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सर्वश्रेष्ठ परीक्षा पथ-प्रदर्शक

B.Sc. **Second Year** (Diploma Course) **Semester-III**

PHYSICS

SYLLABUS

Electromagnetic Theory & Modern Optics

Part-A : Electromagnetic Theory

UNIT-I **Electrostatics :** Electric charge & charge densities, electric force between two charges. General expression for Electric field in terms of volume charge density (divergence & curl of Electric field), general expression for Electric potential in terms of volume charge density and Gauss law (applications included). Study of electric dipole. Electric fields in matter, polarisation, auxiliary field **D** (Electric displacement), electric susceptibility and permittivity.

UNIT-II **Magnetostatics :** Electric current & current densities, magnetic force between two current elements. General expression for Magnetic field in terms of volume current density (divergence and curl of Magnetic field), General expression for Magnetic potential in terms of volume current density and Ampere's circuital law (applications included). Study of magnetic dipole (Gilbert & Ampere model). Magnetic fields in matter, magnetisation, auxiliary field **H**, magnetic susceptibility and permeability.

UNIT-III **Time Varying Electromagnetic Fields :** Faraday's laws of electromagnetic induction and Lenz's law. Displacement current, equation of continuity and Maxwell-Ampere's circuital law. Self and mutual induction (applications included). Derivation and physical significance of Maxwell's equations. Theory and working of moving coil ballistic galvanometer (applications included).

UNIT-IV **Electromagnetic Waves :** Electromagnetic energy density and Poynting vector. Plane electromagnetic waves in linear infinite dielectrics, homogeneous & inhomogeneous plane waves and dispersive & non-dispersive media. Reflection and refraction of homogeneous plane electromagnetic waves, law of reflection, Snell's law, Fresnel's formulae (only for normal incidence & optical frequencies) and Stoke's law.

Part-B : Physical Optics & Lasers

UNIT-V **Interference :** Conditions for interference and spatial & temporal coherence. Division of Wavefront-Fresnel's Biprism and Lloyd's Mirror. Division of Amplitude-Parallel thin film, wedge shaped film and Newton's Ring experiment. Interferometer-Michelson and Fabry-Perot.

UNIT-VI **Diffraction :** Distinction between interference and diffraction. Fresnel's and Fraunhofer's class of diffraction. Fresnel's Half Period Zones and Zone plate. Fraunhofer diffraction at a single slit, n slits and Diffracting Grating. Resolving Power of Optical Instruments-Rayleigh's criterion and resolving power of telescope, microscope & grating.

UNIT-VII **Polarisation :** Polarisation by dichroic crystals, birefringence, Nicol prism, retardation plates and Babinet's compensator. Analysis of polarised light. Optical Rotation-Fresnel's explanation of optical rotation and Half Shade & Biquartz polarimeters.

UNIT-VIII **Lasers :** Characteristics and uses of Lasers. Quantitative analysis of Spatial and Temporal coherence. Conditions for Laser action and Einstein's coefficients. Three and four level laser systems (qualitative discussion). Types of lasers and laser.

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PART-A : ELECTROMAGNETIC THEORY

UNIT-I

Electrostatics

SECTION-A VERY SHORT ANSWER TYPE QUESTIONS

Q.1. Write down the basic properties of electric charge.

Ans. There are following basic properties of electric charge :

1. Charge comes in two varities : positive and negative.
2. Charge is conserved.
3. Charge is quantised.

Q.2. Explain the meaning of the statement 'electric charge of a body is quantized'.

Ans. It is found experimentally that all free charges are integral multiple of a basic unit of charge known as charge of electron and denoted by 'e'. Thus charge q on a body is always given by

$$Q = ne \text{ (where } n \text{ is an integer, positive or negative).}$$

The fact that electric charge is always on integral multiple of e is termed as 'Quantization of charge'.

If there is a transfer of charge from one object to another, a full transfer of electron will take place.

Q.3. Name the different types of charge densities.

Ans. There are three different types of charge densities found in electrostatics :

1. Linear charge density $(\lambda) = \dfrac{\text{Total charge}}{\text{Length}}$
2. Surface charge density $(\sigma) = \dfrac{\text{Total charge}}{\text{Area}}$
3. Volume charge density $(\rho) = \dfrac{\text{Total charge}}{\text{Total volume}}$.

Q.4. State Coulomb's law.

Ans. According to Coulombs law, "The force between two point charges is directly proportional to the product of the charges, and inversely proportional to the square of the distance between them".

Let two point charges q_1 and q_2 are situated at points A and B respectively. The distance between these two points is r, then the force F exerted by q_1 on q_2 is given by

$$F = \frac{1}{4\lambda\varepsilon_0} \frac{q_1 q_2}{r^2}$$

where ε_0 is the permittivity of free space. $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}M^2$.

This force will act along the line joining the two charges.

Q.5. Explain superposition principle.

Ans. If we have n point charges, they act independently in pairs. The force on any one of them, let us say q_1 is given by the vector sum

$$\vec{F_1} = \vec{F_{12}} + \vec{F_{13}} + \vec{F_{14}} + \ldots \vec{F_{1n}}$$

The presence of other charges does not affect the force between any pair.

Q.6. What are the differences between electrostatic force and gravitational force?

Ans. There are two basic differences between electrostatic force and gravitational force :

1. Gravitational force is always attractive, while electrostatic force can be attractive or repulsive both.
2. Gravitational force is weak force as compared to electrostatic force. It is approximately 10^{39} times stronger as compared to gravitational force.

Q.7. Define electric field.

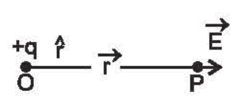
Ans. The electric field due to a charge Q at a point in space may be defined as the force that a unit positive charge would experienced if placed at that point. The charge Q, which is producing the electric field is called a source charge and which test the effect of source charge is known as test charge.

$$\vec{E} = \frac{\vec{F}}{q}$$

Its unit is N/C or volt/meter.

Q.8. Write the expression for the electric field due to a point charge.

Ans. Let P be a point lying in vaccum at a distance r from a point charge q at point O.



Let a test charge q_0 be placed at P. According to coulomb's law. The force F acting on q_0 due to q is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q q_0}{r^2} \hat{r}$$

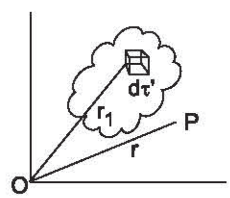
The electric field at the point P is, by definition, given by the force per unit test charge

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

The direction of $\vec{E}$ is along the line joining O and P. Pointing outward if q is positive and inward if q is negative.

Q.9. Give the expression for the electric field in terms of volume charge distribution.

Ans. Suppose charge is distributed in a given volume as shown in Fig. Let us take a small volume element $d\tau'$ at the position vector r'. If the charge contained in this small volume is dq', the quantity



$$\rho(r') = \frac{dq'}{d\tau'}$$ is called the volume charge density.

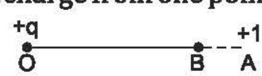
The charge in this element is $dq' = \rho(r')\,d\tau'$

The electric field at the position r due to this volume charge distribution is given by

$$E(r) = \int_{\text{Volume}} \frac{\rho(r')(\vec{r} - \vec{r'})\,d\tau'}{4\pi\varepsilon_0 |\vec{r} - \vec{r'}|^3}$$

Q.10. Define electric potential.

Ans. Let us consider an isolated point charge $+q$ lying at O. A and B are two points in its electric field. Let W_{AB} be the work done by an external agent in moving a unit positive charge from A to B. The potential difference between two points in an electric field is the amount of work done in moving a unit positive charge from one point to another against electric forces.



$$W_{AB} = V_B - V_A$$

If A is at infinity then $V_A = 0$

$$W = W_B$$

Therefore electric potential at a point is defined as the amount of work done in moving a unit positive charge from infinity to that point, without acceleration, against electrical forces. This is a scalar quantity.

Q.11. What do you understand from equipotential surface?

Ans. This is defined as the surface over which all the points have the same potential. If a test charge q is moved between any two points on an equipotential surface through any path, the work done is zero. This is because the potential difference between two points A and B is defined by

$$V_B - V_A = W_{AB/q}$$

If $$V_A = V_B \Rightarrow W_{AB} = 0$$

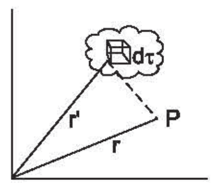
Hence the electric field vector must be everywhere normal to an equipotential surface.

Q.12. Write the formula for electric potential in terms of volume charge density.

Ans. The electric potential at a distance *r* from a point charge '*q*' is given by

$$v = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

If the charge distribution is continuous



Electric potential is given by

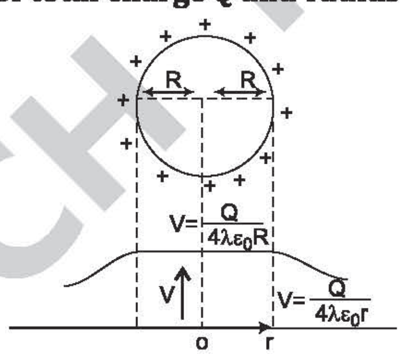
$$V = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

For a volume charge it is given by

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(r')}{|r-r'|}d\tau'$$

Q.13. Draw the variation of electric potential due to a uniformly charged conducting sphere of total charge *Q* and radius *R*.

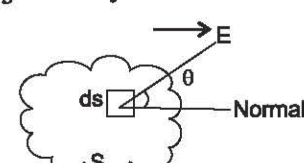
Ans.



Q.14. Define electric flux and give its unit.

Ans. The flux of *E* through a surface *S*, is given by

$$\phi_E = \oint_S \vec{E}\cdot\vec{dS} = \oint E\,\vec{dS}\cos\theta$$



where θ is the angle between area vector and electric field.

It is a measure of the number of field lines passing through *S*. This suggests that the flux through any closed surface is a measure of total charge inside. The unit of electric flux is Newton-meter^2/coulomb.

Q.15. State Gauss's law.

Ans. The total flux of the electric field $\vec{E}$ over any closed surface is equal to $1/\varepsilon_0$ times the total net charge enclosed by the surface.

$$\phi = \oint \vec{E} \cdot \vec{dS} = \frac{Q_{enc}}{\varepsilon_0} \quad ...(1)$$

eq. (1) is the integral form of Gauss law.
The differential form of Gauss law can be written as

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad ...(2)$$

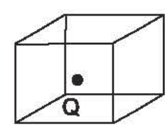
where ρ is the volume charge density and ∇ (delta) is a operator given by

$$\nabla \equiv \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Q.16. A charge Q is placed at the center of a cube. Calculate the electric flux through any one of the face of the cube.

Ans. Since electric flux is a measure of total charge enclosed by the surface *i.e.*,

$$\phi_E = \frac{Q_{enc}}{\varepsilon_0}$$



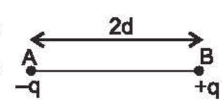
for cube $Q_{enc} = Q$ $\qquad \phi_E = \frac{Q}{\varepsilon_0}$

This is the total flux from all the faces of the cube. Since cube has six faces, therefore flux through any one of the face of the cube will be 1/6th of the total flux.
Flux through one face is

$$\phi = \frac{\phi_E}{6} = \frac{Q}{6\varepsilon_0}$$

Q.17. What is an electric dipole?

Ans. Let us consider two charges $-q$ at point A and $+q$ at B, the distance between them being $2d$. Such a charge configuration is called an electric dipole.



The magnitude of the dipole moment P is given by the product of any one of the charges and the distance between them.

$$P = q \times 2d$$

The direction of P is from $-q$ to $+q$.
The units of P is coulomb-metre.

Q.18. Write the formula for electric field due to a dipole at an axial point and on the equatorial line.

Ans. The expression for electric field due to a dipole at an axial point is given by

$$E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\frac{2P}{r^3} \quad \text{[}P\text{-Dipole moment]}$$

Electric field on an equatorial line due to dipole is

$$E_{eq.} = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^3}$$

Q.19. What is the relation between electric field and electric potential. Calculate the value of electric field, if potential is given by $\phi = 3x^2 yz$.

Sol. The relation between electric field and electric potential is given by

$$E = -\text{grad}\,\phi = -\nabla\phi \qquad \text{...(1)}$$

Thus the electric field at any point is the negative of the gradient of potential at that point. The minus sign indicates that E points in the direction of decreasing ϕ

$$\phi = 3x^2 yz$$

using above relation (1)

$$E = -\nabla\phi = -\frac{\partial\phi}{\partial x}\hat{i} - \frac{\partial\phi}{\partial y}\hat{j} - \frac{\partial\phi}{\partial z}\hat{k}$$

$$\frac{\partial\phi}{\partial x} = 6xyz, \frac{\partial\phi}{\partial y} = 3x^2 z, \frac{\partial\phi}{\partial z} = 3x^2 y$$

$$E = -6xyz\,\hat{i} - 3x^2 z\,\hat{j} - 3x^2 y\hat{k}$$

Q.20. Explain the term potential energy.

Ans. The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system bringing each charge in from an infinite distance. The electric potential energy of system of two charges q_1 and q_2 placed at a distance r is given by

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

If we have n different charges in any arrangement in space. Then

$$U = \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)\sum_{i=1}^{n}\sum_{j\neq 1}\frac{q_i q_j}{r_{ij}}$$

Q.21. What is a dielectric?

Ans. A dielectric (or insulator) is a material in which all the electrons are tightly bound to the nuclei of the atoms. Thus, there are no free electrons to carry current. Hence the electrical conductivity of a dielectric is very low. Glass, eborite, mica, rubber, oils etc. are the examples of dielectric.

Q.22. Define dielectric constant.

Ans. The ratio of the capacitances of a given capacitor with the material filling the entire space between its plates to the capacitance of the same capacitor in vaccum is called the dielectric constant or relative permittivity of the material.

If C is the capacitance with dielectric material and C_0 that in vaccum, then dielectric constant of the material

$$K = C/C_0$$

Q.23. What are polar and non-polar molecules?

Ans. A polar molecule is one in which the centre of gravity of the positive charges is separated from the centre of gravity of the negative charges by a finite distance. The polar molecule is thus have an electric dipole and has an intrinsic permanent electric dipole moment.

A non-polar molecule is one in which the centre of gravity of positive charges coincides with the centre of gravity of the negative charges. Symmetrical molecules (*e.g.*, H_2, N_2, O_2) are non-polar. A non-polar molecules has zero electric dipole moment.

Q.24. Write Gauss law in dielectrics.

Ans. The Gauss law in dielectrics can be written as

$$\oint K\vec{E}\cdot\vec{ds} = \frac{q}{\varepsilon_0}$$

where K is the dielectric constant.

Q.25. Define electric susceptibility.

Ans. When a dielectric material is placed in an electric field, it becomes electrically polarised for isotopic dielectrics the polarisation vector P is proportional to the electric field E. Thus,

$$\vec{P} = \varepsilon_0 \chi \vec{E}$$

where χ (chi), the electric susceptibility, is a dimensionless quantity and is a characteristic of the medium.

Q.26. Explain the terms dielectric polarization.

Ans. When a dielectric is placed in an external electric field, the electrically charged particles with their displacement arrange in such order that dielectric acquires a certain electric moment, this is called dielectric polarization.

Linear dielectrics shows a direct proportionality between the electric moment P (induced moment) acquired by the particle during the process of polarisation and the intensity of the electric field acting on the particle *i.e.*, the local electric field E_{int} inside the dielectric

$$P \propto E_{int}$$

or

$$P = \alpha E_{int}$$

α is known as polarizability of the given dipole.

Q.27. What are the limitations of Coulomb's law?

Ans. The Coulomb's law fails to explain the stability of the nucleus, since in a nucleus, there are several protons all having the positive charge. Since they have the same charge they must repel according to Coulomb's law, but they do not push themselves because nucleus has the stable identity. Hence, here the Coulomb's law fails.

Q.28. What work will done in taking charge q from infinity up to mid point of an electric dipole?

Ans. Work done is defined as change in potential energy

$$W = (V_B - V_A)q$$

Since at infinity potential is zero and at the mid point of dipole potential is also zero. Therefore,

$$W = 0 \qquad [\text{as } V_B = V_A = 0]$$

Therefore no work will done *m* taking a charge from infinity to mid point of dipole.

Q.29. The electric potential at the centre of a spherical conductor of radius 10 cm is 50 V. Find its surface charge density.

Sol. Potential of a spherical conductor is given by

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

$$50 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

$$\Rightarrow \qquad Q = (4\pi\varepsilon_0 R)50$$

Surface charge density is given by

$$\sigma = \frac{Q}{4\pi R^2}$$

$$\sigma = \left(\frac{4\pi\varepsilon_0 R}{4\pi R^2}\right)50$$

$$\sigma = \left(\frac{\varepsilon_0}{R}\right)50$$

$$\sigma = \frac{8.85\times10^{-12}\times50}{10\times10^{-2}} \qquad [R = 10\text{ cm} = 10\times10^{-2}\text{ m}]$$

$$= 44.25\times10^{-10} = 4.425\times10^{-9}\text{ c/m}^2$$

Q.30. Write Laplace and Poisson's equation.

Ans. Laplace equation

$$\nabla^2 V = 0$$

Poisson's equation

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \qquad \text{(for free space)}$$

and

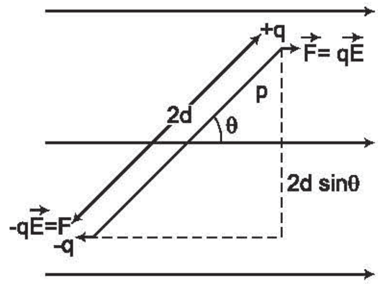
$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

SECTION-B SHORT ANSWER TYPE QUESTIONS

Q.1. Derive the expression for potential energy of a dipole in a uniform electric field.

Ans. Let us consider a dipole of moment $P = 2qd$, placed in a uniform electric field E. Let at any instant dipole makes an angle θ with the electric field. The charge $+q$ experiences a force qE in the direction of the field, while $-q$ experiences a force $-qE$ opposite to the direction of electric field. Since both the forces are in opposite directions therefore net linear force is zero.

As both the forces are not collinear, so they form a couple and there is a net clockwise torque, which tend to set the dipole in the direction of the electric field. The magnitude of the torque is



$$\tau = qE\,(2d\sin\theta)$$
$$= PE\sin\theta \qquad [\because P = q \times 2d]$$

In vector form $\vec{\tau} = \vec{P} \times \vec{E}$

Let W be the work done by an external agent to turn the dipole from its reference orientation θ_0 to angle θ, then

$$W = \int_{\theta_0}^{\theta} \tau\, d\theta = \int_{\theta_0}^{\theta} PE\sin\theta\, d\theta = -[PE\cos\theta]_{\theta_0}^{\theta}$$
$$W = PE\cos\theta_0 - PE\cos\theta = PE[\cos\theta_0 - \cos\theta]$$

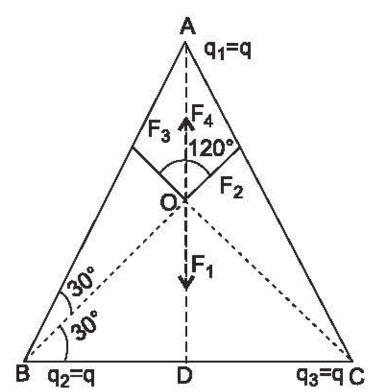
If $\theta_0 = 90°$ then

$$W = -PE\cos\theta \text{ or } W = -\vec{P}\cdot\vec{E}$$

This work done is stored at potential energy in the dipole.

Q.2. Consider three charges q_1, q_2, q_3 each equal to q at the vertices of an equilateral triangle of side 'a'. What is the force on a charge Q (with the same sign as q) placed at the centroid of the triangle.

Ans. In the given equilateral triangle ABC of sides of length 'a'. If we draw a perpendicular AD to the side BC



$$AD = AC\cos 30° = \frac{\sqrt{3}}{2}a$$

and $$AO = \frac{2}{3}AD = \frac{1}{\sqrt{3}}a$$

By symmetry $AO = BO = CO$

Thus, using Coulomb's law

Force F_1 on Q due to charge q at $A = \frac{3}{4\pi\varepsilon_0}\frac{Qq}{a^2}$ along AO

Force F_2 on Q due to charge q at $B = \frac{3}{4\pi\varepsilon_0}\frac{Qq}{a^2}$ along BO

Force F_3 on Q due to charge q at $C = \frac{3}{4\pi\varepsilon_0}\frac{Qq}{a^2}$ along CO

The resultant of forces F_2 and $F_3/F_4 = \sqrt{F_2^2 + F_3^2 + 2F_2F_3\cos 120°}$

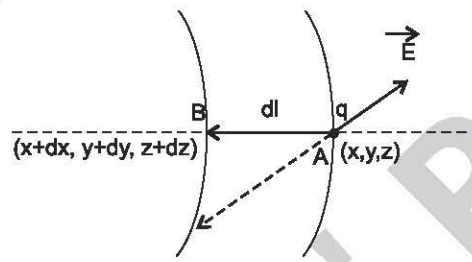
$$= \frac{3}{4\pi\varepsilon_0}\frac{Qq}{a^2} \text{ along } OA$$

Hence the total force on $Q = F_1 - F_4 = 0$

Therefore, the force on charge Q will be zero.

Q.3. Establish the relation $\vec{E} = -\nabla V$.

Sol. Let us consider two neighbouring points $A(x, y, z)$ and $B(x+dx, y+dy, z+dz)$ distance dl apart in the region.



Let the value of potential at A and B be V and $V + dV$ respectively. Let dV be the change in potential in going from A to B, then

$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$= \left\{\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial Z}\right\}\{\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz\}$$

Here, $(\hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz)$ is the displacement vector dl between A and B, thus

$$dV = (\text{grad } V)\cdot dl \qquad ...(1)$$

The work done by the external agent in moving a test charge q from A to B along dl is

$$dW = \vec{F}\cdot\vec{dl} = -q\,\vec{E}\cdot\vec{dl}$$

or

$$\frac{dW}{q} = -\vec{E}\cdot\vec{dl}$$

But, by definition, dW/q is the potential difference dV between the points A and B, thus

$$dV = -\vec{E}\cdot\vec{dl} \qquad ...(2)$$

Comparing eq. (1) and (2)

$$E = -\text{grad } V = -\nabla V$$

Thus, the electric field at any point is the negative of the gradient of potential at that point. The minus sign indicates that E points in the direction of decreasing V.

Q.4. Prove the differential form of Gauss law from integral form.

Ans. The integral form of Gauss Law is given by

$$\oint \vec{E}\cdot\vec{dS} = \frac{Q_{enc}}{\varepsilon_0} \qquad ...(1)$$

Now, suppose the charge is distributed over a volume. Let ρ be the charge density. Then total charge within the closed surface enclosing the volume is given by

$$Q = \int_V \rho \, d\tau \qquad ...(2)$$

from Gauss divergence theorem eq. (1) can be written as

$$\oint \vec{E} \cdot \vec{dS} = \int_V (\vec{\nabla} \cdot \vec{E}) \, d\tau \qquad ...(3)$$

Now eq. (1) can be written as

$$\int_V (\vec{\nabla} \cdot \vec{E}) \, d\tau = \int_V \frac{\rho d\tau}{\varepsilon_0} \qquad ...(4)$$

or $$\int \left(\vec{\nabla} \cdot \vec{E} - \frac{\rho}{\varepsilon_0} \right) d\tau = 0$$

The integral is zero only if integrand is zero. Therefore

$$\vec{\nabla} \cdot \vec{E} - \frac{\rho}{\varepsilon_0} = 0$$

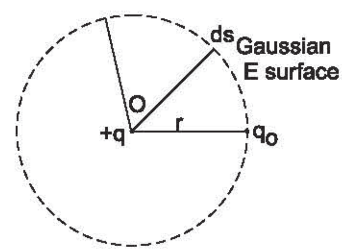
or $$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

This is the differential form of Gauss law.

