^3$

$a^3 = (6)^3 + (8)^3 + (10)^3$

$= 216 + 512 + 1000 = 1728 \quad \Rightarrow \quad a = 12$ cm

8. (i) Volume of cuboid $= 4 \times x \times 15$

$(6)^3 = 4 \times x \times 15 \quad \Rightarrow \quad 216 = 4 \times x \times 15$

$x = 3.6$ cm.

(ii) Total surface of cuboid $= 2(lb + bh + hl)$

$= 2(4 \times 3.6 + 3.6 \times 15 + 15 \times 4)$

$= 2(14.4 + 54 + 60)$

$= 2 \times 128.4 = 256.8 \text{ cm}^2$

(iii) Total surface of cube $= 6(a)^2 = 6(6)^2 = 6 \times 36 = 216 \text{ cm}^2$

9. (i) Ratio between surface area $= \frac{6}{6}\left(\frac{3}{2}\right)^2 = 9 : 4$

(ii) Ratio between volume of cube $= \left(\frac{3}{2}\right)^3 = a^3 = \frac{27}{8} = 27 : 8$

10. Volume of rectangle $= 5.2 \text{ m}^3$

$h \times 2.6 \times 10^4 \text{ cm}^2 = 5.2 \times 100 \times 100$

$h = \frac{5.2}{2.6} = 2 \text{ m}$

Thus, height is 2 m.

11. Volume of cuboid $= l \times b \times h$

$= 120 \times 90 \times 75 = 810000 \text{ cm}^3$

Volume of cube $= (a)^3 = (30)^3 = 27000$

Number of cubes $= \frac{810000}{27000} = 30$

12. Volume of tank $= 480 \text{ kL}$

$10 \times 6 \times h = \frac{480 \times 1000}{1000} \Rightarrow h = 8 \text{ m}$

13. (i) Surface area of cube $= 6(a)^2$

When $a = 2a \Rightarrow$ Surface of cube $= 6(2a)^2 = 4 \times 6(a)^2$

Thus, surface is 4 times increase.

(ii) Volume of cube $= a^3$

When $a = 2a \Rightarrow$ Volume of cube $= (2a)^3 = 8a^3$

Thus, volume is 8 times increase.

14. Area of swimming pool = Area of four walls + Area of base

$= 2h(l + b) + l \times b = 2 \times 3.2(52 + 38) + 52 \times 38$

$= 2 \times 3.2 \times 90 + 1976 = 576 + 1976 = 2552 \text{ m}^2$

Cost of pool $= 56 \times 2552 = ₹ 142912$

15. Area of four walls $= 120 \text{ m}^2$

$2(l + b) \times h = 120 \quad [\because \ l = 2b]$

$2(2b + b) \times 4 = 120 \Rightarrow 2(3b) \times 4 = 120$

$$3b = 15 \quad \Rightarrow \quad b = 5$$
$$l = 2b = 2 \times 5 = 10$$
$$\text{Area of floor} = l \times b = 10 \times 5 = 50 \text{ m}^2$$

16. Circumference of the base of a cylindrical vessel $= 2\pi r$

$$2\pi r = 132 \quad \Rightarrow \quad 2 \times \frac{22}{7} \times r = 132$$
$$r = 21 \text{ cm}$$
$$\text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 21 \times 21 \times 36 = 49896 \text{ cm}^3$$
$$\text{Capacity of cylinder} = \frac{49896}{1000} = 49.896 \text{ L}$$

17. $$\text{Volume of cylinder} = 550 \text{ cm}^3$$
$$\pi r^2 h = 550$$
$$\frac{22}{7} \times 5x \times 5x \times 7x = 550$$
$$x^3 = \frac{550}{22 \times 25} = 1 \quad \Rightarrow \quad x = 1$$
$$\text{Radius} = 5 \times 1 = 5 \text{ cm; Height} = 7 \times 1 = 7 \text{ cm}$$

18. Total surface area of cylinder $= 660 \text{ cm}^2$

$$2\pi r(h + r) = 660 \text{ cm}^2$$
$$2 \times \frac{22}{7} \times 5(h + 5) = 660$$
$$(h + 5) = 21 \quad \Rightarrow \quad h = 21 - 5 = 16 \text{ cm}$$

Thus, height is 16 cm.

19. Volume of earth taken out of pit $= 75 \text{ m}^3 \quad \Rightarrow \quad 8 \times 4.8 \times h = 75$

$$h = \frac{75}{8 \times 4.8} = 1.95 \text{ m}$$

Thus, height/depth of the pit is 1.95 m.

20. $$\text{Number of wheat bags} = \frac{\text{Volume of godown}}{\text{Volume of bags}}$$
$$= \frac{60 \times 40 \times 15 \times 100}{1.8 \times 1.2 \times 40} = \frac{60 \times 15 \times 100}{1.8 \times 1.2}$$
$$= \frac{60 \times 15 \times 100 \times 100}{18 \times 12} = \frac{5 \times 5 \times 100 \times 100}{6}$$
$$= \frac{250000}{6} = 41666$$

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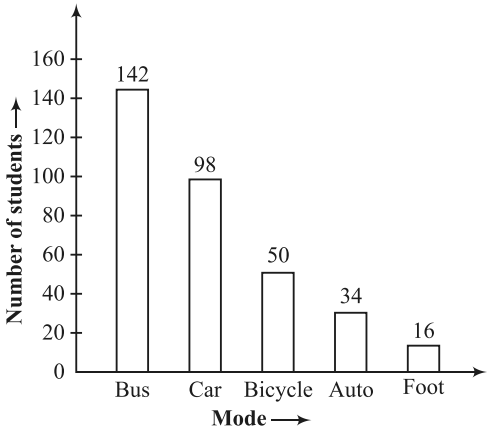
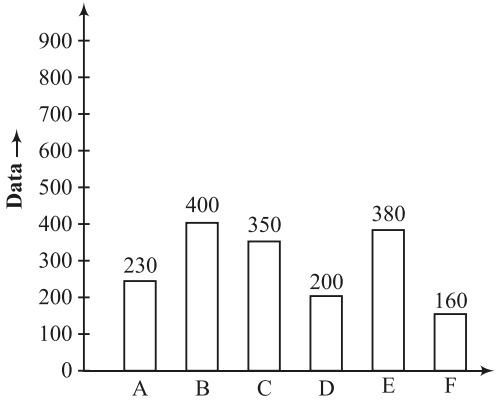
16. Data Handling