Q.5. Derive Coulomb's law from Gauss law.

Ans. Let us consider a positive point charge q placed at θ and another charge q_0 at a distance 'r' from it.

Let us draw a Gaussian surface. (In this case a hallow spherical shell) of radius 'r'. The charge enclosed by this Gaussian surface is 'q'.



Using Gauss law in integral form

$$\oint \vec{E} \cdot \vec{ds} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint E \, ds \cos\theta = \frac{q}{\varepsilon_0}$$

Since electric vector E and area vector dS are parallel therefore angle between them will be zero $(\theta = 0°)$

$$\oint E \, ds = \frac{q}{\varepsilon_0} \qquad [\cos 0° = 1]$$

Again E is constant over the Gaussian surface, hence E will come out from integration

$$E \oint dS = \frac{q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0} \qquad [\oint ds = 4\pi r^2]$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$ [Surface area of spherical shell]

This is the electric field due to a point charge. Therefore the force on charge q_0 placed at r will be

$$F = q_0 E$$

$$\Rightarrow \quad F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}$$ This is Coulomb's law.

Q.6. Derive the expression $\vec{P} = \varepsilon_0 \vec{E}(k-1)$, where symbols have their usual meaning.

Ans. The polarisation vector $\vec{V}$ can be expressed as

$$\vec{P} = \varepsilon_0 \chi \vec{E} \quad ...(1)$$

The constant χ is called the electric susceptibility of the dielectric material. E is the electric field within the dielectric. The electric displacement D is defined by the relation

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \quad ...(2)$$

Substituting eq. (1) into eq. (2)

$$\vec{D} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi \vec{E}$$

$$\vec{D} = \varepsilon_0 (1+\chi)\vec{E} \quad ...(3)$$

We have $$\vec{D} = \varepsilon \vec{E} = \varepsilon_0 KE$$

Putting this value in eq. (3)

$$\varepsilon_0 KE = \varepsilon_0 (1+\chi)\vec{E}$$

or $$K = 1+\chi$$

$$\chi = K-1$$

Substituting this value of χ in eq. (1), we get

$$P = \varepsilon_0 (K-1)\vec{E}$$

Q.7. Show that the force between two charges separated by a distance is reduced by a factor $\left[\frac{1}{1+P/\varepsilon_0 E}\right]$ due to the presence of a dielectric.

Sol. Let us consider two charges q_1 and q_2 are separated by a distance d in free space. Then force between them using Coulomb's law is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{d^2}$$

When a dielectric of dielectric constant K is introduced between them, the force between two charges is given by

$$F' = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{d^2}$$

$$F' = \frac{F}{K} \qquad ...(1)$$

We have the relation $P = \varepsilon_0 E(K-1)$

or $K = 1 + P/\varepsilon_0 E$

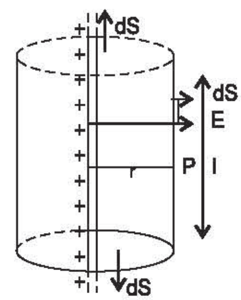
Putting this value of K in eq. (1), we get

$$F' = \frac{F}{\left[1 + \dfrac{P}{\varepsilon_0 E}\right]}$$

The force is reduced in the presence of dielectric.

Q.8. Calculate the electric field at a distance r, due to a infinite line of charge, carrying charge per unit length λ, using Gauss law.

Sol. Let us consider a infinite line of positive charge carrying charge per unit length λ (charge/length).



We have to calculate electric field at point P. Let us draw a Gaussian surface passing through P, which is by symmetry is a cylindrical surface. The length of cylinder is l.

The direction of electric field due to this positive line charge is perpendicular to line charge and away from the charge, first we calculate the flux through this cylinderical surface.

As the cylinder has three surfaces, two caps and one curved surface.

Since area vector of two caps are perpendicular to electric field so the flux through these surfaces will be zero.

The area vector of curved surface and electric field is parallel to each other so the curved surface will contribute to flux through curved surface is

$$\phi = \oint \vec{E}\cdot\vec{dS} = \oint E\,dS\cos 0° = \oint E\,dS$$

Since E is constant over the surface so it will come out from the integral

$$\phi = E\oint dS$$

$$\phi = E\cdot 2\pi r l \qquad [\because \oint dS = 2\pi r l]$$

Now charge enclosed by the cylinder is

$$Q_{enc} = \lambda l$$

Applying Gauss's law

$$\oint \vec{E}\cdot\vec{dS} = \frac{Q_{ene}}{\varepsilon_0}$$

$$E\cdot 2\pi r l = \frac{\lambda l}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

This is the required result. It is clear from the formula that E is inversely proportional to the distance.

SECTION-C LONG ANSWER TYPE QUESTIONS

Q.1. Explain the effect of electric field on dielectric.

Ans.

Effect of Electric Field on Dielectric

Suppose a slab of dielectric material is placed in the uniform electric field E_0 set up between the plates of a charged parallel plate capacitor (Fig. 1). The slab becomes electrically polarised. The polarisation charges inside the body of dielectric cancel each other because the negative side of one polarised molecule is adjacent to the positive side of its neighbour. Thus the net charge in the interior of the slab remains zero. The charges at the ends of dielectric do not cancel. These polarisation charges are called *bound charges* as these cannot be conducted away by a conductor. The charges on the capacitor plates are called free charges. The polarisation charges induced on the two faces of the slab produce their own electric field E', which opposes the external field, E_0. The induced electric field E' is smaller than the applied electric field E_0. Hence the resultant field E within the dielectric is smaller than E_0, but points in the same direction as E_0. The field in the rest of the (free) space is still E_0. Hence we conclude that when a dielectric is placed in an electric field, the field 'within' the dielectric is weakened.

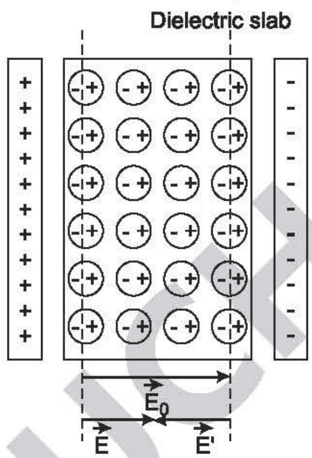


Fig. 1

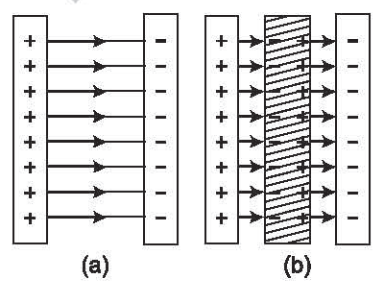


Fig. 2

The weakening of electric field within the dielectric is illustrated in Fig. 2. The Fig. 2(a) shows the original field and in Fig. 2 (b), some of the lines of force leaving the positive plate of the capacitor penetrate the dielectric, others terminate on the charges induced on the dielectric.

The reduction in the magnitude of the electric field from E_0 to E causes a reduction in the potential difference between the plates of the capacitor. Let V_0 and V be the potential difference without dielectric and with dielectric completely filling the space between the plates.

Then we have
$$\frac{E_0}{E} = \frac{V_0}{V}$$

But $\frac{V_0}{V} = \frac{C}{C_0} = K$, where K is dielectric constant of the slab.

$$\frac{E_0}{E} = K$$

Thus the electric field with in the dielectric reduced by a factor k.

Q.2. Calculate the curl of electrostatic field.

Sol. Let us take the simplest possible configuration, a point charge q at the origing. The electric field E at a distance r from the point charge is given by

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r}$$

where $\hat{r}$ is the unit vector.

If we calculate the line integral of this field from some point 'a' to some other point 'b' as shown in fig.

$$\int_a^b \vec{E} \cdot \vec{dl}$$

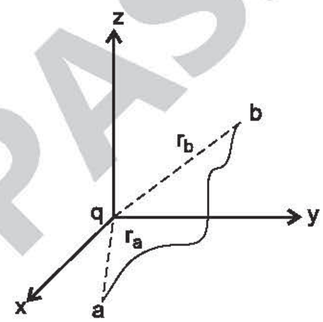


Fig.

In spherical co-ordinates

$$\vec{dl} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r \sin\theta\, d\phi\,\hat{\phi}$$

$$\vec{E} \cdot \vec{dl} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} dr$$

Therefore,
$$\int_a^b \vec{E} \cdot \vec{dl} = \frac{q}{4\pi\varepsilon_0} \int_{r_a}^{r_b} \frac{1}{r^2} dr$$

$$= \frac{1}{4\pi\varepsilon_0} \left[-\frac{1}{r} \right]_{r_a}^{r_b}$$

$$= \frac{q}{4\pi\varepsilon_0} \left[\frac{1}{r_a} - \frac{1}{r_b} \right]$$

where r_a is the distance of point a and r_b is the distance of point b from origin. The integral around the closed path is zero as $r_a = r_b$.

$$\oint E \cdot dl = 0$$

applying Stoke's theorem

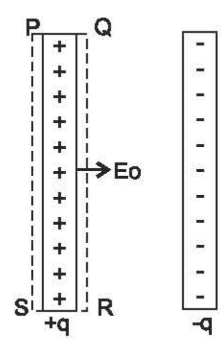
$$\int (\vec{\nabla} \times \vec{E}) \cdot dS = 0 \qquad [\int \vec{A} \cdot \vec{dl} = \int (\vec{\nabla} \times \vec{A})\, dS]$$

or
$$\vec{\nabla} \times \vec{E} = 0$$

The curl of electrostatic field will be zero.

Q.3. Derive Gauss law in dielectrics.

Ans. Let us consider the parallel plate capacitor with plate area A and having vacuum between its plates.



Let $+q$ and $-q$ be the charges on the plates and E_0 the uniform electric field between the plates. Let $PQRS$ be a Gaussian surface.

By Gauss's law

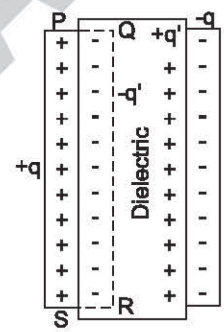
$$\oint \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

Since E_0 and dS are parallel and E_0 is constant over the surface

$$\oint \vec{E_0} \cdot \vec{dS} = \oint E_0 dS = E_0 A$$

$\therefore$
$$E_0 A = \frac{q}{\varepsilon_0} \Rightarrow E_0 = \frac{q}{A\varepsilon_0} \qquad ...(1)$$

Suppose a material of dielectric constant k is introduced in the space between the plates. The dielectric slab gets polarised. A negative charge $-q'$ is induced on one surface and an equal positive charge $+q'$ on the other side. The charges $-q'$ and $+q'$ are known as 'induced charges' or sometimes 'bound charges'. These induced charges produced their own field which opposes the external field E_0. Let E be the resultant field within the dielectric. The net charge within the Gaussion surface is $q-q'$.



$\therefore$ By Gauss's law
$$\oint \vec{E} \cdot \vec{dS} = \frac{q-q'}{\varepsilon_0} \qquad ...(2)$$

or
$$EA = \frac{q-q'}{\varepsilon_0}$$

$$E = \frac{q-q'}{\varepsilon_0 A} \qquad ...(3)$$

Now
$$\frac{E_0}{E} = K,$$

where K is the dielectric constant eq. (1) becomes

$$E = \frac{q}{K\varepsilon_0 A}$$

Inserting this in eq. (3)

$$\frac{q}{K\varepsilon_0 A} = \frac{q-q'}{\varepsilon_0 A}$$

or
$$q - q' = \frac{q}{K}$$

Substituting this value of $q-q'$ in eq. (2)

$$\oint \vec{E} \cdot \vec{dS} = \frac{q}{K\varepsilon_0}$$

or
$$\oint K \vec{E} \cdot dS = \frac{q}{\varepsilon_0}$$

It is the Gauss law in the presence of dielectric.

Q.4. Calculate the electric field (using Gauss law) due to a uniformly charged at sphere of total charge Q and radius R:

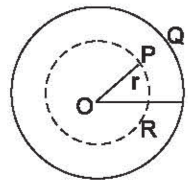
(i) an interior point

(ii) on the surface

(iii) at an exterior point.

Also show the variation of electric field with respect to distance.

Sol. Let us consider a sphere of radius R and total charge Q. The volume charge density (ρ) is given by



$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} \quad ...(1)$$

(i) To find electric field at an interior point at a distance r from the centre. Let us draw a Gaussian surface passes through this point P ($OP = r$).

Since this is a sphere of radius r. The total charge contained in this sphere is

$$q' = \rho \times \frac{4}{3}\pi r^3 \quad [q' = \text{volume} \times \text{charge density}]$$

$$= \frac{Q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$$

$$q' = \frac{Qr^3}{R^3} \quad ...(2)$$

The Gauss law is given by

$$\oint \vec{E} \cdot \vec{dS} = \frac{Q_{enc}}{\varepsilon_0}$$

Since $\vec{E}$ and $\vec{dS}$ are parallel to each other the angle between them will be zero.

Therefore, $$\oint \vec{E} \cdot \vec{dS} = \oint E\, dS \cos 0° = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint E\, dS = \frac{Q_{enc}}{\varepsilon_0}$$

Since E is constant over the surface so it will come out from the integration sign. Hence,

$$E \oint dS = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint dS = 4\pi r^2 \quad \text{(total surface area of the sphere)}$$

So, $$E \cdot 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0}$$

In this case $$Q_{enc} = q' = \frac{Qr^3}{R^3}$$

$$E \cdot 4\pi r^2 = \frac{Qr^3}{\varepsilon_0 R^3}$$

$$E = \frac{Qr}{4\pi\varepsilon_0 R^3} \qquad ...(3)$$

It can also be written as $E = \frac{\rho r}{3\varepsilon_0}$ ...(4) [From (1) $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$]

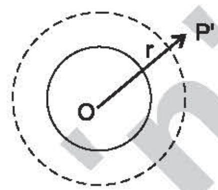
(ii) At the surface $r = R$

using eq. (3) $E = \frac{QR}{4\pi\varepsilon_0 R^3}$

$$= \frac{Q}{4\pi\varepsilon_0 R^2}$$

(iii) At an exterior point.

Let us consider a point P' at a distance r from the center of sphere.



Draw a Gaussian surface passing through this point P'.

Due to symmetry, the field at P' and on every where on the Gaussian surface will be perpendicular to Gaussian surface.

The flux through this surface is given by

$$\oint \vec{E} \cdot \vec{dS} = \oint E\, dS$$ [angle between E and dS is zero]

$$E \oint dS = \frac{q_{enc}}{\varepsilon_0}$$ (Gauss law)

$$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = Q$$

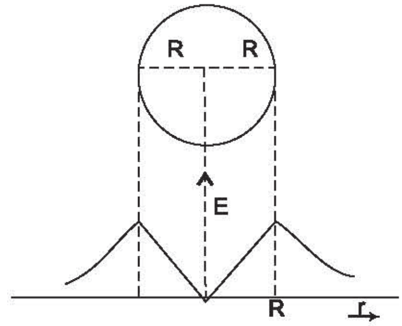
$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

or $$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$ [$\hat{r}$ is the unit vector along the radius]

This is same as the electric field at a distance r from a point charge.

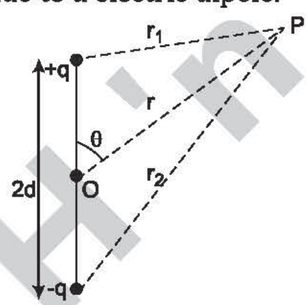
The variation of electric field with respect to distance is as shown in the figure.



Q.5. Derive the expression for field and potential due to an electric dipole.

Ans. An electric dipole consists of two charges q and $-q$ separated by a small distance $2d$. It is characterised by a dipole moment vector $\vec{P}$ whose magnitude is $q \times 2d$ have the direction from $-q$ to $+q$.

We shall calculate the potential due to a electric dipole.



Let us consider a point P at a distance r from the centre of the dipole. The distance of this point P from $+q$ is r_1 and from $-q$ is r_2 as shown in the figure. The potential at P due to this dipole is given by

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r_1} - \frac{q}{r_2}\right] \qquad ...(1)$$

Now by Geometry

and
$$\left.\begin{aligned} r_1^2 &= r_2 + d^2 - 2rd\cos\theta \\ r_2^2 &= r^2 + d^2 + 2rd\cos\theta \end{aligned}\right\} \qquad ...(2)$$

We take r much greater than d ($r >> d$) and retain terms only the first order in d/r

$$r_1^2 = r^2\left(1 + \frac{d^2}{r^2} - \frac{2d}{r}\cos\theta\right)$$

$$\cong r^2\left(1 - \frac{2d\cos\theta}{r}\right) \qquad ...(3)$$

Similarly,
$$r_2^2 \cong r^2\left(1 + \frac{2d\cos\theta}{r}\right) \qquad ...(4)$$

Using the Binomial theorem and retaining terms up to the first order in d/r, we obtain

$$\frac{1}{r_1} \cong \frac{1}{r}\left(1-\frac{2d\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1+\frac{d}{r}\cos\theta\right) \quad ...(5)$$

and

$$\frac{1}{r_2} \cong \frac{1}{r}\left(1+\frac{2d\cos\theta}{r}\right)^{-1/2} \cong \frac{1}{r}\left(1-\frac{d}{r}\cos\theta\right) \quad ...(6)$$

Using eq. (1), (5) and (6) we get

$$V = \frac{q}{4\pi\varepsilon_0}\left\{\frac{1}{r}\left(1+\frac{d}{r}\cos\theta\right)-\frac{1}{r}\left(1-\frac{d}{r}\cos\theta\right)\right\}$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left\{\frac{2d\cos\theta}{r}\right\}$$

$$V = \frac{2qd\cos\theta}{4\pi\varepsilon_0 r^2}$$

Since $P = 2qd$, we have

$$V = \frac{P\cos\theta}{4\pi\varepsilon_0 r^2}$$

This formula is approximately true only for distance large compared to the size of dipole. The important constracting features of electric potential of a dipole are :

(i) The potential due to a dipole depends not just on r but also on the angle between the position vector and dipole moment vector.

(ii) The potential falls off, at large distance, as $\frac{1}{r^2}$ not as $\frac{1}{r}$.

Q.6. Using Gauss's law, obtain the electric field due to a uniform charged infinite plane sheet of charge and hence find an expression for electric field between the plates of a parallel plate capacitor.

Ans.

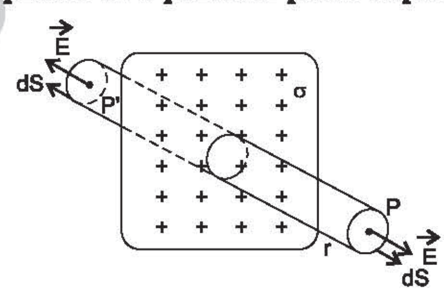


Fig. 1

Let us consider a thin, non conducting infinite plane sheet of charge as shown in Fig. 1. Let the surface charge density is 'σ' (charge/area). Let P be a point at a distance 'r' from the sheet. We have to calculate electric field at P. The Gaussian surface is a pill box extended on both sides of the sheet as shown in Fig. 1, from symmetry, E points at right angles to the end caps and away from the plane sheet, its magnitude is same at P and P'.

The flux through the two plane ends is

$$\begin{aligned}\phi &= \oint \vec{E} \cdot \vec{dS} + \oint \vec{E} \cdot \vec{dS} \\ &= \oint E\, dS + \oint E\, dS \\ &= EA + EA = 2EA\end{aligned}$$

'The flux through the curved surface is zero' as $\vec{E}$ and $\vec{dS}$ are perpendicular to each other.
Hence total flux through the Gaussian cylinder is

$$\phi_T = 2EA + 0 = 2EA$$

The net charge enclosed by the Gaussian cylinder

$$q = \sigma A$$

Applying Gauss's law

$$\phi = \oint \vec{E} \cdot \vec{dS} = \frac{Q_{enc}}{\varepsilon_0}$$

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$\Rightarrow$ $$E = \frac{\sigma}{2\varepsilon_0}$$

This is the electric field due to a plane sheet of charge. It is clear from the above result that E is independent of the distance from the sheet.

Fields due to two parallel sheets of charge (Parallel Plate Capacitor) : Let us consider two parallel intinite sheet of charge with equal and opposite charge densities $+\sigma$ and $-\sigma$.

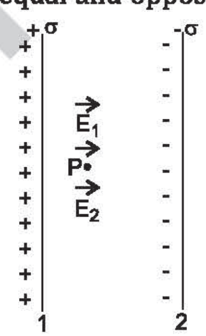


Fig. 2

Let us consider a point P between the plates of parallel plate capacitor.
The electric field at P due to plate 1

$$\vec{E}_1 = \frac{\sigma}{2\varepsilon_0} \qquad \text{(away from plate 1)}$$

The electric field at P due to plate 2

$$\vec{E}_2 = \frac{\sigma}{2\varepsilon_0} \qquad \text{(toward the plate-2)}$$

Since electric field is in the same direction at P. Therefore electric field at P

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} \Rightarrow E = \frac{\sigma}{\varepsilon_0} \text{ toward right.}$$

Q.7. Define electric flux. The volume charge density in sphere of radius R is given by $\rho = \rho_0 \left(1 + \frac{r}{R}\right)$ for $r \le R$.

$\rho = 0$ for $r > R$

Calculate flux linked with the sphere.

Ans.

Electric Flux

Let us consider an infinitesimal surface element dS of surface S as shown in Fig. 1.

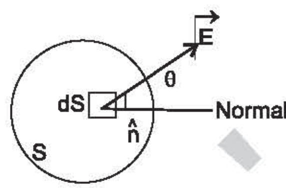


Fig. 1

The flux of E through a surface S is a measure of number of lines passing through surface. Mathematically electric flux is defined as

$$\phi = \oint \vec{E} \cdot \hat{n}\, dS \qquad [\hat{n} = \text{normal unit vector}]$$

This is a measure of total charge enclosed by the surface. Its unit is Newton metre^2/coulomb.

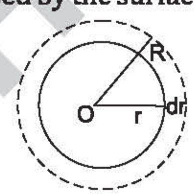


Fig. 2

Let us consider a sphere of radius R whose charge density is given by

$$\rho = \begin{cases} \rho_0 \left(1 - \frac{r}{R}\right) & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$$

The electric flux is a measure of charge enclosed. So we have to calculate the charge enclosed by the sphere.