Exercise 16.1

1. Bar graph between mode and students is shown as follows :



2. Bar graph between given data is shows as follows :



3. $60 + 45 + 42 + 48 + 75 = 270$

$$\text{Central angle for Hindi} = \frac{60}{270} \times 360 = 80°$$

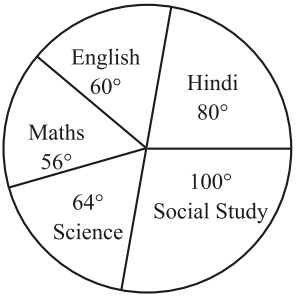
$$\text{Central angle for English} = \frac{45}{270} \times 360 = 60°$$

$$\text{Central angle for Maths} = \frac{42}{270} \times 360 = 56°$$

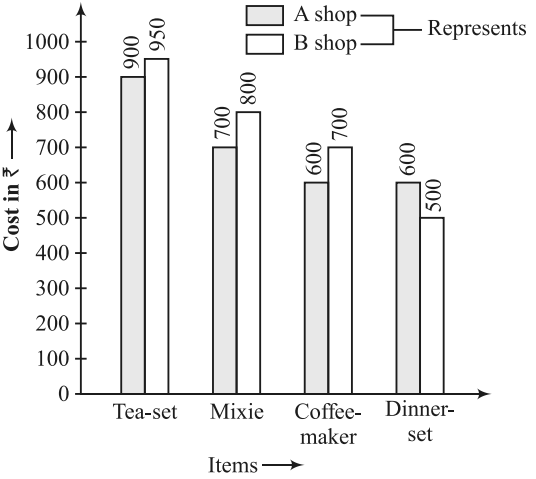
$$\text{Central angle for Science} = \frac{48}{270} \times 360 = 64°$$

Central angle for Social Study $= \frac{75}{270} \times 360 = 100°$

Required pie-graph is shown as follows :



4. Bar graph between cost and items is shown as follows :



5. $35 + 20 + 20 + 15 + 10 = 100$

$$\text{Angle of } A = \frac{35}{100} \times 360° = 126°$$

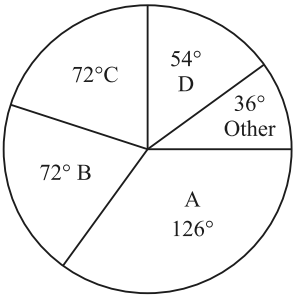
$$\text{Angle of } B = \frac{20}{100} \times 360° = 72°$$

$$\text{Angle of } C = \frac{20}{100} \times 360° = 72°$$

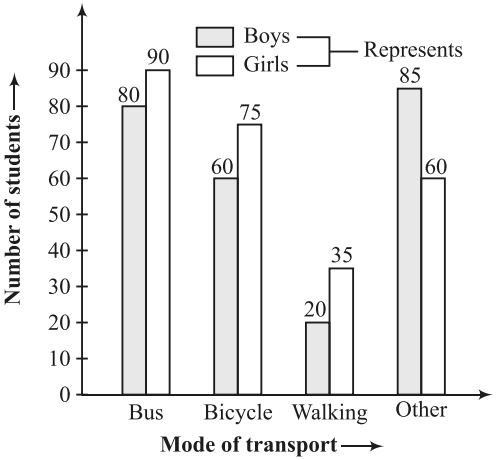
$$\text{Angle of } D = \frac{15}{100} \times 360° = 54°$$

$$\text{Angle of others} = \frac{10}{100} \times 360° = 36°$$

Required pie-chart is shown as follows :



6. Bar graph between students and mode of transport is shown as follows :



Exercise 16.2

1. Do it yourself.

2. (i) An even number and a multiple of 3 = 6
Possible outcomes = 1 , 2, 3, 4, 5, 6 =6
Favourable outcomes = 1
Required probability $(P) = \frac{1}{6}$

(ii) Possible outcomes = 1, 2, 3, 4, 5, 6 = 6
Number less than 5 = 1, 2, 3, 4 = 4
favourable outcomes = 4
Required probability $(P) = \frac{4}{6} = \frac{2}{3}$

(iii) Total number of elementary events 1, 2, 3, 4, 5, 6 = 6
Number greater than 3 = 4, 5, 6
Favourable number of elementary events = 3

Required probability $(P) = \frac{3}{6} = \frac{1}{2}$

(iv) Total number of elementary events $1, 2, 3, 4, 5, 6 = 6$

A number between 3 and $6 = 4, 5$

Favourable number of elementary events $= 2$

Probability $(P) = \frac{2}{6} = \frac{1}{3}$

3. Elementary events associated to random experiment of tossing 3 coins are :

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

Total number of elementary events = 8.