Let us consider a spherical shell of radius r and thickness 'dr'. The charge contained in this spherical shell is

$$dq = \rho \cdot 4\pi r^2 dr$$

Total charge enclosed in the sphere is

$$Q_{enc} = \int dq = \int_0^R \rho \cdot 4\pi r^2 dr$$

$$= \int_0^R \rho_0 \left(1 - \frac{r}{R}\right) \cdot 4\pi r^2 dr$$

$$= 4\pi\rho_0 \int_0^R \left[r^2 - \frac{r^3}{R}\right] dr$$

$$= 4\pi\rho_0 \left[\frac{r^3}{3} - \frac{r^4}{4R}\right]_0^R$$

$$Q_{enc} = 4\pi\rho_0 \left[\frac{R^3}{3} - \frac{R^3}{4}\right]$$

$$Q_{enc} = \frac{4\pi\rho_0 R^3}{12}$$

Hence the electric flux linked with the sphere is

$$\phi = \frac{Q_{enc}}{\varepsilon_0}$$

$$\phi = \frac{4\pi\rho_0 R^3}{12\varepsilon_0}$$

Q.8. Calculate using Gauss law the electric field due to a uniform infinite cylindrical charge and plot a variation of electric field with distance.

Sol. Let us consider a uniformly charged infinite cylinder of radius R and charge density ρ (charge/volume). The task is to find electric field at any point distant r from the axis lying (i) inside (ii) on the surface (iii) outside.

Case (i) : When the point lies inside the charge distribution : Let us consider a point P_1 at a distance r (CR) from cylinder axis. Draw a co-axial cylinder of radius r and length l so that point P_1 lies on the surface of this Gaussian surface.

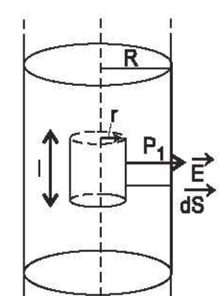


Fig. 1

The electric field $\vec{E}$ is every where normal to the curved surface and has the same magnitude at all points on it. The electric flux due to plane faces is zero. So the total electric flux is due to the curved surface alone.

The electric flux due to curved surface is :

$$\phi = \oint \vec{E} \cdot \vec{dS}$$
$$= \oint E\, ds \cos 0°$$
$$= \oint E\, dS$$
$$= E \oint dS \qquad [E \text{ is constant}]$$
$$= E \cdot 2\pi r l \qquad [ds = 2\pi r l]$$

The net charge enclosed by the Gaussian surface

$$q = (\pi r^2 l) \cdot \rho \qquad [\pi r^2 l = \text{volume of cylinder}]$$

Applying Gauss law $\phi = \oint \vec{E} \cdot \vec{dS} = \dfrac{Q_{enc}}{\varepsilon_0}$

$$E \cdot 2\pi r l = \frac{\pi r^2 l \rho}{\varepsilon_0}$$
$$E = \frac{\rho r}{2\varepsilon_0} \qquad ...(1)$$

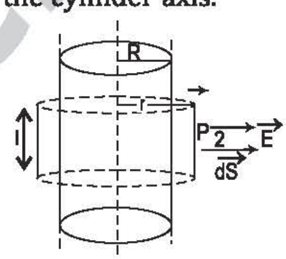
This is the electric field at the interior of cylinder.

Case (ii) : When the point lies on the surface of charge distribution ($r = R$).

From eq. (1) put $r = R$

$$E = \frac{\rho R}{2\varepsilon_0}$$

Case (iii) : When the point lies outside the surface charge distribution : Let us consider a point P_2 at a distance r (> R) from the cylinder axis.



Gaussian surface (Here cylinder of length l) in such a way so that point P_2 lies on the surface. As in the case (i) the flux is contributed by curved surface only. Therefore electric flux due to curved surface is

$$\phi = \oint \vec{E} \cdot \vec{dS} = E \cdot 2\pi r l \qquad [l = \text{length of cylinder}]$$

Total charge enclosed by the Gaussian surface

$$Q_{enc} = (\pi k^2 l)\rho$$

Applying Gauss law $\oint \vec{E} \cdot \vec{dS} = \dfrac{Q_{enc}}{\varepsilon_0}$

$$E \cdot 2\pi r l = \frac{\pi R^2 l \rho}{\varepsilon_0}$$

$$E = \frac{\rho R^2}{2\varepsilon_0 r}$$

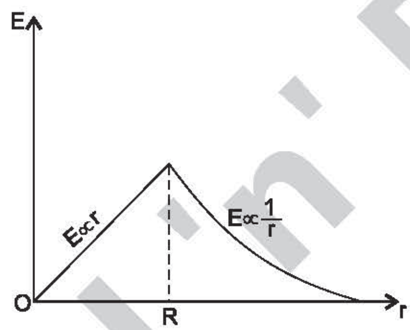
This is the electric field at a point lying outside the cylinder.

Put $r = R$

$$E = \frac{\rho R}{2\varepsilon_0}$$

This is the electric field at the surface as obtained from case (i) result.

The variation of $\vec{E}$ with r :



Q.9. Calculate the potential due to a uniformly charged conducting sphere.

Sol. Potential at a point due to a Uniformly charged Conducting Sphere : When a conducting sphere is given a charge, the charge is distributed uniformly on the surface of the sphere. Let R = radius of the sphere (Fig. 1) The sphere is given a charge $+q$.

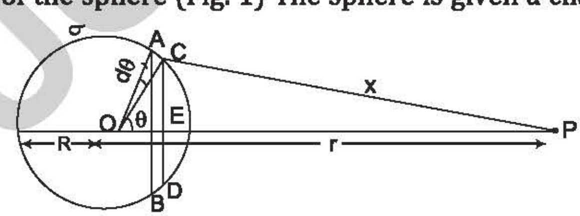


Fig. 1

Surface density of charge $= \sigma = q/(4\pi R^2)$

(i) **Potential at an external Point :** P is a point at a distance r from the centre of the sphere. Draw two parallel planes AB and CED so as to form an annular ring.

$$CP = x, \angle COP = \theta, \angle COA = d\theta, CE = R\sin\theta, AC = Rd\theta$$

$$\text{Area of the ring} = (2\pi R\sin\theta)(Rd\theta) = 2\pi R^2 \sin\theta\, d\theta$$

$$\text{Charge on the ring } ABCD = \sigma\,(2\pi R^2 \sin\theta\, d\theta) = \frac{1}{2} q \sin\theta\, d\theta$$

Each point on this narrow ring is at the same distance x from P.

$\therefore$ Potential at P due to the charge on the ring

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{\left(\frac{1}{2} q \sin\theta \, d\theta\right)}{x} \quad \text{...(1)}$$

From ΔCOP, $$x^2 = R^2 + r^2 - 2Rr\cos\theta \quad \text{...(2)}$$

Here, both x and θ variable. Differentiating,

$$2x\,dx = 2\,Rr\sin\theta\,d\theta$$

or $$\sin\theta\,d\theta = \frac{x\,dx}{Rr} \quad \text{...(3)}$$

Substituting in Eq. (1), we get

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{q\,dx}{2Rr}$$

The whole sphere is divided into such narrow rings.

$\therefore$ Potential at P due to the whole sphere is given by

$$V = \int_{r-R}^{r+R} \frac{1}{4\pi\varepsilon_0} \frac{q\,dx}{2Rr} = \frac{1}{4\pi\varepsilon_0} \frac{q}{2Rr} \int_{r-R}^{r+R} dx$$

$\therefore$ $$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} \quad \text{...(4)}$$

Hence the potential at any point outside a charged sphere is the same as if the whole charge on the sphere is concentrated as its centre.

(ii) If P lies on the surface of the sphere, $r = R$

$\therefore$ $$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R} \quad \text{...(5)}$$

(ii) **Potential at an internal point :** If P lies inside, then the limits of integration are $R - r$ and $R + r$.

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{2Rr} \int_{R-r}^{R+r} dx$$

$\therefore$ $$V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R} \quad \text{...(6)}$$

Thus the potential at all points inside a uniformly charged conducting sphere is the same. The variation of potential inside and outside the sphere is shown in Fig. 2.

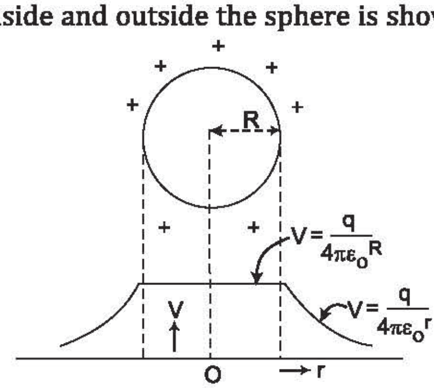


Fig. 2

Q.10. Calculate the potential and electric field due to a uniformly charged non-conducting solid sphere.

Ans. Potential due to a uniformly charged non-conducting solid sphere : In a conducting sphere, the entire charge resides on the surface of the sphere. But in a non-conducting sphere, charge is distributed uniformly in its entire volume. Let R be the radius of the non-conducting sphere. Let q be the total charge on the sphere.

$$\text{Volume charge density} = \rho = q \Big/ \left(\frac{4}{3}\pi R^3\right) \quad \text{...(1)}$$

(i) Potential at an External Point : Let P be a point distant r from the centre O of the sphere (Fig. 1). Divide the sphere into a large number of concentric spherical shells carrying charges $q_1, q_2, q_3, \ldots$.

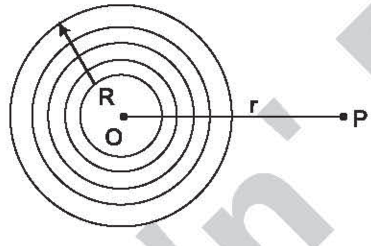


Fig. 1

$$\text{Potential at } P \text{ due to the shell of charge } q_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

The potential V due to the whole sphere is equal to the sum of the potentials due to all the shells.

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r} + \ldots$$

$$= \frac{1}{4\pi\varepsilon_0 r}(q_1 + q_2 + q_3 + \ldots)$$

But $(q_1 + q_2 + q_3 + \ldots) = q$, the charge on the sphere.

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\right) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Thus the charged sphere behaves toward an external point as if its entire charge were concentrated at its centre.

(ii) Potential at an Internal Point : Let the point P be inside the sphere at a distance r from the centre O (Fig. 2). If we draw a concentric sphere through P, the point P is external for the inner solid sphere of radius r, and internal for the outer spherical shell of internal radius r and external radius R.

The charge on the inner solid sphere is $\frac{4}{3}\pi r^3 \rho$.

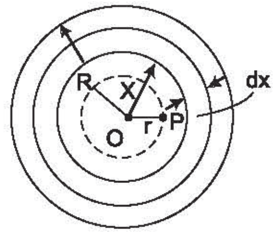


Fig. 2

Potential at P due to the inner solid sphere $= V_1 = \frac{1}{4\pi\varepsilon_0} \frac{\frac{4}{3}\pi r^3 \rho}{r} = \frac{r^2 p}{3\varepsilon_0}$ ...(1)

Let us now find the potential at P due to the outer spherical shell. Divide this shell into a number of thin concentric shells. Consider one such shell of radius x and thickness dx. Charge contained in this shell is $4\pi x^2 (dx)\rho$.

Potential at P due to this shell $= \frac{1}{4\pi\varepsilon_0} \frac{4\pi x^2 (dx)\rho}{x} = \frac{\rho\, x\, dx}{\varepsilon_0}$

The potential at P due to the whole shell of internal radius r and external radius R is

$$V_2 = \int_r^R \frac{\rho x\, dx}{\varepsilon_0} = \frac{\rho\,(R^2 - r^2)}{2\varepsilon_0} \quad ...(2)$$

The total potential at P is

$$V = V_1 + V_2 = \frac{r^2\rho}{3\varepsilon_0} + \frac{\rho\,(R^2 - r^2)}{2\varepsilon_0} = \frac{\rho\,(3R^2 - r^2)}{6\varepsilon_0}$$

But $$\rho = q \Big/ \left(\frac{4}{3}\pi R^3\right)$$

$$V = \frac{3q}{4\pi R^3} \frac{(3R^2 - r^2)}{6\varepsilon_0}$$

$\therefore$ $$V = \frac{1}{4\pi\varepsilon_0} \frac{q\,(3R^2 - r^2)}{2R^3}$$

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left[\frac{1}{4\pi\varepsilon_0} \frac{q\,(3R^2 - r^2)}{2R^3}\right] = \frac{1}{4\pi\varepsilon_0} \frac{qr}{R^3}$$

UNIT-II

Magnetostatics

SECTION-A VERY SHORT ANSWER TYPE QUESTIONS

Q.1. Define electric current. Also write its unit.

Ans. An electric current is a stream of charged particle, such as electrons or ions, moving through an electrical conductor or space. The current in a wire is charge per unit time passing a given point.

$$I = \frac{dq}{dt}$$

where dq is the charge flow in dt time.

By definition, negative charge moving to the left count the same as positive ones to the right.

The unit of current is ampere.

Q.2. Name the different types of current densities.

Ans. There are three types of current densities :

1. Linear current density.
2. Surface current density.
3. Volume current density.

Q.3. Define volume current density.

Ans. Since current is defined as the flow of charges in unit time. If the charges are distributed in a volume and if these charges starts moving in a systematic manner, there will be a current in the volume. Such a current is called 'volume current'. Volume current per unit area is known as volume current density. It is given by the formula

$$\vec{J} = \rho \vec{v}$$

where $\vec{J}$ is the volume current density, ρ is the volume charge density and $\vec{v}$ is the velocity of charges in the volume. It is a vector quantity and its unit is Amp/meter^2.

Q.4. Define surface current density.

Ans. If the charges distributed on a surface and if these charges moves systematically on the surface, there is an electric current throughout the surface, such a current is called 'surface current'.

Surface current is described in terms of vector quantity called 'surface current density', defined at each point of the surface. It is represented by symbol 'κ'.

Surface current per unit length (taken perpendicular to the direction of current) is known as surface current density (κ).

$$\vec{\kappa} = \sigma \vec{v}$$

where σ is surface charge density and $\vec{v}$ is the velocity of charge.
A vector quantity and its units are Amp/meter.

Q.5. Write down the equation of continuity. What is the physical meaning of this equation?

Ans. The equation of continuity is given by

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial s}{\partial t} = 0$$

Here, $\vec{J}$ is the volume current density and ρ is the volume charge density.

This equation tells us about the conservation of charge.

Q.6. Write the formula for the magnetic force between two current carrying elements.

Ans. The expression of force between two current carrying elements is given by

$$\vec{dF} = \frac{\mu_0 I_1 I_2}{4\pi}\left(\frac{\vec{dl_1} \times \vec{dl_2} \times \vec{r}}{r^3}\right)$$

Q.7. Write down the formula for the magnetic force between two parallel straight conductor.

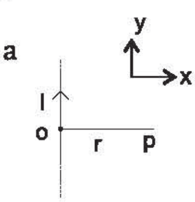
Ans. The force between two parallel straight conductor at a distance x carrying current I_1 and I_2 is given by

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{x}$$

The force is attractive if current flows in same direction. The force is repulsive if current flows in opposite direction.

Q.8. Write the expression for the magnetic field due to a long straight conductor atom carrying a steady current I.

Ans. Magnetic field due to a long straight conductor carrying a current I, at a distance r is given by



$$B = \frac{\mu_0}{2\pi}\frac{2I}{r}\hat{k}$$

Q.9. What is the divergence and curl of a static magnetic field?

Ans. The divergence of magnetic field is

$$\vec{\nabla} \cdot \vec{B} = 0$$

The curl of static magnetic field is

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

where $\vec{J}$ is the volume current density.

Q.10. Write the general expression for the magnetic field in terms of volume current density.

Ans. The general expression for magnetic field at a position $\vec{r}$ due to a current element $I\,dl$ will be

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,\vec{dl} \times \vec{r}}{r^3}$$

If the current is distributed in space with a current density J. Then $I\,dl = J\,d\tau$.
Expression for magnetic field becomes

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J} \times \vec{r}}{r^3}\,d\tau.$$

Q.11. What do you mean by magnetic vector potential?

Ans. Since divergence of magnetic field is always zero.

$$\vec{\nabla} \cdot \vec{B} = 0$$

Therefore we can write $\vec{B} = \vec{\nabla} \times \vec{A}$ $[\nabla \cdot \nabla \times A = 0]$

The vector $\vec{A}$ is called magnetic vector potential.

Hence magnetic vector potential is defined as a vector function whose curl is equal to $\vec{B}$.

Q.12. Write the general expression for magnetic potential in terms of volume current density.

Ans. The expression for magnetic potential at a distance $\vec{r}$ due to volume current density J is given by

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\,d\tau}{r}$$

Q.13. Define Ampere's circuital law.

Ans. It states that line integral of the magnetic induction around a closed path is equal to the total current enclosed by the path multiplied by permeability of free space

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

Q.14. Define magnetisation.

Ans. The electrons in an atom have a magnetic dipole moment due to their orbital motion around the nucleus as well as due to their spin. Due to this fact certain atoms and molecules have a net magnetic dipole moment. So when a substance is magnetised all its magnetic dipoles oriented in the same direction. If the dipole moment of each atomic dipole is $\vec{m}$ and number of dipoles per unit volume is n then

Magnetic moment per unit volume $\vec{M} = \vec{m}\,n$

The magnetic moment per unit volume is known as magnetization or magnetic polarisation.

Q.15. Write the relation between magnetic field ($\vec{B}$), auxiliary field $\vec{H}$ and magnetization $\vec{M}$.

Ans. The relation is given by

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

or
$$\vec{B} = \mu_0 \vec{H} + \vec{M}$$

Q.16. Define magnetic susceptibility.

Ans. The ratio of magnitude of magnetic dipole moment per unit volume $\vec{M}$ and the magnitude of the magnitising field $\vec{H}$ is known as magnetic susceptibility

$$\chi_M = \frac{|\vec{M}|}{|\vec{H}|}$$

Magnetic susceptibility of vaccum is zero.

Q.17. Explain the term permeability.

Ans. The ratio of magnitude of magnetic induction field $\vec{B}$ inside the material and the magnitude of magnetising field $\vec{H}$ is known as magnetic permeability μ

$$\mu = \frac{|\vec{B}|}{|\vec{H}|} = \frac{B}{H}$$

Its unit is Tesla/A/m.

Q.18. Write the relation between relative permeability and susceptibility.

Ans. The relation between relative permeability and susceptibility is given by

$$\mu_r = \frac{\mu}{\mu_0} = (1 + \chi_M)$$

Q.19. Give the classification of magnetic materials on the basis of their permeability and susceptibility.

Ans. $0 < \mu < 1$: Diamagnetic material $-1 < \chi_M < 0$ (negative)

$\mu \simeq 1$: Paramagnetic material $0 < \chi_M < 1$ (positive)

μ (very large) : Ferromagnetic material χ_M-very large (positive).

Q.20. Calculate the magnetic field at a distance of 5 m from an infinite straight conductor carrying current of 50 ampere.

Sol. Given : Current $(I) = 50$ amp, Distance $(r) = 5$ m.

The magnetic field due to a finite long current carrying conductor is given by

$$|\vec{B}| = \frac{\mu_0}{2\pi}\frac{I}{r}$$

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r}$$

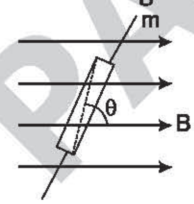
$$= 10^{-7} \times \frac{2 \times 50}{5}$$

$$B = 2 \times 10^{-6} \text{ Tesla.}$$

Q.21. Explain magnetic dipole moment.

Ans. When a magnetic dipole is placed in an external magnetic field, a torque acts on the dipole, which tends to align the axis of dipole along the direction of magnetic field.

The torque on magnetic dipole in magnetic field $\vec{B}$



$$\tau = MB\sin\theta$$

$$M = \frac{\tau}{B\sin\theta}$$

$\theta = 90°, \beta = 1$ Tesla then $M = \tau$.

Thus magnetic moment of magnetic dipole is numbrically equal to torque acting on it, when it is placed perpendicular to a uniform magnetic field of strength 1 Tesla. Its unit B Amp/meter^2.

SECTION-B SHORT ANSWER TYPE QUESTIONS

Q.1. State and prove ampere's circuital law.

Ans.

Ampere's Circuital Law

This law states that line integral $\oint \vec{B} \cdot \vec{dl}$ of the magnetic field $\vec{B}$ along the closed path is equal to μ_0 times the net current i passing through the closed path.

$$\oint \vec{B} \cdot dl = \mu_0 i \qquad (\mu_0 : \text{Permeability of free space})$$

Let us consider an infinitely long straight wire carrying a current i. The magnitude of the magnetic field due to this wire at a distance r as shown in Fig. 1 using Biot-Savart law is

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{i}{r}$$

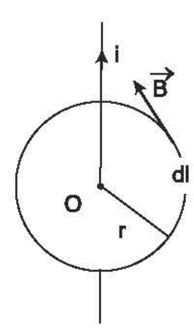


Fig. 1

The magnetic field makes concentric circles around the wire.

Let us take small element $\vec{dl}$ on the circle and it is directed along $\vec{B}$. Therefore the line-integral of this magnetic field is

$$\vec{B}\cdot\vec{dl} = B\,dl\cos 0° \qquad [\theta = 0°]$$
$$= B\,dl$$
$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl = B\oint dl \qquad \text{[Since } B \text{ is constant all around the circle]}$$
$$= \frac{\mu_0}{2\pi}\frac{i}{r}\oint dl$$
$$= \frac{\mu_0 i}{2\pi r}\cdot 2\pi r \qquad [\oint dl = 2\pi r\text{, total circumference}]$$
$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i$$

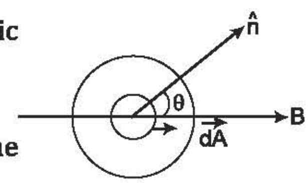
which is Ampere's circuital law.

Q.2. Define magnetic flux.

Ans.

Magnetic Flux

The magnetic flux is defined as the total number of lines of magnetic induction passing through the surface.



Let us consider an element of surface area $\vec{dA}$ of a surface A. Let $\hat{n}$ be the unit vector normal to the area element $\vec{dA}$. The magnetic flux over area $\vec{dA}$ is given by

$$d\phi = \vec{B}\cdot\vec{dA} = \vec{B}\cdot\hat{n}\,dA$$

Total magnetic flux over the surface

$$\phi = \int\phi = \int_A \vec{B}\cdot\hat{n}\,dA$$
$$\phi = \int_A B\cos\theta\,dA$$

[θ = angle between unit vector and magnetic field]

If B is uniform over the area A, then

$$\phi = B\cos\theta\int_A dA$$
$$\phi = BA\cos\theta$$

This is magnetic flux, its unit is Tesla/metre^2.

Q.3. Starting from integral form of Ampere's law obtain the differential form of Ampere's law.

Ans. We have, Ampere's circuital law is

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

or $$\oint \vec{B}\cdot\vec{dl} = \mu_0 \int_S \vec{J}\cdot\vec{ds}$$

applying stokes law on left hand side

$$\int_S (\vec{\nabla}\times\vec{B})\cdot ds = \mu_0 \int_S \vec{J}\cdot\vec{ds} \qquad [\int A\cdot dl = \int_S (\nabla\times A)\,dA]$$

or $$\int_S (\vec{\nabla}\times\vec{B} - \mu_0\vec{J})\cdot\vec{ds} = 0$$

This is only true if integral is zero. Therefore

$$\vec{\nabla}\times\vec{B} - \mu_0\vec{J} = 0$$

or $$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

This is the differential form of Ampere's law.

Q.4. Prove the relation $\mu_r = (1+\chi_m)$. Where μ_r is called relative permeability and χ_m is magnetic susceptibility.