(i) Getting two heads

Favourable number of elementary evnets = 3

Required probability $= \frac{3}{8}$

(ii) Getting one heads

Favourable number of elementary events = 3

Required probability $= \frac{3}{8}$

4. Do it yourself.

5. (i) Required probability $= \frac{26}{52} = \frac{1}{2}$

(ii) Required probability $= \frac{2}{52} = \frac{1}{26}$

(iii) Required probability $= \frac{6}{52} = \frac{3}{26}$

(iv) Required probability $= \frac{2}{52} = \frac{1}{26}$

6. A pair of dice is thrown. Then, total outcomes $= 6^2 = 36$

(i) The event 'getting sum as a prime number' will occure if the elementary event (1, 1) (1, 2) (1, 4) (1, 6), (2, 1) (2, 3) (2, 5) (3, 2) (3, 4) (4, 1) (4, 3) (5, 2) (5, 6) (6, 1) (6, 5)

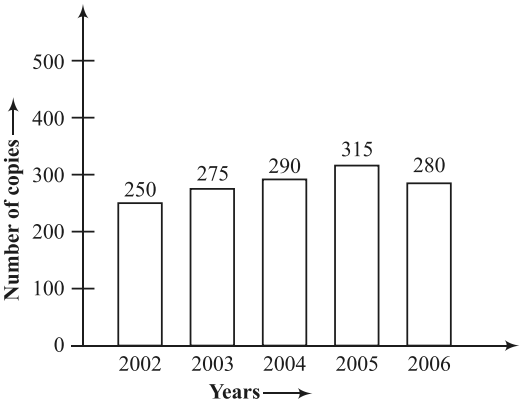
Hence, required probability $= \frac{15}{36} = \frac{5}{12}$

(ii), (iii), (iv) Do it yourself.

Revision Exercise

1. Do it yourself.

2. Bar graph between copies and years is shown as follows :



3. (i) Percentage of students who play badminton

$$P\% = \frac{50}{150} \times 100 = \frac{1}{3} \times 100 = 33\frac{1}{3}\%$$

(ii) Ratio of students who play table tennis and badminton

$$= 40 : 50 = 4 : 5$$

4. Do it yourself.

5. $30 + 25 + 10 + 15 + 20 = 100$

$$\text{Angle of Banana} = \frac{30}{100} \times 360 = 108°$$

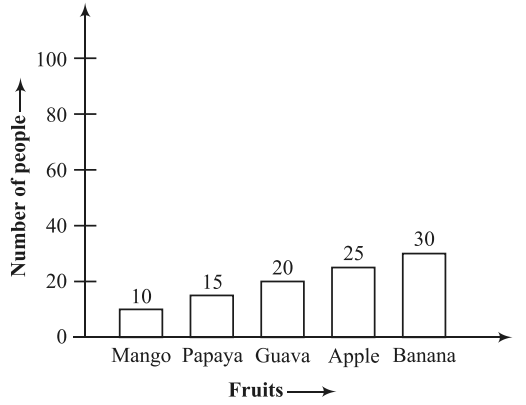
$$\text{Angle of Apple} = \frac{25}{100} \times 360 = 90°$$

$$\text{Angle of Mango} = \frac{10}{100} \times 360 = 36°$$

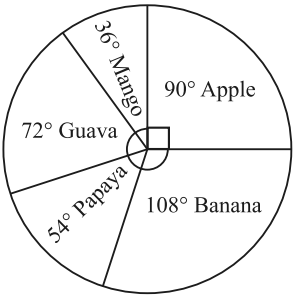
$$\text{Angle of Papaya} = \frac{15}{100} \times 360 = 54°$$

$$\text{Angle of Guava} = \frac{20}{100} \times 360 = 72°$$

(i) (a) Bar graph is shown as follows :

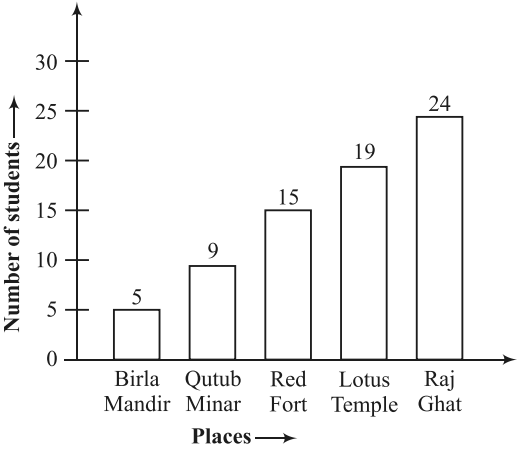


(b) Pie-chart is shown as follows :



(ii), (iii) Do it yourself.

6. Bar graph between place of interest and students.



7. (i) Required probability $= \frac{26}{52} = \frac{1}{2}$

(ii) Probability that card draw is neither a king nor a queen $= \frac{44}{52} = \frac{11}{13}$

(iii) Probability that card draw is a red face card $= \frac{6}{52} = \frac{3}{26}$

(iv) Probability that card draw is a card of spade or an ace

$$= \frac{16}{52} = \frac{4}{13}$$

8. Do it yourself.

9. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17

(i) Probability that the number on card is odd $= \frac{9}{17}$

(ii) Probability that the number on card is even $= \frac{8}{17}$

(iii) Probability that the number on card is prime $= \frac{7}{17}$

(iv) Probability that the number on card is divisible by $3 = \frac{5}{17}$

10. Number of prize = 5

Number of black tickets = 995

Probability of winning prize $= \frac{5}{995 + 5} = \frac{5}{1000} = \frac{1}{200}$

11. Number of days in leap year = 366

Number of Sundays in a leap year $= \frac{366}{7} = 52$ Sunday $+ 2$

Total outcomes with 2 days = (Sunday, Monday), (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday). (Thursday, Friday), (Friday, Saturday), (Saturday, Sunday) = 7

Number of outcomes without Sunday = 5

Probability of leap year with 53 Sunday $= \frac{2}{7}$

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17. Introduction to Graphs

Exercise 17.1

1. Do it yourself.

2. (i) (2, 3)

In the point (2, 3) abscissa and ordinate both are positive.

So, it lies in the I quadrant.

(ii) In point (–5, –4) abscissa and ordinate both are negative.

So, it lies in III quadrant.

(iii) In point (–10, –6) abscissa and ordinate both are negative.

So, it lies in III quadrant.

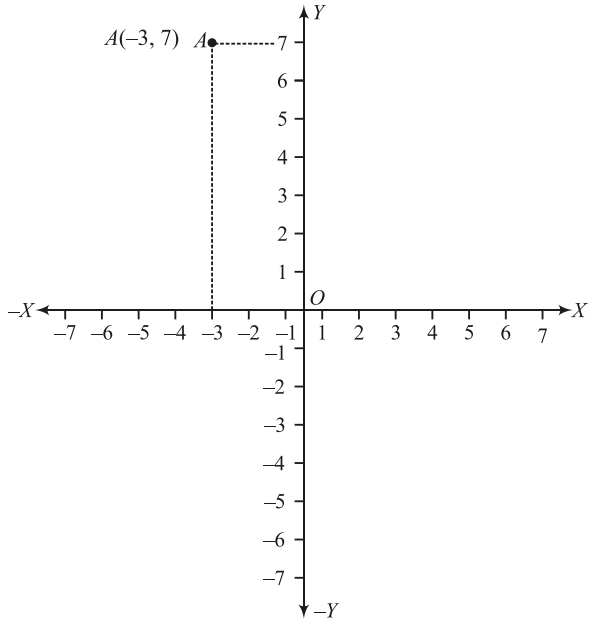
(iv) In point (4, – 5) abscissa is positive and ordinate is negative.

So, it lies in IV quadrant.

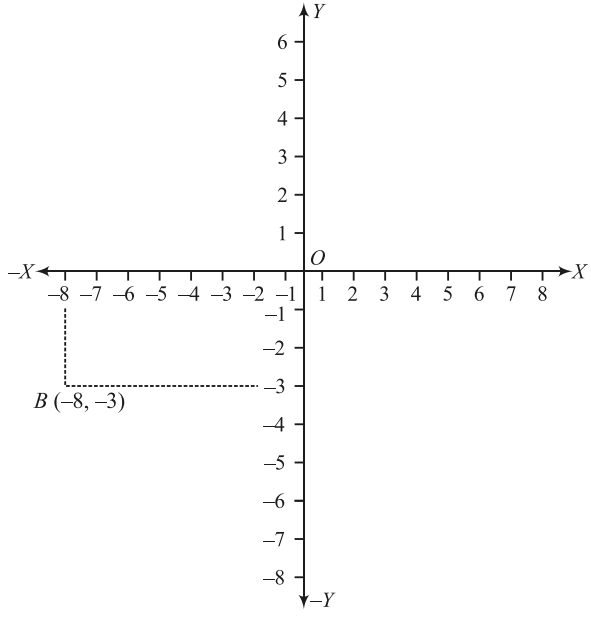
3. (i) (–3, 0) Point lie in *X*-axis. (ii) (0, 7) Point lie on *Y*-axis.

(iii) (0, – 5) Point lie one *Y*-axis. (iv) (6,0) Point lie on *X*-axis.

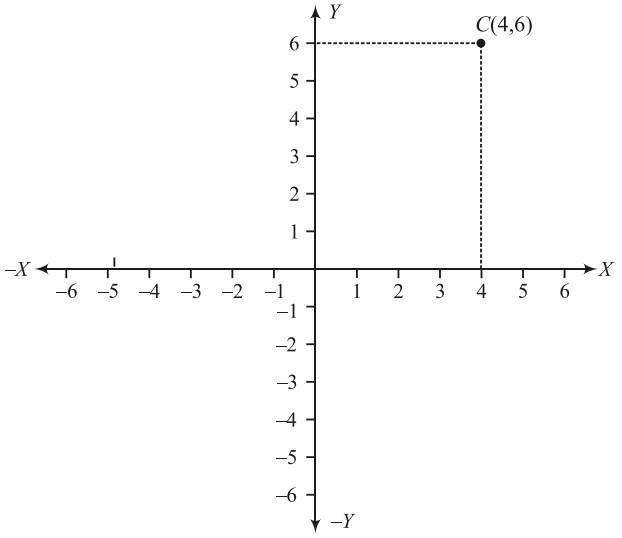
4. (i) $A(-3, 7)$


A(–3, 7)
A
Y
–Y
–X
X
O
7
6
5
4
3
2
1
–1
–2
–3
–4
–5
–6
–7
–7 –6 –5 –4 –3 –2 –1 1 2 3 4 5 6 7

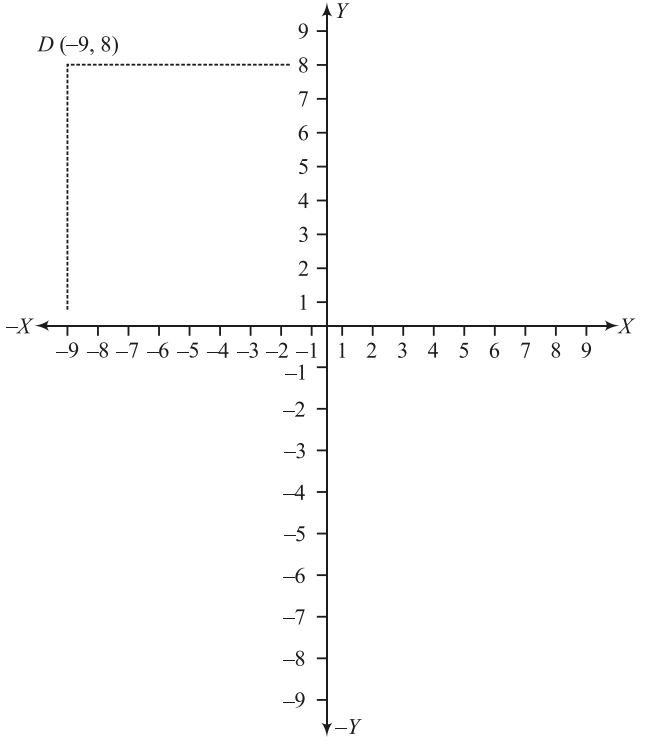
(ii) $B(-8, -3)$


Y
–Y
–X
X
O
6
5
4
3
2
1
–1
–2
–3
–4
–5
–6
–7
–8
–8 –7 –6 –5 –4 –3 –2 –1 1 2 3 4 5 6 7 8
B (–8, –3)