Ans. The magnetic susceptibility is defined as

$$\chi_m = \vec{M}\cdot\vec{H} \qquad ...(1)$$

where $\vec{M}$ is the magnetization and $\vec{H}$ is the magnetising field

We also have $$\vec{B} = \mu H\,\vec{H} \qquad ...(2)$$

But $$\vec{B} = \mu_0(\vec{H}+\vec{M}) \qquad ...(3)$$

Substituting the values of $\vec{B}$ and $\vec{M}$ in terms of $\vec{H}$, we get

$$\vec{B} = \mu\vec{H} = \mu_0(\vec{H}+\chi_m\vec{H})$$

$$= \mu_0(1+\chi_m)\vec{H}$$

or $$\mu = \mu_0(1+\chi_m)$$

or $$\frac{\mu}{\mu_0} = (1+\chi_m)$$

or $$\mu_r = (1+\chi_m)$$

Q.5. Two parallel straight wires are placed at 4 cm apart the current in them are 1A and 3A in same direction respectively. Find the position, where they will produce zero magnetic field.

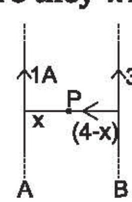


Fig. 1

Ans. Let us consider two wires A and B carrying current 1 A and 3 A flowing in same direction as shown in fig. (1) respectively.

Let at point P at a distance 'x' from wire A magnetic field is zero.

magnetic field at P due to wire A is

$$B_1 = \frac{\mu_0}{2\pi}\frac{I_1}{x}$$

Magnetic field at P due to wire B is

$$B_2 = \frac{\mu_0}{2\pi} \frac{I_2}{(4-x)}$$

Since at P magnetic field is zero, therefore

$$B_1 - B_2 = 0 \text{ or } B_1 = B_2$$

$$\frac{\mu_0}{2\pi} \frac{I_1}{x} = \frac{\mu_0}{2\pi} \frac{I_2}{(4-x)}$$

or

$$\frac{I_2}{I_1} = \frac{4-x}{x}$$

$$\frac{3}{1} = \frac{4-x}{x}$$

$$3x = 4 - x$$

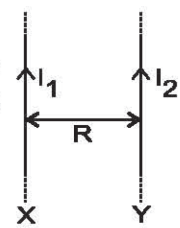
$$4x = 4 \quad \Rightarrow \quad x = 1 \text{ cm}$$

i.e., at a distance $x = 1$ cm from wire A, magnetic field will be zero.

Q.6. Discuss for the force acting between two parallel current carrying wires.

Ans. Let X and Y be two very long parallel straight conductors carrying current I_1 and I_2 respectively. The separation between them is R. The magnitude of the magnetic field induction at any point on Y due to current I_1, in X is given by

$$B = \frac{\mu_0}{2\pi} \frac{I_1}{R} \quad \text{(directed inward)}$$



Thus wire Y is situated in magnetic field B perpendicular to its length. It therefore experience a magnetic force. The magnitude of the force acting on the length l of Y is

$$F = I_2 B l = I_2 \left(\frac{\mu_0}{2\pi} \frac{I_1}{R} \right) l = \left(\frac{\mu_0}{2\pi} \frac{I_1 I_2}{R} \right) l$$

Force per unit length of Y is $\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{R}$

The Fleming's left hand rule shows that the direction of this force is towards X if I_1 and I_2 are in the same direction and away from X if they are in opposite direction.

Q.7. Current of 10 Amp flows through each of the two parallel long wires which are 2 cm apart. Calculate the force exerted per unit length of each wire.

Ans. The force per unit length between two parallel wires carrying current I_1 and I_2 respectively given by

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{R}$$

Since, $I_1 = I_2 = 10$ Amp and $R = 2$ cm $= 2 \times 10^{-2}$ m

So,

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \frac{i^2}{R} = \frac{2 \times 10^{-7} \times 10^2}{(2 \times 10^{-2})}$$

$$\frac{F}{l} = 10^{-3} \text{ Netwon.}$$

Q.8. Calculate the magnetic field due to a long straight current carrying wire, using Amper's circuital law.

Ans. Let us consider a point P at a distance r from the long straight current carrying conductor, carrying current I as shown in fig.

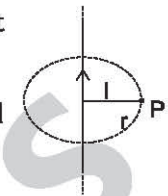


Fig.

Let us draw a circle of radius r around the wire. The magnetic field $\vec{B}$ is tangential to the circle every where.

From Ampere's circuital law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc.}$$

$$\oint B\, dl \cos 0° = \mu_0 I$$

Since B is constant over the circle it will come out from integral. So,

$$B \int dl = \mu_0 I$$

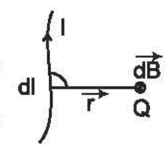
$$B \cdot 2\pi r = \mu_0 I \qquad (\int dl = 2\pi r)$$

$$B = \frac{\mu_0}{2\pi} \frac{I}{r}$$

This is the required result.

Q.9. Define Biot-Sovart law.

Ans. Let us consider a conductor carrying current I as shown in fig. The magnitude $\vec{dB}$ of the magnetic induction at point Q at a distance $\vec{r}$ from the current element dl carrying a current I is given by



$$dB \propto \frac{I\, dl \sin\theta}{r^2}$$

[θ is the angle between current element and radius vector joing the point Q with current element]

or

$$dB = k \frac{I\, dl \sin\theta}{r^2}$$

k is the constant of proportionality. In S.I. units k is written as

$$k = \frac{\mu_0}{4\pi} \qquad (\mu_0\text{-permeability of free space})$$

$$dB = \frac{\mu_0}{4\pi} \frac{I\, dl \sin\theta}{r^2}$$

Vector from Biot-Savart law can be written as

$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\,(\vec{dl} \times \hat{r})}{r^3}$$

$\hat{r}$ is the unit vector along position vector $\vec{r}$.

SECTION-C LONG ANSWER TYPE QUESTIONS

Q.1. Using Biot-Savart law, derive expression for magnetic field at a point on the axis of a current carrying coil.

Ans. **Magnetic Field on the Axis of a Circular Current Loop**

Let us consider a circular coil of radius *a*, carrying a current *i*. Let *P* be a point on diagram of the circular coil distant *x* from the centre at which the field is to be induced.

Consider a current-element of length $\vec{dl}$ at the top of the coil. If $\vec{r}$ be the displacement vector from the current-element to *P*, then from Biot-Savart law, magnetic field induction at *P* due to the current-element $\vec{dl}$ is given by :

$$d\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{i\vec{dl} \times \vec{r}}{r^3}$$

The magnitude of $d\vec{B}$ is :

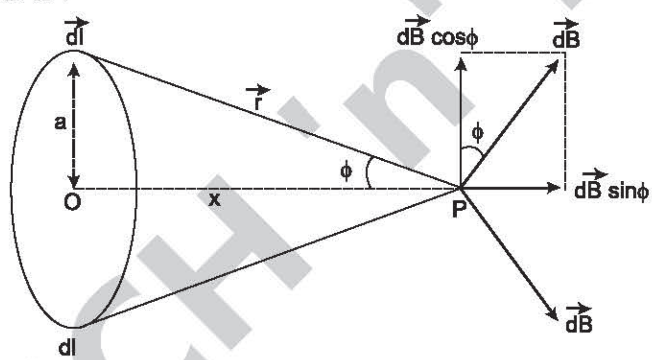


Fig. 1

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{i\,dl \sin 90°}{r^2} = \frac{\mu_0}{4\pi} \cdot \frac{i\,dl}{r^2} \quad ...(1)$$

because angle between $d\vec{l}$ and $\vec{r}$ is 90°.

The direction of *dB* is perpendicular to the plane containing $d\vec{l}$ and $\vec{r}$ and is given by right hand screw rule. The vector $d\vec{B}$ can be resolved into two components; one of magnitude $dB \sin\phi$ along the axis of the coil and the other of magnitude $dB \cos\phi$ perpendicular to the axis. If we now consider another current-element of the same length *dl* at the bottom of the coil, the field $d\vec{B}'$ due to this element will be equal in magnitude to *dB* but directed as shown in fig. 1. It can also be resolved into two components along and perpendicular to the axis. If we consider the magnetic induction produced by whole of the circular coil, then by symmetry, the components of magnetic induction at right angles to the axis are equal and opposite, hence they cancel each other. The components along the axis are in the same direction and they are added. Thus, the resultant field $\vec{B}$ at axial point *P* is along the axis and is given by :

$$B = \int dB \sin\phi$$

Substituting the value of dB from eq. (1), we get

$$B = \frac{\mu_0 i}{4\pi r^2} \int dl \sin\phi$$

$\frac{1}{r^2}$ has been taken out side the integral because r has the same value for all the current-elements. Now from Fig. 1,

$$\sin\phi = \frac{a}{r}$$

$$\therefore \qquad B = \frac{\mu_0\, ia}{4\pi r^3} \int dl.$$

But $\int dl =$ (circumference of the coil) $= 2\pi a$ and $r = (a^2 + x^2)^{1/2}$.

$$\therefore \qquad B = \frac{\mu_0\, ia^2}{2(a^2 + x^2)^{3/2}} \text{ weber/meter}^2$$

If the coil contains N turns, then resultant magnetic field induction is given by

$$B = \frac{\mu_0\, Nia^2}{2(a^2 + x^2)^{3/2}} \text{ weber/meter}^2 \qquad \text{...(2)}$$

The direction of B is along the axis of the coil.

Special Cases

(i) Magnetic field induction at the centre of the circular coil : Value of B at the centre of the coil is obtained by putting $x = 0$ in eq. (2). Thus

$$B_c = \frac{\mu_0 Nia^2}{2(a^2)^{3/2}} = \frac{\mu_0 Ni}{2a} \qquad \text{...(3)}$$

(ii) The Magnetic field due to a small circular coil : If the coil is small, then $x >> a$, therefore

$$B = \frac{\mu_0 Nia^2}{2x^3}$$

Variation of Magnetic Field : The variation of the field B along the axis of the coil is shown in Fig. 2. B is greatest at the centre of the coil where $x = 0$ and decreases as we move away from the centre, becoming zero at infinity. The rate of change of field from point to point along the axis is, however, not constant. The curve has nearly straight line small portions on the both sides. This shows that on each side of the coil there is a point on the axis where the field varies uniformly with distance over a small region.

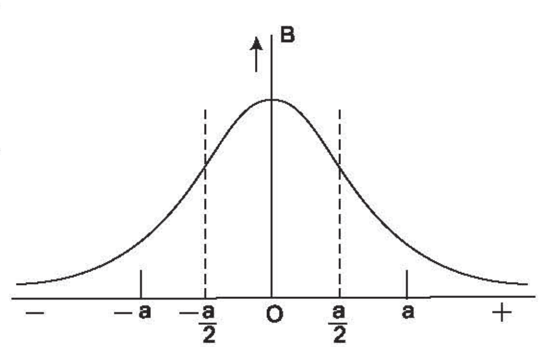


Fig. 2

Mathematically, dB/dx at that point is constant or $\frac{d^2B}{dx^2}=0$. This point is called **'Point of inflexion'**.

Differentiating eq. (2) with respect to x, we obtain the rate of variation of magnetic field B with distance x and

$$\frac{dB}{dx}=\frac{\mu_0 Nia^2}{2}\cdot\frac{d}{dx}(a^2+x^2)^{-3/2}$$

$$=\frac{\mu_0 Nia^2}{2}\left(-\frac{3}{2}\right)(a^2+x^2)^{-5/2}(2x)$$

$$=-\frac{3}{2}\mu_0 Nia^2\,[(a^2+x^2)^{-5/2}x].$$

For uniform variation of B due to circular coil, $\frac{dB}{dx}$ must be constant and $\frac{d^2B}{dx^2}$ must be zero.

Therefore, differentiating again

$$\frac{d^2B}{dx^2}=-\frac{3}{2}\mu_0 Nia^2\frac{d}{dx}[x(a^2+x^2)^{-5/2}]$$

$$=-\frac{3}{2}\mu_0 Nia^2\left[(a^2+x^2)^{-5/2}+x\left(-\frac{5}{2}\right)(a^2+x^2)^{-7/2}(2x)\right]$$

$$=-\frac{3}{2}\mu_0 Nia^2[(a^2+x^2)^{-5/2}-5x^2(a^2+x^2)^{-7/2}]$$

$$=-\frac{3}{2}\mu_0 Nia^2(a^2+x^2)^{-7/2}[(a^2+x^2)-5x^2]$$

Putting $\frac{d^2B}{dx^2}=0$, we get

$$(a^2+x^2)-5x^2=0$$

$$4x^2=a^2$$

$$x=\pm\frac{a}{2}.$$

Thus at a distance $x=\pm\frac{a}{2}$ from the centre of the coil, variation of magnetic field is constant, *i.e.*, magnetic field due to coil varies uniformly along the axis. Therefore, on each side of the coil there is a point on the axis where the field varies uniformly with distance over a small region. This point is called **'Point of inflexion'**. This result is made use of in **Helmholtz galvanometer** in which two identical flat circular coils are mounted on a common axis at a distance equal to their common radius 'a'. Near the point on the axis midway between them the field B is very nearly constant over an appreciable region, and is given by :

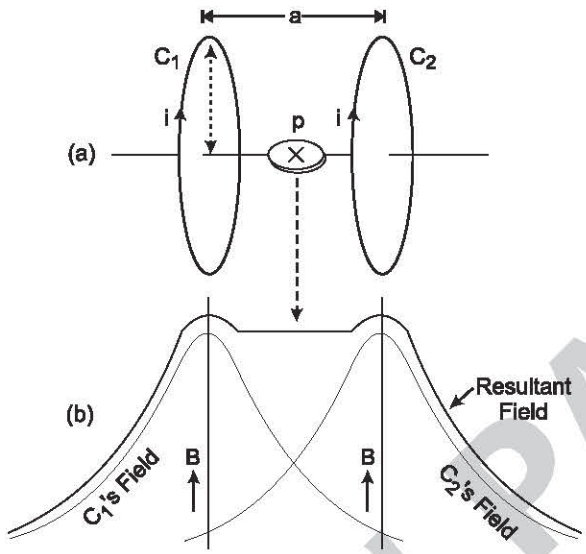


Fig. 3

$$B = 2 \times \frac{\mu_0 \, Nia^2}{2\left(a^2 + \frac{a^2}{4}\right)^{3/2}} = \frac{8\mu_0 \, Ni}{5\sqrt{5}a} \text{ Weber/meter}^2$$

where N is the numbeer of turns in one coil.

It has been shown in Fig. 3.

Q.2. Using Ampere' law, obtain the expression for magnetic field at a point lying inside of the axis of a current solenoid.

Ans. Let us consider a solenoid very long compared with its diameter and carrying a current i_0. Then by right hand rule magnetic field is parallel to the axis of solenoid. Thus the magnetic field is uniform except near the edges.

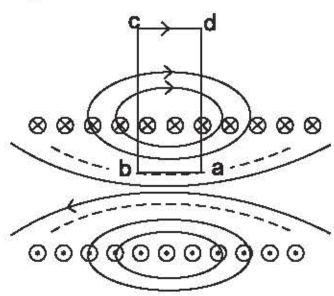


Fig.

Let us consider a closed rectangular path $abcd$, then we see that side ab is parallel to the axis of the solenoid and sides bc and da are very long, so that the side cd is far from the solenoid. Therefore field at side cd is negligibly small. The field is right angles to the side bc and da.

Now applying Ampere's law to the rectangular path *abcd*

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i \qquad ...(1)$$

i being the net current enclosed by the rectangle.

From (1) $$\oint \vec{B} \cdot \vec{dl} = \int_a^b \vec{B} \cdot \vec{dl} + \int_b^c \vec{B} \cdot \vec{dl} + \int_c^d \vec{B} \cdot \vec{dl} + \int_d^a B \cdot \vec{dl}$$

at $$\int_b^c \vec{B} \cdot \vec{dl} = \int_d^a \vec{B} \cdot \vec{dl} = 0$$

along bc and da, field and dl are mutually perpendicular.

Inside the solenoid $\vec{B}$ is parallel to dl along ab

$\therefore$ $$\oint \vec{B} \cdot \vec{dl} = \int_a^b B\, dl \cos 0° = \int B\, dl$$

Since magnetic field is constant inside the solenoid

$\therefore$ $$\oint \vec{B} \cdot \vec{dl} = B \int_a^b dl = Bx \qquad ...(2)$$

where x is the length of side ab.

If n is the number of turns per unit length of the solenoid then number of turns in length x is nx.

Since the current in each turn is i_0, therefore net current enclosed by the rectangle is

$$i = nxi_0 \qquad ...(3)$$

Substituting the value of $\oint \vec{B} \cdot \vec{dl}$

$$\oint \vec{B} \cdot \vec{dl} = Bx + 0 + 0 + 0 = Bx$$

From eq. (1) $$\int \vec{B} \cdot \vec{dl} = \mu_0 i$$

$$Bx = \mu_0 nxi_0 \quad \Rightarrow \quad B = \mu_0 ni_0$$

Thus the field B inside a thin long solenoid is independent of the length and diameter of the solenoid and is constant throughout the central region.

Q.3. What do you mean by magnetic vector potential? Find vector potential due to a long straight current wire.

Ans.

Application of Magnetic Vector Potential : Magnetic Field For a Long Straight Current Wire

Consider a wire of length L, carrying a current I. Let P be a point at distance X from the wire. For convenience, we consider the point P to be situated symmetrically. However the results obtained would hold for any position of P. Let O be the origin of coordinate system, with Y axis along the current and X-axis towards the field point P. The magnetic vector potential at P due to a small element dy at a distance y from O is given by :

$$dA = \frac{\mu_0 I}{4\pi} \frac{dy}{r} \text{ directed along the current}$$

$$= \frac{\mu_0}{4\pi} I \frac{dy}{(x^2 + y^2)} \hat{j}$$

$\hat{j}$ being unit vector along y axis.

The magnetic potential due to whole length of wire is :

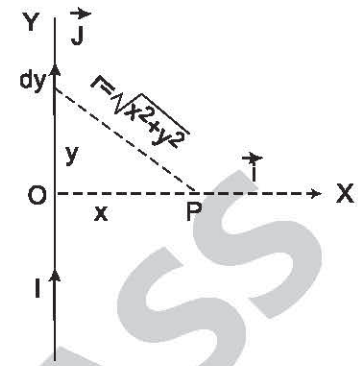


Fig. 1

$$A = \hat{j}\frac{\mu_0 I}{4\pi}\int_{-L/2}^{+L/2}\frac{dy}{\sqrt{(x^2+y^2)}}$$

$$= \hat{j}\frac{\mu_0 I}{4\pi}\left[\log\{y+\sqrt{(y^2+x^2)}\}\right]_{-L/2}^{+L/2}$$

$$\left[\text{Since from standard integral}\int\frac{dx}{(x^2+a^2)} = \sin^{-1}\frac{x}{a}\text{ or }\log\{x+\sqrt{(x^2+a^2)}\}\right]$$

$$\therefore \qquad A = \hat{j}\frac{\mu_0 I}{4\pi}\left[\log\left\{\frac{L}{2}+\sqrt{\left(\frac{L^2}{4}+x^2\right)}\right\} - \log\left\{-\frac{L}{2}+\sqrt{\left(\frac{L^2}{4}+x^2\right)}\right\}\right]$$

$$= \hat{j}\frac{\mu_0 I}{4\pi}\log\left\{\frac{\frac{L}{2}+\sqrt{\left(\frac{L^2}{4}+x^2\right)}}{-\frac{L}{2}+\sqrt{\left(\frac{L^2}{4}+x^2\right)}}\right\} = \hat{j}\frac{\mu_0 I}{4\pi}\log\left[\frac{\frac{L}{2}\left\{1+\left(1+\frac{4x^2}{L^2}\right)^{1/2}\right\}}{\frac{L}{2}\left\{-1+\left(1+\frac{4x^2}{L^2}\right)^{1/2}\right\}}\right]$$

The wire is infinitely long, then $L \to \infty$.

So, $\frac{x'}{L'} << 1$; hence using binomial theorem and neglecting higher order terms, therefore

$$A = \hat{j}\frac{\mu_0 I}{4\pi}\log\left[\frac{1+\left(1+\frac{4x^2}{2L^2}\right)}{-1+\left(1+\frac{4x^2}{2L^2}\right)}\right]$$

$$= \hat{j}\frac{\mu_0 I}{4\pi}\log\left[\frac{\left(2+\frac{2x^2}{L^2}\right)}{\left(\frac{2x^2}{L^2}\right)}\right] = \hat{j}\frac{\mu_0 I}{4\pi}\log\frac{1+\frac{x^2}{L^2}}{\left(\frac{x^2}{L^2}\right)}$$

$$= \hat{j}\frac{\mu_0 I}{4\pi}\log\left(1+\frac{L^2}{x^2}\right)$$

$$= \hat{j}\,\frac{\mu_0 I}{4\pi}\log\left(\frac{L}{x}\right)^2 \qquad \left(\text{since } \frac{L^2}{x^2} >> 1\right)$$

$$A = \frac{\mu_0 I}{2\pi}\log\left(\frac{L}{x}\right)\hat{j} \qquad ...(1)$$

This is the requires expression for magnetic potential due to a long straight current carrying wire.

Q.4. Find the magnetic induction at the center of a square current loop of side 'a' units.

Ans. Let $PQRS$ be a square current-loop of side a with O as the centre of the loop at which the field is required. Let A be the foot of perpendicular from O at the side PQ (Fig. 1.)

If dl is a current-element in the side PQ at a distance l from A and $\vec{r}$ the radius vector from the element to O, then by Biot-Savart law, the magnitude of the field $d\vec{B}$ at O due to $d\vec{l}$ is given by

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{i\,dl\sin\theta}{r^2}$$

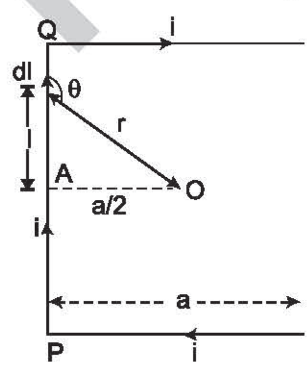


Fig. 1

But from Fig. 1. we get

$$\sin(180° - \theta) = \sin\theta = \frac{a/2}{r}$$

and

$$r^2 = \left(\frac{a}{2}\right)^2 + l^2$$

$\therefore$

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{i\,dl\,a}{2\left(\frac{a^2}{4}+l^2\right)^{3/2}}$$

The magnitude of the field at the whole length of arm PQ is, therefore given by :

$$B_1 = \frac{\mu_0 ia}{\pi}\int_{l=-a/2}^{l=+a/2}\frac{dl}{\left(\frac{a^2}{4}+l^2\right)^{3/2}}.$$

The integral, we put $l = \frac{1}{2}a\tan\phi$ so that $dl = \frac{1}{2}a\sec^2\phi\,d\phi$. Then

$$B_1 = \frac{\mu_0 ia}{8\pi}\int_{-\pi/4}^{\pi/4}\frac{(a/2)\sec^2\phi\,d\phi}{\left(\frac{a^2}{4}\sec^2\phi\right)^{3/2}}$$

$$= \frac{\mu_0 i}{2\pi a}\int_{-\pi/4}^{\pi/4}\cos\phi\,d\phi = \frac{\mu_0 i}{2\pi a}[\sin\phi]_{-\pi/4}^{\pi/4}$$

$$= \frac{\mu_0 i}{2\pi a}\left[\sin\frac{\pi}{4} + \sin\frac{\pi}{4}\right] = \frac{\mu_0 i}{2\pi a}\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right] = \frac{\mu_0 i}{\sqrt{2}\pi a}$$

By symmetry the field at O due to the remaining three sides, will be of the magnitude and direction. Therefore, the field B due to the entire loop will determine the field due to one side only. Thus,

$$B = 4B_1 = \frac{4\mu_0 i}{\sqrt{2}\,\pi a}$$

$$B = \frac{2\sqrt{2}\mu_0 i}{\pi a}$$

Q.5. Derive expression for magnetic field due to a magnetic dipole.