(iii) C (4, 6)



(iv) D (–9, 8)



5. Do it yourself.

Exercise 17.2

1. (i) We know that

Relation of Celsius and Fahrenheit,

Given : $C = 25°, F = ?$

$$\frac{C}{5} = \frac{F-32}{9} \Rightarrow \frac{25}{5} = \frac{F-32}{9}$$

$$5 \times 9 = F - 32 \; F = 45 + 32 = 77°\text{F}$$

(ii) We know that $\frac{C}{5} = \frac{F-32}{9}$

$$\frac{C}{5} = \frac{95-32}{9} = \frac{63}{9}$$

$$\frac{C}{5} = 7$$

$$C = 5 \times 7$$

$$C = 35°\text{C}$$

2. Draw the graph yourself 4 kg apple of price = ₹ 240

1 kg apple of price = ₹ $\frac{240}{4}$ = ₹ 60

22 kg apple of price = 60×22 = ₹ 1320

3. P = ₹ 1,000, $T = 1$, year $R = ?$, SI = ₹ 60

$$\text{SI} = \frac{PRT}{100} \Rightarrow 60 = \frac{1000 \times R \times 1}{100}$$

$\therefore \quad R = 6\%$

(i) $$\text{SI} = \frac{1800 \times 6 \times 1}{100} = ₹\ 108$$

Thus, simple interest of 1800 is ₹ 108.

(ii) $$156 = \frac{P \times 6 \times 1}{100}$$

$$P = \frac{15600}{6} = ₹\ 2600$$

4. (i) In one second distance covered = 10 m

6 second distance covered = 60 m

(ii) Time of 90 m covered distance = $\frac{90}{10}$ = 9 second

5. Cost of 5 Litres diesel = ₹ 350

Cost of 1 Litre diesel = $\frac{350}{5}$ = ₹ 70

Diesel can be purchased in ₹ 1960 = $\frac{1960}{70}$ = 28 Litre

6. (i) In one hour distance covered = 50 km

In $\frac{1}{2}$ hour distance covered = $50 \times \frac{1}{2} = 25$ km

(ii) Time of 175 km = $\frac{175}{50} = 3.5$ hours

Time of 175 am = 8 am + $\frac{1}{2}$ hour = 8 : 30 am

7. Do it yourself.

Revision Exercise

1. (i) In the point $(-7, -8)$ abscissa and ordinate both are negative.
So, it lies in the III quadrant.

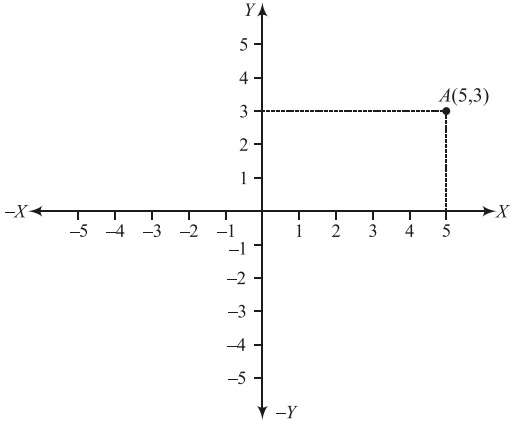
(ii) In the point $(5, 8)$ abscissa and ordinate both are positive.
So, it lies in the I quadrant.

(iii) In the point $(-4, 5)$ abscissa is negative and ordinate is positive.
So, it lies in the II quadrant.

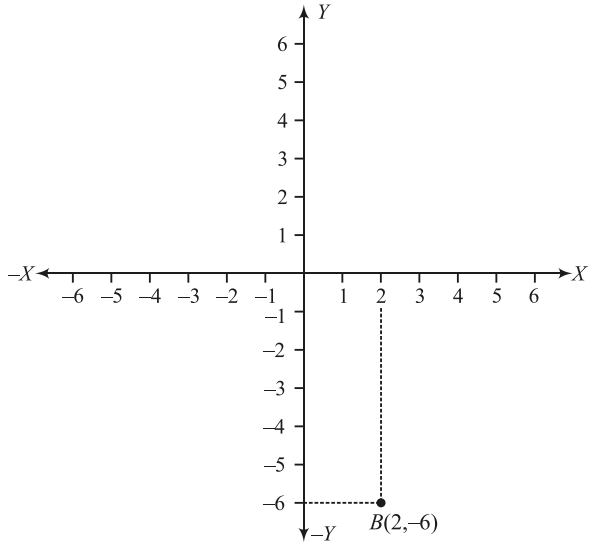
(iv) In the point $(6, -3)$ abscissa is positive and ordinate is negative.
So, it lies in IV quadrant.