Ans.

Magnetic Field at any Point due to Magnetic Dipole

Fig. 1. shows a magnetic dipole SN. P is any point in air at distance r from the centre O of the dipole. Let $\vec{\mu}$ be the magnetic moment of the dipole. Let θ be the angle between OP and the axis SN of the dipole. The dipole $\vec{\mu}$ is a vector quantity, hence we resolve it into two components :

(i) The component along $OP = \mu\cos\theta$

(ii) The component perpendicular to $OP = \mu\sin\theta$.

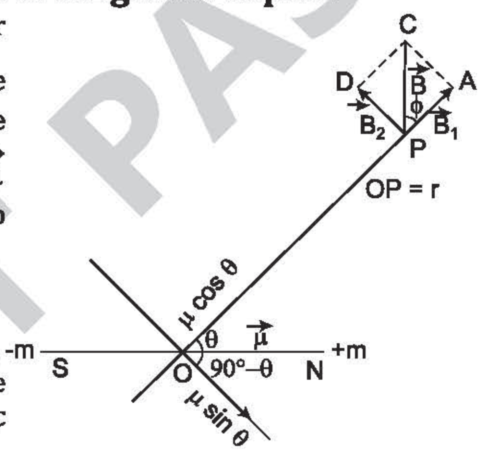


Fig. 1

With respect to $\mu\cos\theta$ the point P is on the axis. Since the dipole is short, the magnitude of the magnetic induction $\vec{B_1}$ at P due to this component is given by

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2\mu\cos\theta}{r^3} \qquad ...(1)$$

where μ_0 $(=4\pi\times10^{-7}$ Wb/Am) is the permeability of free spaces.

The direction of $\vec{B_1}$ is along PA. With respect to $\mu\sin\theta$, the point P is on the equator. Since the dipole is short the magnitude of the magnitude of the magnetic induction $\vec{B_2}$ at P due to this component is given by

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\mu\sin\theta}{r^3} \qquad ...(2)$$

The direction of $\vec{B_2}$ is opposite to that of $\mu\sin\theta$, *i.e.*, along PD which is perpendicular to OP. The magnitude and the direction of the resultant magnetic induction $\vec{B}$ at P is represented by the diagonal of the rectangle $PACD$ in which PA represent $\vec{B_1}$ and PD represents $\vec{B_2}$. The magnitude of $\vec{B}$ is given by

$$B^2 = B_1^2 + B_2^2$$

$$B = \sqrt{B_1^2 + B_2^2}$$
$$= \frac{\mu_0}{4\pi} \cdot \frac{\mu}{r^3} \sqrt{4\cos^2\theta + \sin^2\theta}$$

Substituting $\sin^2\theta = 1 - \cos^2\theta$, we get

$$B = \frac{\mu_0}{4\pi} \cdot \frac{\mu}{r^3} \sqrt{1 + 3\cos^2\theta} \qquad \text{...(3)}$$

The equation gives the magnitude of the resultants magnetic induction at P due to the magnetic dipole.

The direction of $\vec{B}$ is along the diagonal PC. Let ϕ be the angle between $\vec{B}$ and $\vec{B_1}$. This angle is given by

$$\tan\phi = \frac{B_2}{B_1} = \frac{\frac{\mu_0}{4\pi} \cdot \frac{\mu \sin\theta}{r^3}}{\frac{\mu_0}{4\pi} \cdot \frac{2\mu\cos\theta}{r^3}}$$
$$= \frac{\sin\theta}{2\cos\theta} = \frac{1}{2}\tan\theta \qquad \text{...(4)}$$

This equation gives the angle ϕ which the direction of the resultant magnetic induction at P makes with the line joining the point and the centre of the dipole.

Particular Cases

(i) **Magnetic induction at a point on the axis of the dipole :** If the point P is on the axis, $\theta = 0°$ or 180°. Then $\cos 0° = 1$ and $\cos 180° = -1$. Therefore, from Eq. (3) the magnitude of the magnetic induction is given by

$$B_{axis} = \frac{\mu_0}{4\pi} \cdot \frac{\mu}{r^3} \cdot \sqrt{1+3}$$
$$= \frac{\mu_0}{4\pi} \cdot \frac{2\mu}{r^3} \qquad \text{...(5)}$$

The direction of magnetic induction is along the axis of the dipole.

(ii) **Magnetic induction at a point on the equatorial line of the dipole :** If the point P is on the equatorial line, $\theta = 90°$ or 270°. Then $\cos 90° = \cos 270° = 0$. Therefore from Eq. (3), the magnitude of the magnetic induction is given by

$$B_{eq} = \frac{\mu_0}{4\pi} \cdot \frac{\mu}{r^3} \qquad \text{...(6)}$$

The direction of the magnetic induction is perpendicular to the equatorial line and is opposite to the direction $\vec{\mu}$.

UNIT-III

Time Varying Electromagnetic Fields

SECTION-A VERY SHORT ANSWER TYPE QUESTIONS

Q.1. Write Faraday's law of electromagnetic induction in differential and integral forms.

Ans. The integral form of Faraday's law is

$$\oint_c \vec{E}\cdot\vec{dl} = -\frac{d}{dt}\int_S \vec{B}\cdot\vec{dS}$$

The differential form of Faraday's law is

$$\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

where $\vec{E}$ is electric field and $\vec{B}$ is magnetic flux density.

Q.2. Define Lenz's law.

Ans. Whenever the magnetic flux through a conductor is changed, an emf induced in the conductor. The magnitude of the induced emf is equal to the rate of change of magnetic flux through the circuit.

Lenz tells about the direction of induced emf and gave the statement, "The direction of the induced emf or current is such as to oppose the change that produced it".

Q.3. Write Maxwell-Ampere's circuital law in differential form.

Ans. The differential form of Maxwell-Ampere circuital law is

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

where $\vec{H}$ is magnetic field intensity, $\vec{J}$ is conduction current density and $\vec{D}$ is electric displacement.

Q.4. What is a displacement current?

Ans. Maxwell introduced the new concept of displacement current. Faraday discovered that a changing magnetic field produces an electric field, Maxwell suggested that a changing electric field must produced magnetic field and develop the idea of displacement current. Maxwell proposed that the changing electric field between the plates serves the purpose of conduction current inside the gap. The displacement current in the gap found to be equal the conduction current in lead wires.

Displacement current density J_D defined as the time rate change of the electric displacement.

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t} \qquad \text{(displacement current)}$$

$$I_d = A\vec{J}_D = A\frac{\partial \vec{D}}{\partial t} \qquad \text{(where } A \text{ is the area)}$$

Q.5. Write down the equation of continuity.

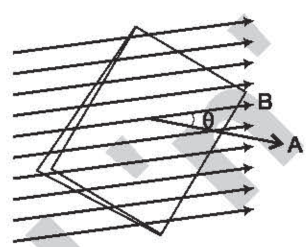
Ans. The equation of continuity is

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

where $\vec{J}$ is the current density and ρ is the charge density.

Q.6. Define magnetic flux and give its units.

Ans. The magnetic flux through a plane of area A placed in a uniform magnetic field can be written as



$$\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

where θ is the angle between B and A. The SI unit of magnetic flux is Wbm^2.

Q.7. Write Maxwell's equations in material media.

Ans. There are four Maxwell's equation of electromagnetism, which may be written in differential form as

$$\vec{\nabla} \cdot \vec{D} = \rho \qquad \text{(Gauss law)}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \text{(No name)}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{(Faraday's law)}$$

$$\vec{\nabla} \times \vec{G} = \vec{J} + \frac{\partial D}{\partial t} \qquad \text{(Ampere's law)}$$

where, $\vec{D}$ = electric displacement, E = electric field intensity, B = magnetic induction

H = magnetic field intensity, ρ = charge density, J = current density.

Q.8. State Faraday's law of electromagnetic induction.

Ans. Faraday's law of electromagnetic inductions are stated below.

"The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit."

Mathematically, the induced emf is given by

$$\varepsilon = -\frac{d\phi}{dt} \qquad [\phi = \text{flux}]$$

The negative sign indicates the direction ε and direction of current in a closed loop.
If there are closely wound *N* turns, the total induced emf is given by

$$\varepsilon = -N\frac{d\phi}{dt}$$

Q.9. Explain self inductance. Write its unit.

Ans. The magnetic flux ϕ produced in a coil is directly proportional to the current *I* flowing in the coil *i.e.*,

$$\phi \propto I$$

or $$\phi = LI$$

The proportionality constant *L* is called the coefficient of self induction or self inductance of the coil.
When the flux changes, the back emf induced in the coil is given by Faraday's law

$$\varepsilon = -\frac{d\theta}{dt} = -L\frac{dI}{dt}$$

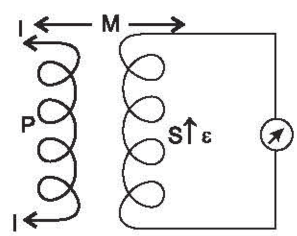
or $$L = -\frac{\varepsilon}{dI/dt}$$

If $$\frac{dI}{dt} = 1, \ |\varepsilon| = L$$

Therefore self inductance of a coil is numerically equal to the induced emf when current in it is changing at unit rate.
The S.I. unit of self inductance is Henry (H) or Newton meter/Amp^2.

Q.10. Explain mutual inductance between two coils.

Ans. Let us consider two coils *P* and *S* placed closed to each other. When a current *I* flows through the coil *P* changes the magnetic flux through coil *S* also changes. So there is a induced emf in the coil *S*. This is called mutual induction.



The magnetic flux through the second coil *S* depends upon *I*

$$\phi \propto I$$
$$\phi = MI$$

M is called mutual inductance of two coils.
The induced emf is given by

$$\varepsilon = -\frac{dQ}{dt} = -MdI/dt \text{ or } M = -\frac{\varepsilon}{dI/dt}$$

When $\frac{dI}{dt} = 1, \; |\varepsilon| = M$

The mutual inductance of two circuits is numerically equal to the emf in one circuit when the rate of change of current in the other is unity.

S.I. unit of mutual inductance is Henry or Newton-meter/Amp^2.

Q.11. Define coefficient of coupling between two coils, give its significance.

Ans. The ratio $K = M/\sqrt{L_1 L_2}$ is known as coefficient of coupling between two coils, its value lies between 0 and 1

i.e., $0 \le K \le 1$

If $k = 0$, there is no coupling between the coils

If $k = 1$, there is no leakage of flux *i.e.*, all the flux produced in one coil linked to another.

Q.12. Calculate the work done in increasing the current from 1 A to 2 A in a coil of self inductance 5 mH.

Ans. The energy stored in a inductor of self inductance L is given by

$$E = \frac{1}{2} Li^2 \qquad \text{(where '}i\text{' is the current)}$$

The work done when current changes from 1 A to 2 A is given by change in energy stored in the inductor.

$$W = \frac{1}{2} Li_2^2 - \frac{1}{2} Li_1^2 = \frac{1}{2} L (2^2 - 1^2)$$

$$= \frac{1}{2} \times 5 \times 10^3 \, (4 - 1)$$

$$= \frac{15}{2} \times 10^{-3}$$

$$W = 7.5 \times 10^{-3} \text{ Joule.}$$

Q.13. Two identical coils having self inductance 20 mH are coupled, so the mutual inductance between them is 10 mH. What is the coefficient of coupling?

Ans. Self inductance of coils $L_1 = L_2 = 20$ mH

Mutual inductance of the coils $M = 10$ mH

Coefficient of coupling is given by

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$k = \frac{10}{\sqrt{20 \times 20}} \Rightarrow k = \frac{10}{20} = 0.5$$

Q.14. Explain the physical significance of $\vec{\nabla} \cdot \vec{B} = 0$.

Ans. This is one of the Maxwell's equation in differential form. It says that divergence of magnetic induction is zero. It also signifies that magnetic monopoles does not exists. The magnetic poles always exists in pairs.

SECTION-B SHORT ANSWER TYPE QUESTIONS

Q.1. Starting from Maxwell's equations prove the continuity equation.

Ans. We have the Maxwell's equation

$$\vec{\nabla} \times \vec{H} = J + \frac{\partial D}{\partial t} \qquad ...(1)$$

Taking divergence on both sides of this equation

$$\vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{H}) = \vec{\nabla} \cdot \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$$

Since divergence curl of any vector is zero above equation reduces to

$$\vec{\nabla} \cdot \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) = 0$$

or

$$\vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \left(\frac{\partial \vec{D}}{\partial t} \right) = 0$$

$$\nabla \cdot J + \frac{\partial}{\partial t}(\nabla \cdot D) = 0 \qquad ...(2)$$

(Space and time operations are interchangable)

We have Gauss law is given by

$$\nabla \cdot D = \rho$$

Putting this value in eq. (2), we get

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

This is the required result.

Q.2. Derive the formula for self inductance of a long solenoid.

Ans. Let l be the length of long solenoid with an air core and have N number of turns. When i current flows through it the magnetic field inside it is given by equation

$$B = \mu_0 n i \qquad \left(\text{Where } n = \frac{N}{l}\right)$$

If A be the area of each turn then magnetic flux through each turn

$$\phi_B = BA = \mu_0 n i A$$

Total flux through the solenoid

$$\phi_T = N\phi_B = \mu_0 N n i A$$

When the current i varies, the flux changes giving rises to the induced emf

$$\varepsilon = -\frac{d\phi_T}{dt} = -\mu_0 N_n A \frac{di}{dt}$$

$$= -L\frac{di}{dt} \qquad \text{(from definition of } L\text{)}$$

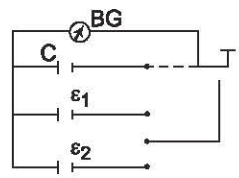
Therefore, $$L = \mu_0 N_n A$$

or $$L = \frac{\mu_0 N^2 A}{l}$$

This is the self inductance of solenoid.

Q.3. How to compare the emf's of two cells with the help of Ballistic Galvanometer?

Ans. The circuit to compare the emf's of two cells is shown in the figure below :



A suitable condensor C is first charged with cell of emf ε_1 and then discharge through the Ballistic Galvanometer (BG) and observed the throw say it is θ_1. Similarly charged with another cell of emf ε_2 and find out the throw θ_2, then

$$q_1 = c\varepsilon_1 k\theta_1 \left(1 + \frac{\lambda}{2}\right)$$

and $$q_2 = c\varepsilon_2 k\theta_2 \left(1 + \frac{\lambda}{2}\right)$$

Since the capacitor is charged with same charge therefore

$$q_1 = q_2$$

$$c\varepsilon_1 k\theta_1 \left(1 + \frac{\lambda}{2}\right) = c\varepsilon_2 k\theta_2 \left(1 + \frac{\lambda}{2}\right)$$

$$\varepsilon_1\theta_1 = \varepsilon_2\theta_2$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\theta_2}{\theta_1}$$

Q.4. Explain the construction of a transformer.

Ans.

The Transformer

It is a device for converting a low alternating voltage at high current into a high alternating voltage at low current and vice-versa. It is an electrical device based on the principle of mutual induction between the coils.

Construction

A transformer consists of two coils, called the primary P and secondary S, which are insulated from each other and wound on a common soft-iron laminated core (Fig.)

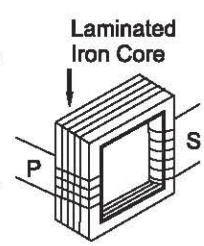


Fig.

The alternating voltage to be transformed is connected to the primary while the load is connected to the secondary. Transformers which convert low voltages into higher voltages are called step-up transformers. Transformers which convert high voltages into lower voltages are called step-down transformers.

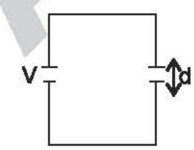
In a step-up transformer, the primary coil consists of a few turns of thick insulated copper wire of large current carrying capacity and secondary consists of a very large number of turns of thin copper wire. In a step-down transformer, the primary consists of a large number of turns of thin copper wire and the secondary of a few turns of thick copper wire.

Now when an ac is applied to the primary coil, it sets up an alternating magnetic flux in the core which also gets linked with the secondary. This change in flux linked with the secondary coil induces an alternating emf in the secondary coil. Thus the energy supplied to the primary is transferred to the secondary through the changing magnetic flux in the core.

Q.5. If a constant current changes a capacitor, show that the displacement current will be given by $I_d = C\dfrac{dV}{dt}$.

Sol. We have displacement current J_D is given by

$$J_D = \frac{I_d}{A} = \frac{\partial D}{\partial t}$$



or $$I_d = A\frac{\partial D}{\partial t}$$

as $$D = \varepsilon E$$

So, $$I_d = A\varepsilon\frac{\partial E}{\partial t} \quad ...(1)$$

The electric field between the plates of a capacitor and voltage are related by

$$E = \frac{V}{d}$$

from eq. (1) $$I_d = \frac{A\varepsilon}{d}\frac{\partial V}{\partial t}$$

$$I_d = C\frac{\partial V}{\partial t} \quad \left[\text{as } C = \frac{\varepsilon A}{d}\right]$$

This is the required result.

Q.6. Justify the following statement : The addition of displacement current results in to unification of electrical and magnetic phenomena.

Ans. The addition of displacement current *i.e.*, $\dfrac{\partial D}{\partial t}$ to Ampere's law results in

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

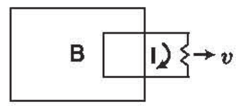
It means that displacement current relates the electric field vector $\vec{E}$ to magnetic field vector $\vec{H}$. This in turn implies that in case of time dependent field it is not possible to deal with electric and magnetic field separately but the two fields are interlinked and give rise to what are known as electromagnetic fields.

Hence the addition of displacement current to Ampere's law results in the unification of electric and magnetic phenomena.

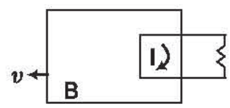
Q.7. Explain Faraday's Law of electromagnetic induction.

Ans. In 1831 Michael Faraday reported on a series of experiments including three that can be characterized as follows.

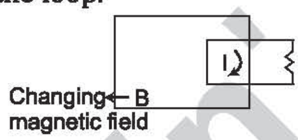
Experiment 1. He pulled a loop of wire to the right through a magnetic field. A current flowed in the loop.



Experiment 2. He moved the magnet to the left, holding the loop still again a current flowed in the loop.



Experiment 3. With both the loop and the magnet at rest, he changed the strength of the field, once again the current flowed in the loop.



Faraday summed up the above facts into two laws known as Faraday's laws of electromagnetic induction.

First law : When the magnetic flux linked with a circuit changes, an emf is always induced in it.

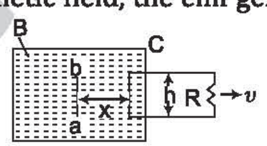
Second law : The magnitude of induced emf is equal to the rate of change of flux linkage.

Q.8. Explain the motional (or dynamically) induced emf.

Ans.

Motional emf

When a conductor moves in a magnetic field, the emf generated is known as motional emf.



Let us consider a rectangular loop moving with velocity v away from magnetic field. Let at any instant length x is inside the magnetic field, h be the breadth of rectangular loop. The magnetic field is $\vec{B}$.

The flux through the loop

$$\phi = \int \vec{B} \cdot \vec{da}$$

$$\phi = Bhx \qquad [hx = \text{area of loop inside magnetic field}]$$

as the loop moves, the flux decreases

$$\frac{d\phi}{dt} = \frac{d}{dt}(Bhx) = Bh\frac{dx}{dt} = Bhv$$

Induced emf is the negative rate change of magnetic flux.

Hence, $\varepsilon = -Bhv$

This is the formula for motional emf.

SECTION-C LONG ANSWER TYPE QUESTIONS

Q.1. Write down Maxwell's equations in differential and integral forms and explain their physical significance (meaning).

Ans. **Differential form of Maxwell's Equations**

There are four fundamental equations of electromagnetism and corresponds to generalisation of certain experimental observations regarding electricity and magnetism. The differential form of Maxwell's equations are :

(i) Gauss's law for the electric field of charge yield

$$\vec{\nabla} \cdot \vec{D} = \rho$$

where $\vec{D}$ is electric displacement in coulomb/m^2 and ρ is volume charge density.

(ii) Gauss law for magnetic field yields

$$\vec{\nabla} \cdot \vec{B} = 0$$

where $\vec{B}$ is magnetic induction in web/m^2.

(iii) Ampere's law in circuital form for the magnetic field accompaying a current when modified by Maxwell yields

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

where $\vec{H}$ is magnetic field intensity and J is the current density.

(iv) Faraday's law in circuital form for the induced electromotive force produced by the rate of change of magnetic flux linked with the path yields

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

where $\vec{E}$ is the electric field intensity in volts/m.

Physical Significance (or Integral Form)

By means of Gauss and strokes theorem we can write the Maxwell's field equation in integral form and hence obtain their physical significance.

(i) Integrating Maxwell's first equation $\vec{\nabla} \cdot \vec{D} = \rho$ over an arbitrary volume τ, we get

$$\int_\tau (\vec{\nabla} \cdot \vec{D})\, d\tau = \int_\tau \rho\, d\tau$$

using Gauss divergence theorem on left hand side and keeping in mind

$$\int \rho\, d\tau = Q$$

$$\oint \vec{D} \cdot \vec{ds} = Q$$

"So Maxwell's first equation signifies that the total electric displacement linked with a closed surface is equal to the total charge enclosed by the closed surface.

(ii) Integrating second Maxwell's equation $\vec{\nabla} \cdot \vec{B} = 0$ over the arbitrary volume τ, we get

$$\int (\vec{\nabla} \cdot \vec{B})\, d\tau = 0$$

using Gauss divergence theorem

$$\oint \vec{B} \cdot \vec{ds} = 0$$

This equation signifies that the total flux of magnetic induction linked with a closed surface is zero.

(iii) Integrating Maxwell's third equation $\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ over a surface S bounded by the loop C, we get

$$\int (\vec{\nabla} \times \vec{H}) \cdot dS = \int \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot dS$$

using stokes theorem, we get

$$\oint \vec{H} \cdot \vec{dl} = \int \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot dS$$

which signifies that magnetomotive force around a closed path [$\oint \vec{H} \cdot \vec{dl}$] is equal to conduction current plus displacement current linked with that path.

(iv) Integrating Maxwell's fourth equation $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ over a surface S bounded by the loop C, we get

$$\int (\vec{\nabla} \times \vec{E}) \cdot \vec{dS} = -\int \frac{\partial \vec{B}}{\partial t} \cdot \vec{dS}$$

using stokes theorem on L.H.S.

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t} \int \vec{B} \cdot \vec{dS}$$

This law signifies that the electromotive force *i.e.*, the line integral of electric intensity around a closed path is equal to the negative rate of change of magnetic flux linked with the path.

Q.2. Write Maxwell's equations of electromagnetism and derive them from basic laws.