2. Do it yourself.

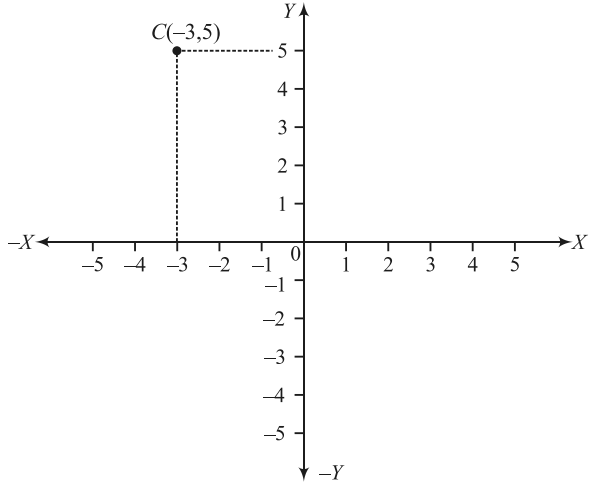
3. (a) $A(5, 3)$



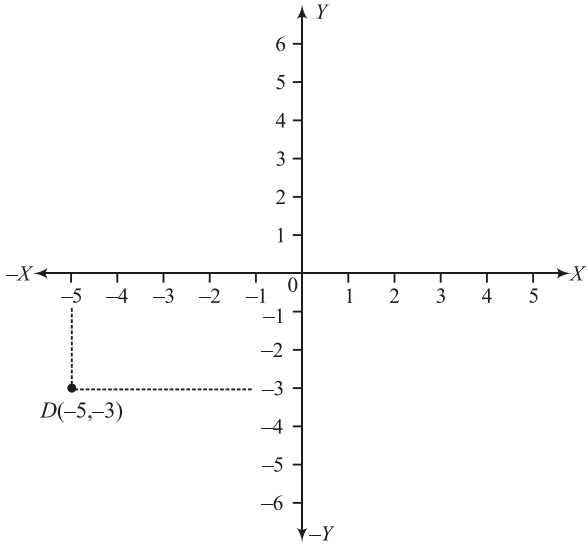
(b) $B(2, -6)$



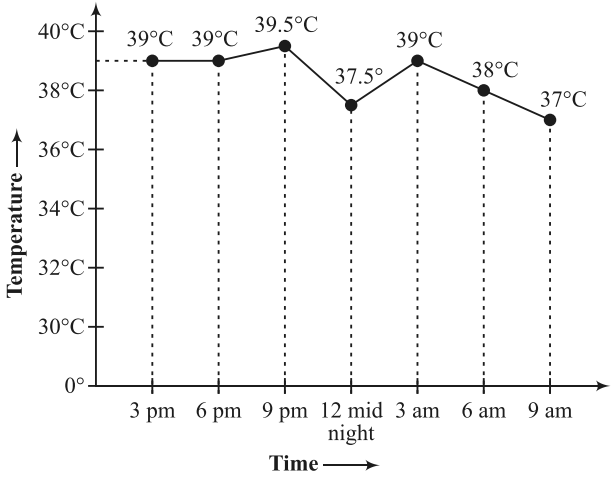
(c) $C(-3, 5)$



(d) $D(-5, -3)$



4. (i) Line graph between Time and Temperature.



(ii) Patients temperature are 5 pm and at 1 am = 39° C, 38°C

5. Do it yourself.

6. Do it yourself.

❑

Model Test Paper-I

1. Do it yourself.

2. (i) $\frac{2}{5} \times \frac{-3}{7} - \frac{1}{14} - \frac{3}{7} \times \frac{3}{5} = \frac{-6}{35} - \frac{1}{14} - \frac{9}{35} = \frac{-6}{35} - \frac{9}{35} - \frac{1}{14}$

$= \frac{-15}{35} - \frac{1}{14} = \frac{-15 \times 2 - 5 \times 1}{70}$

$= \frac{-30 - 5}{70} = \frac{-35}{70} = \frac{-1}{2}$

(ii) $\frac{8}{9} \times \frac{4}{5} + \frac{5}{6} - \frac{9}{5} \times \frac{8}{9} = \frac{32}{45} + \frac{5}{6} - \frac{72}{45}$

$= \frac{32}{45} - \frac{72}{45} + \frac{5}{6} = \frac{32 - 72}{45} + \frac{5}{6}$

$= \frac{-40}{45} + \frac{5}{6} = \frac{-40 \times 2 + 5 \times 15}{90}$

$= \frac{-80 + 75}{90} = \frac{-5}{90} = \frac{-1}{18}$

(iii) $\frac{-3}{7} \times \frac{14}{15} \times \frac{7}{12} \times \left(\frac{-30}{35}\right) = \frac{-14}{60} \times \frac{-30}{35} = \frac{7}{35} = \frac{1}{5}$

3. (i) 9.67×10^5

Number in usual form $= \frac{967}{10^2} \times 10^5 = 967 \times 10^{5-2}$

$= 967 \times 10^3 = 967000$

(ii) 8.37×10^8

Number in usual form $= \frac{837}{10^2} \times 10^8 = 837 \times 10^{8-2}$

$= 837 \times 10^6 = 837000000$

(iii) 9.42×10^{-4}

Number in usual form $= \frac{942}{10^2} \times 10^{-4}$

$= 942 \times 10^{-4} \times 10^{-2} = 942 \times 10^{-6}$

$= \frac{942}{10^6} = 0.000942$

(iv) 6.75×10^{-7}

Number in usual form $= \frac{675}{10^2} \times 10^{-7}$

$$= 675 \times 10^{-7} \times 10^{-2} = 675 \times 10^{-9}$$
$$= \frac{675}{10^9} = 0.000000675$$

4. A school collected fees = ₹ 2304

Factors of $2304 = \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 2} \times \underline{2 \times 3} \times \underline{3}$

Square root of $2304 = 2 \times 2 \times 2 \times 2 \times 3 = 48$

2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Hence, students in school = 48.

5. (i) 81

Square root of $81 = \sqrt{81} = 9$

Number of digits in square root = 1

(ii) 169

Square root of $169 = \sqrt{169} = 13$

Number of digits in square root = 2

(iii) 4761

Square root of $4761 = \sqrt{4761} = 69$

Number of digits in square root = 2

(iv) 27889

Square root of $27889 = \sqrt{27889} = 167$

Number of digits in square root = 3

(v) 525625

Square rot of $525689 = \sqrt{525625} = 725$

Number of digits in square root = 3

6. Do yourself.

(i) Cubes of all even natural number

$2^3 = 8, 4^3 = 64, 6^3 = 216, 8^3 = 512, 10^3 = 1000$

$12^3 = 1728, 14^3 = 2744, 16^3 = 4096,$

$18^3 = 5832, 20^3 = 8000$

So, cubes of all even natural number are even.

(ii) Do yourself.