Ans. Maxwell's equation of electromagnetism are

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \text{(Gauss's law)}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \text{(no name)}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t} \qquad \text{(Faraday's law)}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \text{(Ampere's law with Maxwell's correction)}$$

Together with the force law

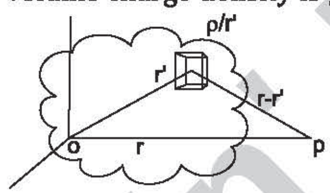
$$\vec{F} = q(\vec{E} + \vec{V} \times \vec{B})$$

and continuity equation

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Derivation of Maxwell's equation : Gauss law :

1. The electric field due to a volume charge density is given by



$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\hat{r}}{r^2} \rho(r)\, d\tau' \qquad ...(1)$$

Take the divergence

$$\vec{\nabla} \cdot \vec{E} = \frac{1}{4\pi\varepsilon_0} \int \nabla \cdot \left(\frac{\hat{r}}{r^2}\right) \rho(r')\, d\tau'$$

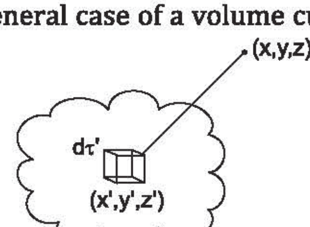
$$= \frac{1}{4\pi\varepsilon_0} \int 4\pi\delta^3(r - r')\rho(r')\, d\tau'$$

[$\delta^3(r - r')$ = Dirac Delta function in three dimension]

$$\nabla \cdot E = \frac{\rho(r)}{\varepsilon_0}$$

which is Gauss law in differential form.

(ii) The Biot-savart law for general case of a volume current reads.



$$\vec{B}(r) = \frac{\mu_0}{4\pi} \int \frac{J(r') \times \hat{r}}{r^2}\, d\tau' \qquad ...(2)$$

This formula gives the magnetic field at a point $\vec{r} = (x, y, z)$ in terms of an integral over the current distribution $J(x', y', z')$.

B is a function of (x, y, z)

J is a function of (x', y', z')

$$\vec{r} = (x - x')\hat{i} + (y - y')\hat{j} + (z - z')\hat{k}$$

$$d\tau' = dx'\,dy'\,dz'$$

The integration is over the primed coordinates. Applying the divergence of eq. (2)

$$\vec{\nabla} \cdot \vec{B} = \frac{\mu_0}{4\pi} \int \nabla \cdot \left(J(r') \times \frac{\hat{r}}{r^2} \right) d\tau'$$

Now, $$\nabla \cdot \left(J(r') \times \frac{\hat{r}}{r^2} \right) = \frac{\hat{r}}{r^2} \cdot (\vec{\nabla} \times \vec{J}) - \vec{J} \left(\vec{\nabla} \times \frac{\hat{r}}{r^2} \right)$$

[using $\{\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{A}) - A \cdot (\vec{\nabla} \times \vec{B})\}$]

$$\vec{\nabla} \times \vec{J} = 0,$$

because $\vec{J}$ does not depend on the unprimed variables (x, y, z), where as $\vec{\nabla} \times \frac{\hat{r}}{r^2} = 0$.

Hence $$\vec{\nabla} \cdot \vec{B} = 0$$

The divergence of the magnetic field is zero.

Faraday's Law : According to Faraday's law of electromagnetic induction, we know that the induced emf is proportional to the rate of change of flux. *i.e.*,

$$\varepsilon = -\frac{d\phi}{dt} \quad \text{...(1)}$$

Now if $\vec{E}$ be the electric intensity at a point, the work done in moving a unit charge through a small distance dl is $\vec{E} \cdot \vec{dl}$. So the work done in moving the unit charge once round the circuit is $\oint \vec{E} \cdot \vec{dl}$.

Emf is defined as the amount of work done in moving a unit charge once round the electric circuit.

$$\varepsilon = \oint \vec{E} \cdot \vec{dl} \quad \text{...(2)}$$

Comparing eq. (1) and (2), we get

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi}{dt}$$

But $$\phi = \int \vec{B} \cdot \vec{ds}$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt} \int \vec{B} \cdot \vec{ds}$$

using stokes theorem, line integral converts to surface integral

$$\int (\vec{\nabla} \times \vec{E}) \cdot ds = -\frac{d}{dt} \int_S \vec{B} \cdot \vec{ds} \quad \text{or} \quad \int \left(\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} \right) \cdot ds = 0$$

As the above integral is true for any arbitrary surface the integrand must vanish

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Ampere's Circuital Law : The work done in carrying unit magnetic pole once round a closed arbitrary path linked with the current I is expressed by

$$\oint \vec{H} \cdot \vec{dl} = I$$

or

$$\oint \vec{H} \cdot \vec{dl} = \int_s J \cdot ds \qquad ...(1) \quad (\text{as } I = \int \vec{J} \cdot \vec{dS})$$

where S is the surface bounded by the closed path C using stokes theorem, we get

$$\int \vec{H} \cdot \vec{dl} = \int \vec{\nabla} \times \vec{H} \cdot \vec{dS}$$

So eq. (1) becomes

$$\int (\vec{\nabla} \times \vec{H}) \cdot \vec{dS} = \int \vec{J} \cdot \vec{dS} \quad i.e., \quad \vec{\nabla} \times \vec{H} = \vec{J}$$

we know that $H = B/\mu_0$

So, above eq. becomes

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \qquad ...(2)$$

Maxwell found it to be incomplete for changing electric fields and assumed that a quantity

$$J_D = \varepsilon_0 \frac{\partial E}{\partial t}$$

called displacement current must be included in it so that it must satisfy continuity equation. So, eq. (2) becomes

$$\vec{\nabla} \times \vec{B} = \mu_0 (J + J_D)$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + N_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

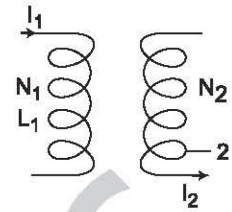
This is Amphere's law.

Q.3. What is meant by coefficient of coupling? Obtain an expression for the coefficient of coupling between two coils.

Ans.

Coefficient of Coupling

The coefficient of coupling between two coils gives an idea about that how much flux is linked between the two coils. It is a measure of flux linked.



Let us consider two coils having self inductance L_1 and L_2 and have number of turns N_1 and N_2 respectively. I_1 and I_2 are the current flowing through the two coils.

Let ϕ_1 and ϕ_2 be the magnetic flux linked with each turn of coils 1 and 2 due to their own currents I_1 and I_2 respectively.

The self inductance of the coils is given by

$$L_1 = \frac{N_1\phi_1}{I_1} \quad ...(1)$$

and

$$L_2 = \frac{N_2\phi_2}{I_2} \quad ...(2)$$

Let ϕ_{12} be the flux per turn in the coil 1 due to current F_2 in coil 2. Similarly ϕ_{21} is flux per turn linked with coil 2 due to current I_1 in coil 1. Then the mutual inductance between them is given by

$$M = \frac{N_1\phi_{12}}{I_2} = \frac{N_2\phi_{21}}{I_1} \quad ...(3)$$

The whole of the flux from one coil is linked with the other coil. Then

$$\phi_{12} = \phi_2$$

and

$$\phi_{22} = \phi_1$$

From eq. (3)

$$M = \frac{N_1\phi_2}{I_2} = \frac{N_2\phi_1}{I_1}$$

$$M^2 = \frac{N_1 N_2 \phi_1 \phi_2}{I_1 I_2} \quad ...(4)$$

from eq. (1) and eq. (2)

$$L_1 L_2 = \frac{N_1 N_2 \phi_1 \phi_2}{I_1 I_2} \quad ...(5)$$

Hence,

$$M^2 = L_1 L_2$$

or

$$M = \sqrt{L_1 L_2} \quad ...(6)$$

In practice whole of the flux from one coil is not necessarily completely linked with the other. The ratio $M/\sqrt{L_1 L_2}$ is known as the coefficient of coupling between the coils. It is denoted by K. Thus

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

K is a number between 0 and 1, depending upon the geometry of the coils and their relative positions.

If $K = 1$, there is no leakage of flux *i.e.*, all the flux from one coil linked to another coil.

If $K = 0$ there is no coupling between the coils.

K lies between 0 and 1.

Q.4. Give the theory and working of moving coil Ballistic galvanometer.

Ans.

The Moving Coil Ballistic Galvanometer

A ballistic galvanometer consists essentially of a rectangular coil suspended in a magnetic field as shown in Fig. 1. Let the current i flow through the coil. There is no resultant force along bc and which are parallel to $\vec{B}$. There is equal to opposite force on ab and dc. The torque is given by

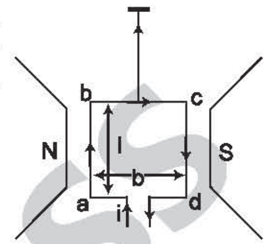


Fig. 1

$$\tau = iAB$$

where A is area of coil lb.

If the current i acts for a short interval dt, then the moment of impulse is $iABdt$ and total moment of impulse,

$$\int_0^T iAB\,dt = qAB, \text{ since } q = \int_0^T idt$$

$$\begin{aligned}\text{But moment of impulse} &= \text{mass} \times \text{velocity} \times \text{distance from the axis of rotation}\\ &= \text{moment} \times \text{distance from the axis of rotation}\\ &= \text{angular moment.}\end{aligned}$$

If I is moment of inertia of the coil about the suspension fibre and ω is angular velocity then angular momentum $= I\omega$

$$qAB = I\omega \qquad \text{...(1)}$$

The kinetic energy of the system is $\frac{I\omega^2}{2}$ and is used up in twisting the suspension fibre. If c is the restoring force per unit twist, the couple on twist θ is $c\theta$ and work done for an additional twist $d\theta$ is, $c\theta\, d\theta$ so that the whole work done in twisting the suspension fibre is,

$$\int_0^\theta c\theta\, d\theta = \frac{1}{2}c\theta^2$$

therefore,

$$\frac{1}{2}c\theta^2 = \frac{1}{2}I\omega^2$$

or

$$\omega^2 = \frac{c}{I}\theta^2 = \frac{q^2A^2B^2}{I^2} \qquad \text{[from eq. (1)]}$$

$$\frac{C}{I}\theta^2 = \frac{q^2A^2B^2}{I^2}$$

$$q^2 = \frac{c^2}{B^2A^2}\frac{1}{c}\theta^2$$

Now the period of vibration of a body of moment of inertia I, when c is the restoring couple per unit twists is given by,

$$T = 2\pi\sqrt{\frac{I}{c}}$$

$$\frac{I}{c} = \frac{T^2}{4\pi^2}$$

Therefore, $$q^2 = \frac{c^2}{B^2A^2}\frac{T^2}{4\pi^2}\theta^2$$

$$q = \frac{c}{BA}\frac{T}{2\pi}\theta \qquad ...(2)$$

$$q = K\frac{T}{2\pi}\theta \qquad ...(3)$$

where $$K = \frac{c}{BA}$$

This gives a relation between the charge flowing and the ballistic throw θ of the galvanometer, K is the galvanometer constant. The torque or couple on suspension fibre is $c\theta$. Therefore,

$$iAB = c\theta$$

$$i = \frac{c}{BA}\theta \qquad ...(4)$$

Current Sensitivity : Qualitatively the current sensitivity of a galvanometer is defined as the deflection, as read from the per unit current. So, from equation (4),

$$\text{current sensitivity} = \frac{\text{deflection}}{\text{current}} = \frac{\theta}{i} = \frac{BA}{c} \qquad ...(5)$$

As we know that mirror and scale arrangement is used for reading the deflection, if the current i in amp., the scale deflection θ in millimeters and scale distance L in metres, the current sensitivity,

$$S_i = \frac{\theta}{Li}$$

If i =1 amp, then Si is a large number for a given value of L. Consequently for convenience we prefer microampere which gives microampere current sensitivity.

Micro-ampere sensitivity is the deflection in mm produced on a scale 1000 mm away galvanometer mirror, when a current of one micro ampere (10^{-6} amp.) is passed in the galvanometer.

Charge Sensitivity of Quantity Sensitivity : The charge sensitivity of the B.G. is defined as the deflection per unit charge. So from equation (5),

$$\text{Charge sensitivity} = \frac{\text{deflection}}{\text{charge}}$$

$$= \frac{\theta}{q} = \frac{2\pi}{T}\cdot\frac{BA}{c}$$

But from equation (5), $\frac{BC}{c}$ is current sensitivity.

Hence, $$\text{charge sensitivity} = \frac{2\pi}{T} \times \text{current sensitivity.} \qquad ...(6)$$

Hence the charge sensitivity of a B.G. is $\frac{2\pi}{T}$ times the current sensitivity.

Voltage Sensitivity : The voltage sensitivity under any given conditions is the deflection per unit voltage and is consequently current sensitivity divided by the resistance, *i.e.*,

$$\text{Voltage sensitivity} = \frac{\text{Current sensitivity}}{\text{Resistance}}$$

The resistance is that of entire circuit and not that of the instrument alone. Voltage sensitivity is large number, hence for convenience, we prefer micro-volt sensitivity.

Micro-volt Sensitivity : Micro-volt sensitivity is the deflection in millimeter on a scale 1 meter away from the galvanometer mirror, when a potential difference of one micro-volt (10^{-6} volt) maintained across the galvanometer terminals.

Figure of merit of galvanometer : Figure of merit of galvanometer is the current which will produce a deflection of one scale division, when we use lamp and scale arrangement, it is the current which will produce a deflection of 1 mm on a metre scale one metre away from the galvanometer mirror.

It may be noted here that the current sensitivity or charge sensitivity is sometimes defined as current or charge per unit deflection and then it is reciprocal of what have been defined equation (5) and (6). We prefer to call it figure of merit of the galvanometer.

$$\text{Figure of merit} = \frac{q}{\theta} \text{ or } \frac{i}{\theta}.$$

Q.5. Write down differential equation of motion of a coil in a ballistic galvanometer and derive the condition under which its motion is oscillatory. What is logarithmic decrement?

Ans. When we derive the equation $q = k\dfrac{T}{2\pi}\theta$, we have assumed that whole of kinetic energy of the coil is used up in twisting the fibre and neglected the retarding forces presents namely (1) due to friction of air (2) viscosity of suspension fibre and (3) induced current in any neighboring mass or metal. The first two of these factors constitute mechanical damping, while the third factor accounts for electro-magnetic damping. The retarding couple due to damping varies as the angular velocity $= d\theta/dt$ may be taken as $P\,d\theta/dt$, where P is the coefficient of mechanical damping.

When the circuit is closed, we have in addition the induced current produced in the coil which varies as the rate of change of displacement and inversely as the total resistance of the circuit, hence the retarding couple is $\dfrac{m}{R}\dfrac{d\theta}{dt}$, where the constant m involves the magnetic flux due to the magnet and the area of coil. Thus, the total retarding couple due to damping.

$$\left(P + \frac{M}{R}\right)\frac{d\theta}{dt}$$

Now the angular acceleration is $\dfrac{d^2\theta}{dt^2}$.

According to Newton's second law, equation of motion is,

$$I\frac{d^2\theta}{dt^2} = -c\theta - \left(P + \frac{M}{R}\right)\frac{d\theta}{dt}$$

or
$$I\frac{d^2\theta}{dt^2} + \left(\frac{M}{R} + P\right)\frac{d\theta}{dt} + c\theta = 0$$

or
$$\frac{d^2\theta}{dt^2}+\frac{1}{I}\left(\frac{M}{R}+P\right)\frac{d\theta}{dt}+\frac{c\theta}{I}=0$$

or
$$\frac{d^2\theta}{dt^2}+2b\frac{d\theta}{dt}+k^2\theta=0 \qquad ...(1)$$

where
$$2b=\frac{1}{I}\left(\frac{M}{R}+P\right) \qquad ...(2)$$
$$K^2=\frac{c}{I} \qquad ...(3)$$

Let the solution of this equation be $\theta=Ae^{\alpha t}$ then $\frac{d\theta}{dt}=A\alpha e^{\alpha i}$ and $\frac{d^2\theta}{dt^2}=A\alpha^2 e^{\alpha i}$

Putting these values in equation (1), $\alpha^2+2b\alpha+k^2$
$$\alpha=-b\pm\sqrt{b^2-K^2}$$

This gives $\theta=A_1e^{(-b+\sqrt{b^2+K^2})t}+A_2e^{(-b-\sqrt{b^2-K^2})t}$ where, A_2 and A_1 are constants.

The quantity b^2-K^2 may be positive, zero or negative depending upon whether
$$b>k, b=K \text{ or } b<k$$

Case 1 : When $b>K$, we have two real values of α, then solution of equation (1) is,
$$\theta=A_1e^{\alpha_1 t}+A_2e^{\alpha_2 t}$$

The α_1 and α_2 are negative. The deflection θ will go on decreasing with increasing time. The motion non-oscillating or dead beat and the oscillation is said to be over-damped.

Case 2 : Critical damping : When $b=K$; the two values of α are equal to $-b$ and solution of (1) becomes
$$\theta=(A_1+A_2)e^{-bt}=A_3e^{-bt}$$

That the coil after detection comes to rest in minimum possible time and the galvanometer is said to be critically damped. However, the motion of the coil is just non-oscillatory.

Case 3 : When $b<K$, the quantity $\sqrt{b^2-K^2}$ is imaginary so the values of α are complex *i.e.*,
$$\alpha=-b\pm jg, \text{ where } g^2=K^2-b^2$$

Hence, solution of equation (1) becomes,
$$\theta=A_1e^{(-b+jg)t}+A_2e^{(-b-jg)t}=e^{-bt}[A_1e^{jgt}+A_2e^{-jgt}]$$
$$=e^{-bt}[A_1\{\cos gt+j\sin gt\}+A_2\{\cos gt-j\sin gt\}]$$
$$=e^{-bt}[(A_1+A_2)\cos gt+j(A_2-A_2)\sin gt]$$

Putting $A_1+A_2=B\sin\beta$

and $j(A_1-A_2)=B\sin\beta$
$$\theta=Be^{-bt}\sin(gt+\beta)=Be^{-bt}\sin\{(\sqrt{K^2-b^2})t+\beta\}$$

The motion is oscillatory or ballistic and period of oscillation,
$$T=\frac{2\pi}{g}=\frac{2\pi}{\sqrt{K^2-b^2}}$$

In order that the whole charge may pass through the ballistic galvanometer before it has from its zero position, it is essential that its periodic time be large. Since we have,

$$T = 2\pi\sqrt{\frac{1}{c}},$$

therefore, I should be large and c should be small. As we have $b < K$

$$\frac{1}{2I}\left(\frac{m}{R}+\rho\right) < \frac{c}{I} < \frac{2\pi}{T}$$

Hence to make b small, I should be large, R should be large.
m should be small and so the coil should be wound round a non-conducting frame, *e.g.*, paper, should be bamboo or plastic. p should be small, and so the air resistance should be small and the fibre very fine.
We just see that the requirement of dead-beat galvanometer are just opposite to those for the ballistic $b > K$. Hence the opposite conditions should prevail. The coil is made of thick copper wire and has smaller number of turns so to have smaller resistance and is wound round a metallic or conducting frame.
Damping : The oscillatory motion in B.G. is given by

$$\theta = Be^{-bt}\sin(\sqrt{k^2 - b^2 t + \beta})$$

The effort is made in the B.G. to make '*b*' as small as possible so that the motion is simple harmonic, but in actual practice b can never be zero so the amplitude of oscillation decreases with time.
The amplitude of motion is Be^{-bt} and

for $t = 0$, we have amplitude $\theta_0 = B$

for $t = \frac{T}{4} = \frac{\pi}{2\sqrt{k^2 - b^2}}$ amplitude $\theta_1 = Be^{-\frac{b\pi}{2\sqrt{k^2-b^2}}}$

for $t = \frac{3T}{4} = \frac{3\pi}{2\sqrt{k^2 - b^2}}$ amplitude $\theta_2 = Be^{-\frac{3bz}{2\sqrt{k^2-b^2}}}$

for $t = \frac{5T}{4} = \frac{5z}{2\sqrt{k^2 - b^2}}$ amplitude $\theta_3 = Be^{-\frac{5bz}{2\sqrt{k^2-b^2}}}$

and $$\frac{\theta_1}{\theta_2} = \frac{\theta_2}{\theta_3} = \frac{\theta_3}{\theta_4} \ldots = e^{\frac{bz}{\sqrt{k^2-b^2}}} = e^{\lambda} = d.$$

We see that the successive amplitude $\theta_1, \theta_2, \theta_3$ etc., are continuously decreasing and the ratio of one value to the next is always a constant d. This constant ratio d is called decrement and $\log_e d = \lambda$ is called the logarithmic decrement λ.

UNIT-IV

Electromagnetic Waves

SECTION-A VERY SHORT ANSWER TYPE QUESTIONS

Q.1. State Poynting theorem.

Ans. This is the work-energy theorem of electrodynamics. It states that, "the work done on the charges by the electromagnetic force is equal to the decrease in energy stored in the field, less the energy stored in the field."

$$\frac{dw}{dt} = -\frac{d}{dt}\int_V \frac{1}{2}\left(\epsilon_0 E^2 + \frac{B^2}{\mu_0}\right) d\tau - \frac{1}{\mu_0}\oint_S (\vec{E} \times \vec{B}) \cdot \vec{da}$$

The first integral on the right is the total energy stored in the fields. The second term represents the rate at which energy is carried out of volume V.

Q.2. Define Poynting vector.

Ans. This gives the direction of energy propagation and is defined as

"The energy per unit time, per unit area, transported by the fields is called the poynting vector"

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B})$$

From the above expression it is evident that energy will be transport in a direction perpendicular to both $\vec{E}$ & $\vec{B}$ the term $\vec{S} \cdot \vec{da}$ is the energy per unit time crossing the infinitesimal surface da.

Q.3. What do you mean by plane electromagnetic wave?

Ans. Waves are means of transporting energy or information. A wave is a function of both space and time. Typical example of EM waves included, radio waves, TV signals, radar beams etc. Electromagnetic waves consists of time and space varying electric and magnetic field in mutually perpendicular direction.

A uniform plane wave, in which both fields, E and B lie in the transverse plane *i.e.*, the plane whose normal is the direction of propagation.

Q.4. Write down the wave equation for electric and magnetic fields in free space.

Ans. The wave equation for electric & magnetic field is

$$\left.\begin{aligned} \nabla^2 E &= \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} \\ \nabla^2 B &= \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} \end{aligned}\right\} \qquad ...(1)$$

In general we can write

$$\nabla^2 f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2} \qquad \left\{\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right\}$$

where v is the speed of electromagnetic wave,

$$v - \frac{1}{\sqrt{\mu_0 \in_0}}.$$

Q.5. Write down the expression for electric and magnetic fields for a plane wave.

Ans. Since the fields in a EM wave are uniform over every plane perpendicular to the direction of propagation. We are interested in fields of the form

$$\tilde{E}(z,t) = \tilde{E}_0 \, e^{i(kz-\omega t)}$$

and

$$\tilde{B}(z,t) = \tilde{B}_0 \, e^{i(kz-\omega t)}$$

These are the electric and magnetic fields travelling in the z-direction. Here k is the propagation constant and ω is the angular frequency.

$\tilde{E}_0$ and $\tilde{B}_0$ are the complex amplitudes. The argument $(kz - \omega t)$ is known as the phase of the wave.