7. 137592

2	137592
2	68796
2	34398
7	1799
7	2457
13	351
3	27
3	9
3	3
	1

Factorising 137592 by division method.

$$137592 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times 7 \times 7 \times 13$$

Which have factors in equal pairs and 49×13 is left.

Hence, multiplied $= 7 \times 13 \times 13 = 1183$

Perfect cube $= \sqrt[3]{137592 \times 1183} = 546$

8. (i) $8x^2y, 6yx^2, -3x^2y^2, 8y^2x^2, 3x^2y$

$8x^2y + 6yx^2 + (-3x^2y^2) + 8y^2x^2 + 3x^2y$

$= 8x^2y + 6x^2y + 3x^2y - 3x^2y^2 + 8x^2y^2$

$= 17x^2y + 5x^2y^2$

(ii), (iii), (iv) Do yourself.

9. (i) $10xy^4 - 10x^4y = 10xy(y^3 - x^3)$

$= 10xy(y - x)(y^2 + xy + x^2)$

(ii) $54x^6y + 2x^3y^4 = 2x^3y(27x^3 + y^3)$

$= 2x^3y[(3x)^3 + (y)^3]$

$= 2x^3y[(3x + y)(9x^2 - 3xy + y^2)]$

(iii) $(p - 2q)^3 - (8q)^3$

$= (p - 2q - 8q)[(p - 2q)^2 + (p - 2q)(8q) + (8q)^2]$

$= (p - 10q)(p^2 + 4q^2 - 4qp + 8pq - 16q^2 + 64q^2)$

$= (p - 10q)(p^2 + 52q^2 - 4qp + 8pq)$

$= (p - 10q)(p^2 + 52q^2 + 4pq)$

(iv) $27(x - 1)^3 + m^3 = [3(x - 1)^3 + m^3]$

$= [3(x - 1) + m][3(x - 1)^2 - 3m(x - 1) + m^2)]$

$= (3x - 3 + m)[9(x^2 + 1 - 2x) - 3mx + 3m + m^2]$

$= (3x - 3 + m)[9x^2 + 9 - 18x - 3mx + 3m + m^2]$

$= (3x - 3 + m)[9x^2 + m^2 - 18x - 3mx + 3m + 9]$

10. (i) $-39pq^2r^5 \div -24p^3q^3r = \dfrac{-39pq^2r^5}{-24p^3q^3r} = \dfrac{13r^4}{8p^2q}$

(ii) $\dfrac{\frac{-3}{4}a^2b^3}{\frac{6}{7}a^3b^2} = \dfrac{-3}{4} \times \dfrac{7}{6}\dfrac{b}{a} = \dfrac{-7}{8}\dfrac{b}{a}$

(iii) $\dfrac{9x^4 - 8x^3 - 12x + 3}{3x} = 3x^3 - \dfrac{8}{3}x^2 - 4 + \dfrac{1}{x}$

(iv) $\dfrac{14p^2q^3 - 32p^3q^2 + 15pq^2 - 22p + 18q}{-2p^2q}$

$= -7q^2 + 16pq - \dfrac{15}{2}\dfrac{q}{p} + \dfrac{11}{pq} - \dfrac{9}{p^2}$

11. (i)

$$
\begin{array}{r|l}
a+1 & a^2 - 6a + 14 \\
 & a^3 + a^2 \\
 & \hline
 & -6a^2 + 8a \\
 & -6a^2 - 6a \\
 & \hline
 & 14a + 15 \\
 & 14a + 14 \\
 & \hline
 & 1
\end{array}
$$

Quotient $= a^2 - 6a + 14$ and Remainder $= 1$

(ii)

$$
\begin{array}{r|l}
x-3 & 3x^3 + 15x^2 + 39x + 119 \\
 & 3x^4 - 9x^3 \\
 & \hline
 & 15x^3 - 6x^2 \\
 & 15x^3 - 45x^2 \\
 & \hline
 & 39x^2 + 2x \\
 & 39x^2 - 117x \\
 & \hline
 & 119x - 7 \\
 & 119x - 357 \\
 & \hline
 & 350
\end{array}
$$

Quotient $= 3x^3 + 15x^2 + 39x + 119$ and Remainder $= 350$

(iii)
$$3x+5\overline{)\,6x^2+x-15}\;\;2x-3$$
$$\underline{-\;6x^2\;+10x}$$
$$-9x-15$$
$$\underline{+\;\;+}$$
$$0$$

Quotient $= 2x-3$ and Remainder $= 0$

12. (i) $\dfrac{2y+6}{y+4}=1$

$\Rightarrow \quad 2y+6=(y+4)\times 1$

$\Rightarrow \quad 2y+6=y+4$

$\Rightarrow \quad 2y-y=4-6 \;\Rightarrow\; y=-2$

(ii) $\dfrac{3x+5}{2x+7}=4$

$\Rightarrow \quad (3x+5)=4(2x+7)$

$\Rightarrow \quad 3x+5=8x+28$

$\Rightarrow \quad -28+5=8x-3x$

$\Rightarrow \quad 5x=-23 \;\Rightarrow\; x=\dfrac{-23}{5}$

(iii) $\dfrac{2x+1}{3x-2}=\dfrac{5}{9}$

$\Rightarrow \quad (2x+1)\,9=5\,(3x-2)$

$\Rightarrow \quad 18x+9=15x-10$

$\Rightarrow \quad 18x-15x=-10-9$

$\Rightarrow \quad 3x=-19 \;\Rightarrow\; x=\dfrac{-23}{5}$

13. Let value of car after two year $= x$

$$x=4{,}20{,}000\left(1-\frac{20}{100}\right)\left(1-\frac{90}{100}\right)$$
$$=4{,}20{,}000\left(\frac{80}{100}\right)\left(\frac{90}{100}\right)$$
$$=42\times 80\times 90=₹\,3{,}02{,}400$$