Q.6. Define propagation constant.

Ans. The propagation constant is defined as

$$k = \frac{\omega}{c} \text{ rad/m}$$

or

$$k = \frac{2\pi}{\lambda}$$

This is the measure of that how phase of a wave changes as it traverse a wavelength.

Q.7. Show that the refractive index of the medium is given by $n = \sqrt{\frac{\mu \in}{\mu_0 \in_0}}$, where symbols have their usual meaning.

Sol. We know that the speed of an EM wave in free space is given by

$$C^2 = \frac{1}{\mu_0 \in_0} \quad \Rightarrow \quad C = \frac{1}{\sqrt{\mu_0 \in_0}}$$

The speed of an EM wave in medium is given by

$$v^2 = \frac{1}{\mu \in} \quad \Rightarrow \quad v = \frac{1}{\sqrt{\mu \in}}$$

The refractive index is defined as the ratio of speed of light in vacuum (or free space) to speed of light in medium

$$n = \frac{c}{v}$$

$$= \frac{\frac{1}{\sqrt{\mu_0 \epsilon_0}}}{\frac{1}{\sqrt{\mu \epsilon}}}$$

or
$$n = \sqrt{\frac{\mu \epsilon}{\mu_0 \epsilon_0}}.$$

Q.8. What is dispersive and non-dispersive media.

Ans. If the refractive index of the medium changes with the wavelength (or frequency) of incident wave, then we say that medium is dispersive otherwise it is a non-dispersive medium. So in a dispersive medium the shape of wave changes because different frequencies travel at different speeds.

Q.9. Write the formula for characteristic impedance of free space.

Ans. The characteristic impedance (or intrinsic impedance) is given by

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377 \Omega.$$

Q.10. Show that in a plane electromagnetic wave

$$\vec{K} \cdot \vec{E} = \vec{K} \cdot \vec{B} = \vec{E} \cdot \vec{B} = 0$$

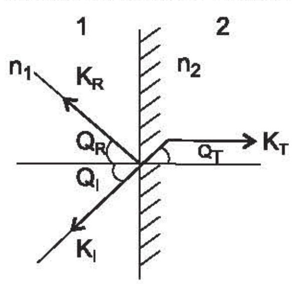
where symbols have their usual meaning $\vec{K}$ is propagation vector.

Ans. Since in a plane electromagnetic wave, electric vector magnetic vector and propagation vector are mutually perpendicular to each other *i.e.*, the angle between $\vec{E}$, $\vec{B}$ and $\vec{K}$ is 90°,

$$\left.\begin{aligned} \vec{K} \cdot \vec{E} &= KE \cos 90° \\ \vec{K} \cdot \vec{B} &= KB \cos 90° \\ \vec{E} \cdot \vec{B} &= EB \cos 90° \end{aligned}\right\} = 0 \qquad \{\cos 90° = 0\}$$

Q.11. Write down snell's law.

Ans. In case of refraction the ratio of the 'sin' of the angle of refraction to the 'sin' of angle of incidence is equal to the ratio of refractive indices of the two media *i.e.*,



$$n_1 \sin \theta_I = n_2 \sin \theta_T$$

where n_1 is the index of refraction of medium (1) and n_2 is the refractive index of medium (2).

SECTION-B SHORT ANSWER TYPE QUESTIONS

Q.1. The electric field in free space is given by

$$\vec{E} = 50\cos(10^8 t + kx)\hat{a}_y \ v/m$$

where $\hat{a}_y$ is unit vector in y-direction.

(a) Find the direction of wave propagation.

(b) Calculate k.

(c) The direction of electric field.

Ans. (a) From the positive sign in $(\omega t + kx)$, we infer that the wave is propagating along $-x$ direction.

(b) Compare the above equation with

$$\vec{E} = E_0 \cos(\omega t + kx)\hat{a}_y$$

we get $\omega = 10^8$

Since in free space, the speed of em wave is equal to the speed of light *i.e.*, $u = c$

$$k = \frac{\omega}{c}$$

$$= \frac{10^8}{3\times 10^8}$$

$$= 0.3333 \text{ rad/m}$$

(c) The polarisation of electric field in y-direction.

Q.2. Calculate the value of the poynting vector at the surface of the sun, if the power radiated by the sun is 3.8×10^{26} watts and its radius is 7×10^8 m.

Sol. The energy per unit time passing in through the surface of the sun is

$$P = \int \vec{S}.\vec{da} \qquad \{\vec{S}\text{- Poynting vector}\}$$

If we assume that $\vec{S}$ is constant over the surface of the sun then

$$P = S\cdot\int da \qquad \{S \text{ and da are parallel to each other}\}$$

$$P = S\cdot 4\pi r^2$$

$$S = \frac{P}{4\pi r^2} \qquad [r \text{ –radius of sun}]$$

$$= \frac{3.8\times 10^{26}}{4\times 3.14\times (7\times 10^8)^2}$$

$$S = 6.174\times 10^7 \text{ watt/m}^2$$

Q.3. Find the relation between electric field $\vec{E}$ and magnetic field $\vec{B}$ in a electromagnetic wave.

Sol. Let us consider a plane *EM* wave propagation in *x* direction.

Let the electric field vector is given by

$$\vec{E} = E_0 \cos(\beta x - \omega t) \qquad \{\beta\text{-propagation vector}\}$$

[The polarization of electric field in *y*-direction]

using the Faraday's law

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{E} = -\hat{K} E_0 \sin(\beta x - \omega t) \qquad [\hat{K} \text{ unit vector in } z \text{ direction}]$$

Therefore

$$\frac{\partial \vec{B}}{\partial t} = \hat{K} E_0 \sin(\beta x - \omega t)$$

$$\vec{B} = \hat{K} E_0 \frac{\beta}{\omega} \cos(\beta x - \omega t)$$

or

$$\vec{B} = \hat{K} B_0 \cos(\beta x - \omega t) \qquad \text{...(1)}$$

Where

$$B_0 = \frac{E_0 \beta}{\omega}$$

or

$$B_0 = \frac{E_0}{c} \qquad \text{...(2)}$$

eq. (1) is the required result. It can be sun that electric field in *y*-direction, magnetic field in *z* direction and wave is propagating in *x*-direction. We can also write

$$\vec{B} = \frac{\vec{\beta} \times \vec{E}}{\omega}$$

we are using β as propagation constant to avoid confusion in *K* and $\hat{k}$.

Q.4. Calculate the reflectance and transmittance, when light travelling in air falls normally on a glass surface ($n = 1.5$).

Sol. Here $n_1 = 1, n_2 = 1.5$

So,

$$R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 = \left(\frac{0.5}{2.5}\right)^2 = \frac{1}{25}$$

Transmittance

$$T = 1 - R = 1 - \frac{1}{25} = \frac{24}{25}$$

Thus only 4% of the incident light energy gets reflected and 96% is transmitted. This is the reason we can see through the window glass so clearly.

Q.5. Write down the boundary conditions by electromagnetic fields at the interface between two media of different permeabilities and permittivities.

Ans. The boundary condition for electromagnetic field at the interface between two media are as follows :

1. The normal component of the electric displacement is discontinues by an amount equal to the free surface density of charge at the boundary *i.e.*,

$$D_{1n} - D_{2n} = \sigma$$

$D_{1n} \rightarrow$ electric displacement in medium 1

$D_{2n} \rightarrow$ electric displacement in medium 2

2. The normal component of the magnetic induction is continuous across a surface of discontinuity *i.e.*,

$$B_{1n} - B_{2n} = 0$$

3. The tangential component of magnetic intensity is discontinuous by an amount equal to the free surface current density J. *i.e.*,

$$H_{1t} - H_{2t} = J$$

4. The tangential component of $\vec{E}$ is continuous across a surface of discontinuity *i.e.*,

$$E_{1t} - E_{2t} = 0$$

SECTION-C LONG ANSWER TYPE QUESTIONS

Q.1. Derive the electromagnetic wave equation from maxwell field equations in free space.

Ans. We know that maxwell's equations are

$$\left.\begin{aligned} \vec{\nabla}\cdot\vec{D} &= \rho && \text{......Gauss's Law} \\ \vec{\nabla}\cdot\vec{B} &= 0 && \text{......No Name} \\ \vec{\nabla}\times\vec{H} &= \vec{J} + \frac{\partial\vec{D}}{\partial t} && \text{......Ampere's Law} \\ \nabla\times E &= -\frac{\partial B}{\partial t} && \text{......Faraday's Law} \end{aligned}\right\} \quad ...(1)$$

along with constitutive relations

$$\left.\begin{aligned} \vec{D} &= \in\vec{E} \\ \vec{B} &= \mu\vec{H} \end{aligned}\right\}$$

and ohm's law $\qquad \vec{J} = \sigma\vec{E}$

In free space $\qquad \delta = 0 \qquad \sigma = 0$

$$\in_r = 1 \qquad \mu_r = 1$$

So Maxwell's equations reduces to

$$\left.\begin{array}{ll}\vec{\nabla}\cdot\vec{E}=0 & \text{......}(a)\\ \vec{\nabla}\cdot\vec{H}=0 & \text{......}(b)\\ \vec{\nabla}\times\vec{H}=\epsilon_0\dfrac{\partial\vec{E}}{\partial t} & \text{......}(c)\\ \vec{\nabla}\times\vec{E}=-\mu_0\dfrac{\partial\vec{H}}{\partial t} & \text{......}(d)\end{array}\right\}\quad\text{...(2)}$$

Now we take the curl of equation 2 (c), then

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H})=\epsilon_0\,\vec{\nabla}\times\frac{\partial\vec{E}}{\partial t}$$

or
$$[\vec{\nabla}\,(\vec{\nabla}\cdot\vec{H})-\nabla^2 H]=\epsilon_0\,\frac{\partial}{\partial t}\,(\nabla\times E)$$

[using $\vec{A}\times(\vec{B}\times\vec{C})=\vec{B}\,(\vec{A}\cdot\vec{C})-\vec{C}\,(\vec{A}\cdot\vec{B})$]

from eq. 2 (b) and 2 (d)

$$\vec{\nabla}\cdot\vec{H}=0 \text{ and } \vec{\nabla}\times\vec{E}=-\mu_0\,\frac{\partial H}{\partial t}$$

So, eq. (3) reduces to

$$\nabla^2\vec{H}-\frac{1}{\mu_0\epsilon_0}\frac{\partial^2\vec{H}}{\partial t^2}=0$$

or
$$\nabla^2\vec{H}-\frac{1}{C^2}\frac{\partial^2 H}{\partial t^2}=0\qquad\left[\mu_0\epsilon_0=\frac{1}{C^2}\right]$$

or
$$\nabla^2\vec{H}=\frac{1}{C^2}\frac{\partial^2 H}{\partial t^2}\qquad\text{...(A)}$$

Similarly taking the curl of eq. 2(d), then we get

$$\nabla^2\vec{E}=\frac{1}{C^2}\frac{\partial^2 E}{\partial t^2}\qquad\text{...(B)}$$

eq. (A) and (B) are identical to the wave equation

$$\nabla^2\psi=\frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}\qquad\text{...(C)}$$

This is the equation of wave travelling with speed v so, we conclude that field vectors $\vec{E}$ and $\vec{H}$ are propagated in free space as waves at a speed

$$C = \frac{1}{\sqrt{\mu_0 \in_0}} \sqrt{\left(\frac{4\pi}{4\pi_0\mu_0}\right)}$$

$$= \sqrt{9\times10^9 \times 10^7}$$

$$C = 3\times10^8 \text{ m/s}$$

i.e., the velocity of light.

Q.2. Obtain the poynting's theorem for the conservation of energy in an electromagnetic field. Discuss significance of poynting's vector.

Ans. We know that Ampere's law and Faraday's law in differential forms

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t} \quad ...(1)$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad ...(2)$$

If we take the scalar product of eq. (1) with $\vec{E}$ and of eq. (2) with $(-\vec{H})$, we get

$$\vec{E}\cdot\vec{\nabla}\times\vec{H} = \vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\partial\vec{D}}{\partial t} \quad ...(3)$$

and

$$-\vec{H}\cdot\vec{\nabla}\times\vec{E} = \vec{H}\cdot\frac{\partial\vec{B}}{\partial t} \quad ...(4)$$

adding eq. (3) and (4) we get

$$-\vec{H}\cdot\vec{\nabla}\times\vec{E} + \vec{E}\cdot\vec{\nabla}\times\vec{H} = \vec{J}\cdot\vec{E} + \left[\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t}\right]$$

using the vector identity

$$\vec{H}\cdot\vec{\nabla}\times\vec{E} - \vec{E}\cdot\vec{\nabla}\times\vec{H} = \nabla\cdot(\vec{E}\times\vec{H})$$

The above equation reduces to

$$-\nabla\cdot(\vec{E}\times\vec{H}) = \vec{J}\cdot\vec{E} + \left(\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} + \vec{H}\cdot\frac{\partial\vec{B}}{\partial t}\right) \quad ...(5)$$

Now $\vec{E}\cdot\frac{\partial\vec{D}}{\partial t} = \in_r\in_0 \vec{E}\,\frac{\partial\vec{E}}{\partial t} = \frac{1}{2}\in_r\in_0\frac{\partial}{\partial t}(\vec{E}\cdot\vec{E}) = \frac{1}{2}\frac{\partial}{\partial t}(\vec{E}\cdot\vec{D}) = \frac{1}{2}$

and $\vec{H}\cdot\frac{\partial\vec{B}}{\partial t} = \mu_r\mu_0\,\vec{H}\cdot\frac{\partial\vec{H}}{\partial t} = \frac{1}{2}\mu_r\mu_0\frac{\partial}{\partial t}(\vec{H}\cdot\vec{H}) = \frac{1}{2}\frac{\partial}{\partial t}(\vec{H}\cdot\vec{B})$

eq. (5) reduces to

$$\vec{J}\cdot\vec{E}+\frac{1}{2}\frac{\partial}{\partial t}[\vec{E}\cdot\vec{D}+\vec{H}\cdot\vec{B}]+\vec{\nabla}\cdot(\vec{E}\times\vec{H})=0 \qquad ...(6)$$

each term in the above equation can be given same physical meaning, if it is multiplied by an element of volume $d\tau$ and integrated over a volume τ whose enclosing surface s. Thus the result is

$$\int_\tau \vec{J}\cdot\vec{E}\,d\tau+\int_\tau \frac{1}{2}\frac{\partial}{\partial t}(\vec{E}\cdot\vec{D}+\vec{H}\cdot\vec{B})\,d\tau+\int_\tau \vec{\nabla}\cdot(\vec{E}\times\vec{H})\,d\tau=0$$

But $$\int \vec{\nabla}\cdot(\vec{E}\times\vec{H})\,d\tau=\oint_s(\vec{E}\times\vec{H})\cdot\vec{da}$$

So, $$\int_\tau(\vec{J}\cdot\vec{E})\,d\tau+\int_\tau\frac{1}{2}\frac{\partial}{\partial t}(\vec{E}\cdot\vec{D}+\vec{H}\cdot\vec{B})\,d\tau+\int_s(\vec{E}\times\vec{H})\cdot da=0 \qquad ...(7)$$

Interpretation of $\int_\tau(\vec{J}\cdot\vec{E})\,d\tau$

According to Lorentz force law, the work done on a charge q is

$$\vec{F}\cdot\vec{dl}=q\,(\vec{E}+\vec{V}\times\vec{B})\cdot\vec{V}\,dt=q\,\vec{E}\cdot\vec{V}\,dt$$

Now, $q=\rho\,d\tau$ and $\rho V=J$. So the rate at which work is done on all the charges in volume τ. is

$$\frac{dw}{dt}=\int_\tau(\vec{E}\cdot\vec{J})\,d\tau$$

This represents the rate at which work is done by the field on the charges

eq. (7) can be written as

$$\frac{dw}{dt}=-\frac{1}{2}\int_\tau\frac{\partial}{\partial t}(\vec{E}\cdot\vec{D}+\vec{H}\cdot\vec{B})\,d\tau-\int(\vec{E}\times\vec{H})\cdot ds \qquad ...(8)$$

This is poynting's theorem. It is the work-energy theorem of electrodynamics.

Poynting theorem states that, "the work done on the charges by the electromagnetic force is equal to the decrease in energy stored in the field, less the energy that flowed out through the surface."

Poynting vector : The energy per unit time, per unit area transported by the field is called the poynting vector;

$$\vec{S}=\vec{E}\times\vec{H}$$

Specifically $\vec{S}\cdot\vec{da}$ is the energy per unit time crossing the infinitesimal surface $\vec{da}$-the energy flux.

we can write poynting theorem more compactly

$$\frac{dw}{dt} = -\frac{dU_{em}}{dt} - \oint \vec{S}\cdot\vec{da}$$

where U_{em} is the energy density of the fields

$$U_{em} = \frac{1}{2}(\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H})$$

$$= \frac{1}{2}(\epsilon_0 \cdot E^2 + \frac{1}{\mu_0}B^2)$$

Q.3. State maxwell's equation for the electromagnetic field and obtain the equation for $\vec{E}$ and $\vec{B}$ in homogeneous isotropic dielectric medium.

Ans.

Propagation of *EM* Waves in Isotropic Dielectrics

"A non conducting medium whose properties are same in all directions is called isotropic Dielectric."

The Maxwell's field equations are

$$\left.\begin{aligned} \vec{\nabla}\cdot\vec{D} &= \rho \\ \vec{\nabla}\cdot\vec{B} &= 0 \\ \vec{\nabla}\times\vec{H} &= \vec{J} + \frac{\partial\vec{D}}{\partial t} \\ \vec{\nabla}\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \end{aligned}\right\} \quad ...(1)$$

with

$$\left\{\begin{aligned} J &= \sigma E \\ B &= \mu H \\ D &= \epsilon E \end{aligned}\right\} \quad ...(2)$$

In an isotropic dielectric

$$\sigma = 0, \qquad \rho = 0$$

So, Maxwell's equation reduce to

$$\left.\begin{aligned} \vec{\nabla}\cdot\vec{E} &= 0 && ......(a) \\ \vec{\nabla}\cdot\vec{H} &= 0 && ......(b) \\ \vec{\nabla}\times\vec{H} &= \epsilon\frac{\partial\vec{E}}{\partial t} && ......(c) \\ \vec{\nabla}\times\vec{E} &= -\mu\frac{\partial H}{\partial t} && ......(d) \end{aligned}\right\} \quad ...(3)$$

We take the curl of eq. 3(c) then

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{H}) = \in \nabla\times\left(\frac{\partial E}{\partial t}\right)$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{H}) - \nabla^2\vec{H} = \in\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E}) \quad ...(4)$$

from eq. 3(b) and 3(d), eq. (4) reduces to

$$\nabla^2\vec{H} - \mu\in\frac{\partial^2\vec{H}}{\partial t^2} = 0 \quad ...(5)$$

i.e.,

$$\nabla^2\vec{H} - \frac{1}{v^2}\frac{\partial^2\vec{H}}{\partial t^2} = 0 \quad ...(6)$$

with

$$\mu\in = \frac{1}{v^2}$$

Similarly we get the wave equation for electric field

$$\nabla^2\vec{E} - \frac{1}{v^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0 \quad ...(7)$$

In general we can write eq. (6) and eq. (7) in the form

$$\nabla^2\psi - \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} = 0 \quad ...(8)$$

eq. (8) is a standard wave equation representing an unattenuated wave travelling at a speed v. The electric and magnetic field (E & H) propagates in isotropic dielectric as waves given by

$$\begin{Bmatrix} E \\ H \end{Bmatrix} = \begin{Bmatrix} E_0 \\ H_0 \end{Bmatrix} e^{-i(\omega t - \vec{k}\cdot\vec{r})} \quad ...(9)$$

at a speed

$$v = \frac{1}{\sqrt{\mu\in}} = \frac{1}{\sqrt{\in_r\mu_r\ \in_0\mu_0}}$$

$$v = \frac{c}{\sqrt{\in_r\mu_r}}$$

The speed of *EM* wave in isotropic dielectrics is less than the speed of *EM* waves in free space.

The refractive index is defined as

$$n = \frac{c}{v}$$

$$n = \frac{c}{c/\sqrt{\mu_r\ \in_r}} = \sqrt{\mu_r\in_r}$$

in a non magnetic medium $\mu_r = 1$

$$n = \sqrt{(\epsilon_r)} \quad \Rightarrow \quad n^2 = \epsilon_r \qquad ...(10)$$

The form of field vectors is given by eq. (9). This eq.
Suggests that we can replace

$$\nabla \to i\vec{K} \quad \text{and} \quad \frac{\partial}{\partial t} \to -i\omega$$

So in terms of these operators Maxwell's eq. reduces to

$$\left.\begin{aligned} \vec{K}\cdot\vec{E} &= 0 \quad ......(a) \\ \vec{K}\cdot\vec{H} &= 0 \quad ......(b) \\ -\vec{K}\times\vec{H} &= \omega\epsilon E \quad ......(c) \\ \vec{K}\times\vec{E} &= \omega\mu\epsilon \quad ......(d) \end{aligned}\right\} \qquad ...(11)$$

From this form of Maxwell's equation it is evident that in a plane electromagnetic wave propagating through isotropic dielectric.

(i) The vectors, $\vec{E}, \vec{H}$ and $\vec{K}$ are orthogonal *i.e.*, the electromagnetic wave is transverse in nature and in it the electric and magnetic vectors are also mutually orthogonal.

from 11 (a) dot product is zero, so $\vec{K}$ and $\vec{E}$ are perpendicular

from 11 (b) $\vec{K}$ and $\vec{H}$ are mutually perpendicular

from 11 (c) $\vec{E}$ is perpendicular to both $\vec{K}$ & $\vec{H}$

from 11 (d) $\vec{H}$ is perpendicular to both $\vec{K}$ & $\vec{E}$

(ii) The vectors $\vec{E}$ and $\vec{H}$ are in phase and their magnitudes are related to each other by the relation

$$\left|\frac{\vec{E}}{\vec{H}}\right| = \frac{E_0}{H_0} = \sqrt{\left(\frac{\mu_r}{\epsilon_r}\right)} Z_0 = Z$$

where Z is called the impedance of medium and Z_0 is the impedance of free space

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}$$

(iii) The direction of flow of energy is the direction in which the wave propagates and the poynting vector is $\left(\frac{n}{\mu_r}\right)$ times of the poynting vector if the same wave propagates through free space.

(iv) The electromagnetic energy density is equally distributed in electric field and magnetic field. This is because electromagnetic energy is transmitted with the same velocity with which the fields do.

$$\frac{U_e}{U_m} = \frac{\frac{1}{2} \in E^2}{\frac{1}{2}\mu H^2} = \frac{\in}{\mu}\left(\frac{E^2}{H^2}\right) = \frac{\in}{\mu}(Z^2)$$

$$= \frac{\in}{\mu} \times \frac{\mu}{\in} = 1 \quad \left(\left|\frac{E}{H}\right| = Z\right)$$

$$U_e = U_m$$

Q.4. Assuming that the electric vector of an electromagnetic wave is given by

$$\vec{E} = E_0 e^{-i(\omega t - \vec{k}\cdot\vec{r})}$$

and in crossing a boundary the tangential component of electric intensity is continuous, prove that various laws of reflection and refraction.