14. Let total number of voters $= x$

Loser polled $=42\%$; Winner polled $=58\%$

According to question,

58% of $x-42\%$ of $x=14400$

$$16\%\,x=14400 \;\Rightarrow\; \frac{x}{100}\times 16=14400$$
$$x=900\times 100 \;\Rightarrow\; x=90{,}000.$$

Percentage of voters did not vote = x

$$x = \left(\frac{100{,}000 - 90{,}000}{100{,}000}\right) \times 100$$

$$= \frac{10{,}000}{100{,}000} \times 100 = 10\%$$

❑

Model Test Paper-II

1. CP_1 of tea $= 40 \times 150 = ₹\ 6000$

CP_2 of tea $= 60 \times 100 = ₹\ 6000$

Total CP $= 6000 + 6000 = ₹\ 12000$

SP of tea $= (40 + 60) \times 130 = ₹\ 13000$

$$\text{Profit\%} = \frac{(13000 - 12000)}{12000} \times 100 = \frac{1000}{12000} \times 100$$

$$= \frac{100}{12} = 8.33\%$$

2. CP of Suit = ₹ 16,000, Discount = 20%, Profit% = 30%

$$\text{SP} = \frac{16000 \times (130)}{100} \quad \ldots(i)$$

$$\text{SP} = \frac{\text{MP} \times 80}{100} \quad \ldots(ii)$$

Taking eqs. (i) and (ii),

$16000 \times 130 = \text{MP} \times 80$

$\text{MP} = ₹\ 26{,}000$

3. Principal for the first year = ₹ 46, 875

Interest for the first year $= \frac{46875 \times 4 \times 1}{100} = ₹\ 1875$

(i) Interest for the first year = ₹ 1875

Amount at the end of the first year = 46875 + 1875 = ₹ 48750

(ii) Principal for the second year = ₹ 48,750

Interest for the second year $= \frac{48750 \times 4 \times 1}{100} = ₹\ 1950$

Amount at the end of the second year = 48, 750 + 1950 = ₹ 50, 700

(iii) Principal for the third year = ₹ 50,700

Interest for the third year $= \frac{50700 \times 4 \times 10}{100} = ₹\ 2028$

4. $P = ₹\ 8192, R = 12.5\%, T = 1\frac{1}{2}$ years (half-yearly)

$$R = \frac{12.5}{2} = \frac{25}{4}\%, T = 3 \text{ half-years}$$

$$\text{Amount} = P\left(1 + \frac{R}{100}\right)^T = 8192\left(1 + \frac{25}{4 \times 100}\right)^3$$

$$= 8192\left(1 + \frac{1}{4 \times 4}\right)^3 = 8192 \times \left(\frac{17}{16}\right)^3$$

$$= \frac{8192 \times 17 \times 17 \times 17}{16 \times 16 \times 16} = ₹\ 9826$$

5. Let the cost of 105 m plastic sheet = ₹ x

It is a direct variation.

$$\frac{93}{1395} = \frac{105}{x}$$

$$\Rightarrow \quad x = \frac{105 \times 1395}{93} = ₹\ 1575$$

6. One days's work of Rajesh and Rahul $= \frac{1}{20}$ parts

$$\text{After 8 days} = \frac{8}{20} \text{ parts}$$

$$\text{Remaining work} = 1 - \frac{8}{20} = \frac{12}{20}$$

Rahul can do the remaining work in $= \frac{12}{20} \times 30 = 18$ days

7. Two angles of quadrilateral is 75° and 105°

Let one angle $= x$; Other angle $= 2x$

Sum of angles of quadrilateral $= 360°$

$$75 + 105 + x + 2x = 360°$$

$$3x = 360° - 180°$$

$$3x = 180° \quad \Rightarrow \quad x = 60°$$

Thus, remaining angles 60°, 120°.

8. Let the two adjacent angles are x and y.

Then, $x - y = 30°$ ...(i)

$x + y = 180°$ ...(ii)

Taking eq. (i) and (ii),

$2x = 210 \quad \Rightarrow \quad x = 105°$

$y = 75°$ [from eq. (ii)]

Thus, angles are 105°, 75°, 105°, 75°.

9. Area of quadrilateral $= \frac{1}{2}(a + b) \times h$

$= \frac{1}{2}(7.2 + 8.8) \times 28.4$

$= \frac{1}{2} \times 16 \times 28.4 = 227.2 \text{ m}^2$

10. (i) Area of triangle $= \frac{1}{2} \times 4.8 \times 6 = 14.4 \text{ m}^2$

(ii) $\frac{1}{2} \times 6.4 \times h = \frac{1}{2} \times 4.8 \times 6 \quad \Rightarrow \quad h = \frac{4.8 \times 6}{6.4} = 4.5 \text{ m}$

11. (i) Area of the track $= \pi r_1^2 - \pi r_2^2 = \frac{22}{7}[(70)^2 - (63)^2]$

$= \frac{22}{7}(4900 - 3969) = \frac{22}{7} \times 931 = 2926 \text{ m}^2$

(ii) Difference between the lengths of two circumference of the track $= 2\pi r_1 - 2\pi r_2 = 2 \times \frac{22}{7}(70 - 63) = 2 \times \frac{22}{7} \times 7 = 44 \text{ m}$

12. Capacity of water in water tank $= 4.8$ L

$= 4.8 \times 1000 = 4800 \text{ cm}^3$

Volume of water tank $= l \times b \times h$

$4800 = 20 \times 15 \times h \quad \Rightarrow \quad h = 16 \text{ cm}$

Total surface area of cuboid $= 2(lb + bh + hl)$

$= 2(20 \times 15 + 15 \times 16 + 16 \times 20)$

$= 2(860) = 1720 \text{ cm}^2$

13. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17

(i) Probability that the number on card is odd $= \frac{9}{17}$

(ii) Probability that the number on card is even $= \frac{8}{17}$

(iii) Probability that the number on card is prime $= \frac{7}{17}$

(iv) Probability that the number on card is divisible by 3 $= \frac{5}{17}$

14. Do it yourself.

❑