Ans. Let us consider two medium have permittivity and permeability $(\in_1, \mu_1)$ and $(\in_2, \mu_2)$ respectively separated by a plane at $z = 0$ shown in fig (1),

If the plane wave with vector K_I in the $x - z$ plane and frequency ω, is incident from medium –1.

K_R and K_J are the wave vector of reflected and transmitted wave respectively. The frequencies of reflected and transmitted waves are ω_R and ω_T respectively.

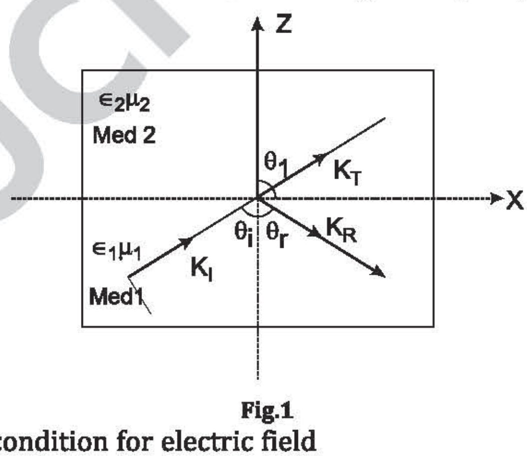


Fig.1

Applying the boundary condition for electric field

$$[E_I]_t + [E_R]_t = [E_T]_t$$

This yields $\quad (E_I)_x + (E_R)_x = (E_T)_x \quad$ ...(1)

by using plane wave equation

$$\vec{E} = \vec{E_0}\, e^{-i(\omega t - \vec{k}\cdot\vec{r})}$$

eq. (1) reduces to

$$(E_{I_0})_x\, e^{-i(\omega_I t - \vec{k_I}\cdot\vec{r})} + (E_{R_0})_x e^{-i(\omega_I t - \vec{k_R}\cdot\vec{r})} = (E_{T_0})_x\, e^{-i(\omega_T t - \vec{k_T}\cdot\vec{r})}$$

for all values of x, y and t.

The equation (2) can only be satisfied if the time and space varying components of the phases are equal. So equating the time varying components of the phases in eq. (2), we get

$$\omega_I t = \omega_R t = \omega_T = t$$

i.e., $$\omega_I = \omega_R = \omega_T = \omega \qquad \text{(say)} \quad ...(3)$$

Above equation shows that the frequency of the wave remains unchanged by reflection and refraction.

Now equating the space varying components we get

$$(\vec{K}_I\cdot\vec{r})_{z=0} = (\vec{K}_R\cdot\vec{r})_{z=0} = (\vec{K}_T\cdot\vec{r})_{z=0} \qquad ...(4)$$

The incident beam is in $x-z$ plane, n_y is zero. It then follows that y–terms in the other expression of eq. (4) are also zero *i.e.*, the reflected and transmitted waves are in the same plane as the incident ray and the normal.

from eq. (4) we have

$$\vec{K}_I\cdot\vec{r} = K_I\,(x\sin\theta_1 + Z\cos\theta_I) \qquad ...(5)$$

$$\vec{K}_R\cdot\vec{r} = K_R\,(x\sin\theta_R - Z\cos\theta_R) \qquad ...(6)$$

$$\vec{K}_T\cdot\vec{r} = K_T\,(x\sin\theta_T + Z\cos\theta_T) \qquad ...(7)$$

In the light of eq. (4), eq. (5) and (6) yield

$$K_I\sin\theta_1 = K_R\sin\theta_R$$

but as $$\omega_I = \omega_R \text{ and } v_I = v_R$$

$$K_I = K_R$$

so $$\sin\theta_I = \sin\theta_R$$

i.e., $$\theta_I = \theta_R$$

In the light of eq. (3), eq. (5) and (7) yield

$$K_I\sin\theta_I = K_T\sin\theta_T$$

$$\frac{\omega}{v_I}\sin\theta_I = \frac{\omega}{v_T}\sin\theta_T \qquad \left\{k = \frac{\omega}{v}\right\}$$

or $$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_T}\sin\theta_T$$

$$n_1\sin\theta_I = n_2\sin\theta_T$$

This is snell's Law.

Q.5. A plane electromagnetic wave is incident on a dielectric surface. Find the amplitudes of the reflected and refracted wave and discuss their phase change [Fresnel formulae].

Ans.

Fresnel Formulae

The formulae relating the amplitudes of the reflected and transmitted electromagnetic waves with that of incident on when the boundary is between two dielectrics are called Fresnel Formulae. These are contained in the boundary conditions *i.e.*,

$$(D_i)_n + (D_R)_n = (D_T)_n \quad ...(1)$$

$$(B_i)_n + (B_R)_n = (B_T)_n \quad ...(2)$$

$$(E_i)_t + (E_R)_t = (E_T)_t \quad ...(3)$$

and

$$(H_i)_t + (H_R)_t = (H_T)_t \quad ...(4)$$

The conditions (1) and (2) when coupled with Snell's law yield no information not included in the conditions (3) and (4). So it necessary to consider only conditions (3) and (4).

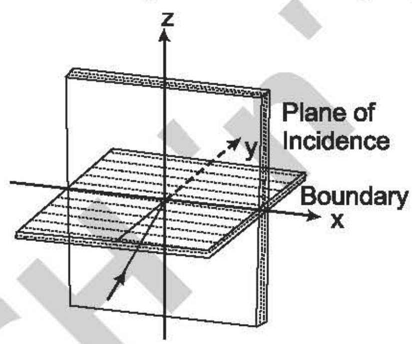


Fig. 1

Now to derive the desired formulae we consider a plane E.M.W. is $x - z$ plane (plane of incidence) incident on a plane boundary (here $x - y$ plane) and consider it as a superposition two waves one with the electric vector parallel to the plane of incidence and the other with electric vector perpendicular to the plane of incidence. Therefore it is sufficient to consider these two cases separately. The general result may be obtained from the appropriate linear combination of the two cases.

Case I : E–parallel to the Plane of Incidence.

The situation is shown in fig.2. The electric and propagation vectors in two media are indicated. The directions of **H** vector are chosen so as to give a positive flow of energy in the direction of wave vectors. In this situation the magnetic vectors are all parallel to the boundary surface.

and $\begin{bmatrix} (H_i)_t = H_i \\ (H_R)_t = H_R \\ (H_T)_t = H_r \end{bmatrix}$ and $\begin{bmatrix} (E_i)_t = E_i \cos\theta_i \\ (E_R)_t = -E_R \cos\theta_R \\ (E_T)_t = E_T \cos\theta_T \end{bmatrix}$

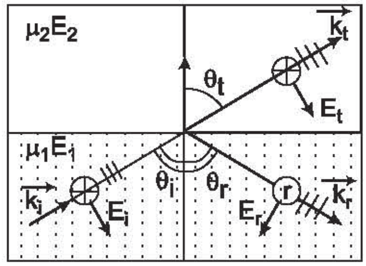


Fig.2

So boundary conditions (3) and (4) reduce to

$$E_i \cos\theta_i - E_R \cos\theta_R = E_T \cos\theta_T \quad ...(5)$$

and $$H_i + H_R = H_T \quad ...(6)$$

In equations (5) and (6) we have omitted the zero subcripts on E and H, it being understood that the phases now cancel and equations are relations between amplitudes.

Now as $\theta_i = \theta_R$ and $H = (E/Z) = (n/\mu_r Z_0)E$ $\left\{\text{as } Z = \frac{\mu_r}{n} Z_0\right\}$

i.e., $H = (n/Z_0)E$ (as $\mu_r = 1$ for dielectrics)

So equations (5) and (6) reduce to

$$E_i \cos\theta_i - E_R \cos\theta_R = E_T \cos\theta_r \quad ...(7)$$

and $$n_1 E_i + n_1 E_R = n_2 E_R \quad ...(8)$$

The interest lies in the fraction of the incident amplitudes which are reflected and transmitted.

So eliminating E_r from equation (7) with the help of (8) we get

$$(E_i - E_R)\cos\theta_i = \frac{n_1}{n_2}(E_i + E_R)\cos\theta_T$$

i.e., $$\left(\frac{E_R}{E_i}\right)_{\|} = \frac{\frac{n_2}{n_1}\cos\theta_i - \cos\theta_T}{\frac{n_2}{n_2}\cos\theta_i + \cos\theta_T} \quad ...(A)$$

or $$\left(\frac{E_R}{E_i}\right)_{\|} = \frac{\left(\frac{\sin\theta_i}{\sin\theta_T}\right)\cos\theta_i - \cos\theta_T}{\left(\frac{\sin\theta_i}{\sin\theta_T}\right)\cos\theta_i + \cos\theta_T}$$ (as $n_1 \sin\theta_i = n_2 \sin\theta_T$)

i.e., $$\left(\frac{E_R}{E_i}\right)_{\|} = \frac{\sin\theta_i \cos\theta_i - \sin\theta_T \cos\theta_T}{\sin\theta_i \cos\theta_i + \sin\theta_T \cos\theta_T} = \frac{\sin 2\theta_i - \sin 2\theta_T}{\sin 2\theta_i + \sin 2\theta_T}$$

i.e., $$\left(\frac{E_R}{E_i}\right)_{\|} = \frac{\tan(\theta_i - \theta_T)}{\tan(\theta_i + \theta_T)} \quad ...(B)$$

Similarly eliminating E_R from eq. (7) with the help of (8) we get

$$E_i \cos\theta_i - \left(\frac{n_2}{n_1} E_T - E_i\right)\cos\theta_i = E_T \cos\theta_T$$

i.e.,
$$\left(\frac{E_T}{E_i}\right)_{\parallel} = \frac{2\cos\theta_i}{\left(\frac{n_2}{n_1}\cos\theta_i + \cos\theta_T\right)} \qquad ...(C)$$

or
$$\left(\frac{E_T}{E_i}\right)_{\parallel} = \frac{2\cos\theta_i}{\left(\frac{\sin\theta_i}{\sin\theta_T}\cos\theta_i + \cos\theta_T\right)}$$

or
$$\left(\frac{E_T}{E_i}\right)_{\parallel} = \frac{2\cos\theta_i \sin\theta_T}{\sin\theta_i \cos\theta_i + \sin\theta_T \cos\theta_T}$$

i.e.,
$$\left(\frac{E_T}{E_i}\right)_{\parallel} = \frac{2\cos\theta_i \sin\theta_T}{\sin(\theta_i + \theta_T)\cos(\theta_i - \theta_T)} \qquad ...(D)$$

Case II : E-perpendicular to the Plane of Incidence.

The situation is shown in fig. 3. The magnetic field vector and the propagation vector are indicated. The electric vectors all directed into the plane of the figure.

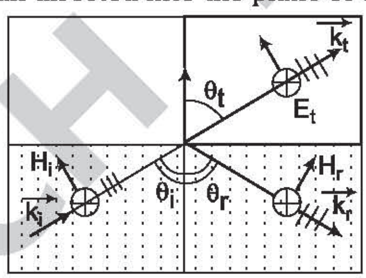


Fig. 3

Since the electric vector are all parallel to the boundary surface

$$\left.\begin{aligned}(E_i)_t &= E_i\\ (E_R)_t &= E_R\\ (E_T)_t &= E_T\end{aligned}\right\}$$

and
$$\left\{\begin{aligned}(H_i)_t &= -H_i \cos\theta_i\\ (H_R)_t &= H_R \cos\theta_R\\ (H_T)_t &= -H_T \cos\theta_T\end{aligned}\right.$$

So boundary conditions (3) and (4) reduce to

$$E_i + E_R = E_r \qquad ...(9)$$

And
$$H_i \cos\theta_i - H_R \cos\theta_R = H_T \cos\theta_T \qquad ...(10)$$

Now as $\theta_i = \theta_R$ and $H = (E/Z) = (nE/Z_0)$

So equation (10) reduces to

$$n_1 E_T \cos\theta_i - n_1 E_R \cos\theta_i = n_2 E_T \cos\theta_T \qquad \text{...(11)}$$

Now eliminating E_T from equation (11) with the help of (9) we get

$$(E_i - E_R) n_1 \cos\theta_i = n_2 \cos\theta_T (E_i + E_R)$$

i.e.,

$$\left(\frac{E_R}{E_i}\right)_\perp = \frac{\cos\theta_i - \dfrac{n_2}{n_1}\cos\theta_T}{\cos\theta_i + \dfrac{n_2}{n_1}\cos\theta_T} \qquad \text{...(E)}$$

or

$$\left(\frac{E_R}{E_i}\right)_\perp = \frac{\cos\theta_i - \dfrac{\sin\theta_i}{\sin\theta_T}\cos\theta_T}{\cos\theta_i + \dfrac{\sin\theta_i}{\sin\theta_T}\cos\theta_T} \qquad (\text{as } n_1 \sin\theta_i = n_2 \sin\theta_T)$$

or

$$\left(\frac{E_R}{E_i}\right)_\perp = \frac{\sin\theta_T \cos\theta_i - \cos\theta_T \sin\theta_i}{\sin\theta_T \cos\theta_i + \cos\theta_T \sin\theta_i}$$

i.e.,

$$\left(\frac{E_R}{E_i}\right)_\perp = -\frac{\sin(\theta_i - \theta_T)}{\sin(\theta_i + \theta_T)} \qquad \text{...(F)}$$

Similarly eliminating E_R from equation (11) with the help of (9) we get

$$n_1 E_1 \cos\theta_i - n(E_T - E_i)\cos\theta_i = n_2 E_T \cos\theta_T$$

i.e.,

$$\left(\frac{E_T}{E_i}\right)_\perp = \frac{2\cos\theta_i}{\cos\theta_i + \dfrac{n_2}{n_1}\cos\theta_T} \qquad \text{...(G)}$$

or

$$\left(\frac{E_T}{E_1}\right)_\perp = \frac{2\cos\theta_i}{\cos\theta_i + \dfrac{\sin\theta_i}{\sin\theta_T}\cos\theta_T}$$

or

$$\left(\frac{E_T}{E_i}\right)_\perp = \frac{2\cos\theta_i \sin\theta_T}{\cos\theta_i \sin\theta_T + \sin\theta_i \cos\theta_T}$$

i.e.,

$$\left(\frac{E_T}{E_i}\right)_\perp = \frac{2\cos\theta_i \sin\theta_T}{\sin(\theta_i + \theta_T)} \qquad \text{...(H)}$$

Discussion :

Equation (A), (B), (C) and (D) are the desired results known as Fresnel formulae. These equations are very general as these hold good for all angles of incidence and all types of dielectric interfaces. The interesting coefficients (E_R/E_i) and $(E_R/E_i)_\perp$ are plotted against the angle of incidence for a dielectric-dielectric interface in fig.4

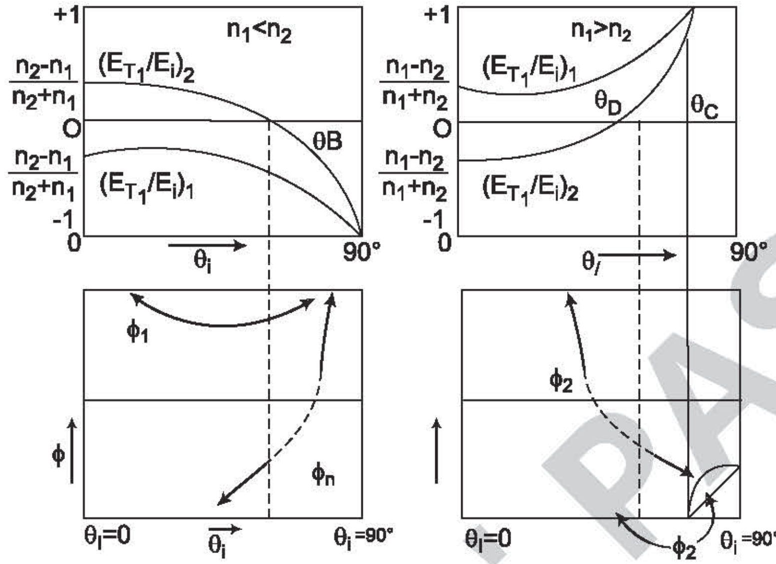


Fig.4

From the above curves it is clear that if E.M.W. incident on the boundary from a rare medium $(i.e., n_1 < n_2)$

(i) For all angles of incidence there is a phase change of π on reflection for E.M.W. whose vibrations are perpendicular to the plane of incidence as $(E_R/E_i)_\perp$ is always negative.

(ii) For all angles of incidence less than the angle θ_B * there is no change in phase of $(E_R)_1$ as $(E_R/E_t)_\perp$ is positive till $\theta_i < \theta_B$: for angles of incidence greater than $\theta_B, (E_r/E_i)_1$ is negative so there is a change of phase π.

(iii) At $\theta_i = \theta_B.(E_R/E_i)_1 = 0 i.e., (E_R)_1 = 0$. This in turn implies that if the angle of incidence is equal to the angle θ_B (called Brewster's angle), the reflected light has only $E_\perp$ component. *i.e.*, it is plane polarised.

However if the E.M.W. are incident on the boundary from a denser medium $(i.e., n_1 > n_2)$.

(i) For all angles of incidence less than θ_B $(E_R/E_i)_1$ is negative while $(E_R E_i)_\perp$ is positive; thus in turn implies that in reflection $(E_R)_1$ suffers a phase change of π w.r.t. $(E_i)_1$ while the $(E_R)_\perp$ in phase with the incident $(E_i)_\perp$.

(ii) For anlges $\theta_B < \theta_i < \theta_c$ * $(E_R E_i)_\perp$ and $(E_R/E_i)_1$ both are positive so the $(E_R)_\perp$ and $(E_R)_1$ components are in phase with the incident $(E_i)_\perp$ and $(E_i)_1$ components respectively.

(iii) At angles $\theta_i > \theta_c$ the phase of both $(E_R)_\perp$ and $(E_R)_1$ are different from that of respective incident components of E_i and as $(E_R)_1$ and $(E_R)_\perp$ equals $(E_i)_1$ and $(E_i)_\perp$ respectively, the total incident light is reflected back into the same medium.

PART-B : PHYSICAL OPTICS & LASERS

UNIT-V

Interference

SECTION-A VERY SHORT ANSWER TYPE QUESTIONS

Q.1. Is it possible to observe interference fringes with light emanating from two independent sources? If not, why?

Ans. No, it is not possible to observe interference fringes with two independent sources, because the two sources will not be coherent. Sources obtained from single source will give the interference pattern.

Q.2. What are coherent sources of light?

Ans. Coherent sources of light are those which produces light waves having sharply defined phase difference that remains constant with time.

Q.3. Name three methods of producing coherent sources.

Ans. Three methods of producing coherent sources are :

(i) Fresnel's Biprism

(ii) Lloyd's Mirror

(iii) Fresnel's Double Mirror

Q.4. What is the effect on the fringe width in the biprism experiment if the distance between the slit and the eyepiece is increased?

Ans. The fringe width is given by

$$w = \frac{D\lambda}{2d}$$

So w will increase if D is increased *i.e.*, the distance between the slit and eyepiece.

Q.5. Write the formula for determining wavelength (λ) in Newton's Rings experiment.

Ans. The formula for determining λ in Newton's Rings experiment is given by

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR}$$

where D_{n+p} – Diameter of $(n+p)^{th}$ ring

D_n – Diameter of n^{th} ring

R – Radius of curvature of lens

P – Integer

Q.6. What is the effect of increasing the distance between lens and plate?

Ans. As the distance between the lens and the plate is increased, the order of the ring at a given point increases. The rings, therefore, come closer and closer, until they can no longer be separately observed.

Q.7. Write the formula for the determination of refractive index of transparent liquid by Newton's Rings.

Ans. The formula for the determination of refractive index of transparent liquid by Newton's Rings is given by

$$\mu = \frac{(D_{n+p}^2 - D_n^2)_{\text{air}}}{(D_{n+p}^2 - D_n^2)_{\text{liquid}}}$$

Q.8. Can colours of thin films be seen by a point source of light?

Ans. The colours of thin films can not be seen by a point source of light. An extended source is necessary for colours in thin films.

Q.9. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. Calculate the maximum and minimum intensities in the resulting beam.

Ans. The amplitudes corresponding to these intensities are

$$a_1 = \sqrt{I} \qquad a_2 = \sqrt{4I} = 2\sqrt{I}$$

$$\text{maximum intensity} = (a_1 + a_2)^2 = (\sqrt{I} + 2\sqrt{I})^2$$

$$= (3\sqrt{I})^2 = 9I$$

$$\text{minimum intensity} = (a_1 \sim a_2)^2 = (2\sqrt{I} - \sqrt{I})^2$$

$$= (\sqrt{I})^2 = I$$

Q.10. Does energy remains conserved in interference?

Ans. Yes, energy remains conserved in interference only the redistribution of energy takes place.

Q.11. Why are circular fringes (rings) obtained in Newton's arrangements?

Ans. Each fringe is the locus of points of equal thickness of the air film, and these loci are concentric circles with point of contact as centre.

Q.12. What is an interferometer?

Ans. A interferometer is an interference-based device used to measure wavelengths of spectral lines and wavelength-differences between fine-structure components of spectral lines.

Q.13. What is the difference between interferometer and etalon?

Ans. In the interferometer, the distance between the two silvered plates is variable, while in an etalon the plates are held rigidly fixed by invar spacers.

Q.14. Write formula for determine the thickness of mica plate by michelson's interferometer.

Sol. The thickness of mica sheet in Michelson's Interferometer is given by

$$t = \frac{x}{\mu - 1}$$

Q.15. Calculate the visibility of the fringes for a reflection of 80% in multiple beam interferometry.

Sol. The visibility of fringes, V in terms of reflectivity R is given by

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2R}{1 + R^2}$$

Here $R = 80\% = 0.8$

$\therefore$ $V = \dfrac{2 \times 0.8}{1 + 0.64} = 0.976$

SECTION-B SHORT ANSWER TYPE QUESTIONS

Q.1. What do you mean by interference of light?

Ans.

Interference of Light

When two waves of the same frequency travel in approximately the same direction and have a phase difference that remains constant with time, the resultant intensity of light is not distributed uniformly in space. "The non uniform distribution of the light due to the superposition of two waves is called Interference" at some points intensity is maximum and the interference is called the "Constructive Interference", at some other point the intensity is a minimum and the interference at these points is called "Destructive Interference".

Q.2. What are coherent sources? Why the two sources must be coherent for sustained interference fringes?

Ans.

Coherent Sources

If two interfering sources emits wave so that they have the constant phase difference between them, then the two sources are called coherent sources.

To obtained a sustained interference pattern the two sources must be coherent. It the phase difference between two sources does not remains constant then the places of maxima and minima shifts and sustained interference is not possible for phase difference to remain the same, the two source must be derived from the same source.

Q.3. Two coherent sources of intensity ratio α interfere. Prove that in the interference pattern.

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2\sqrt{\alpha}}{1 + \alpha}$$

Ans. Let I_1 and I_2 be intensities, and a_1 & a_2 the amplitudes of the disturbances from the two coherent sources. Since intensity is proportional to the square of the amplitude, we can write

$$\frac{a_1}{a_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\alpha}$$

The maximum and minimum intensities in the interference pattern are given by

$$I_{\max} = (a_1 + a_2)^2 \quad \text{and } I_{\min} = (a_1 - a_2)^2$$

$$\therefore \quad \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{(a_1 + a_2)^2 - (a_1 - a_2)^2}{(a_1 + a_2)^2 + (a_1 - a_2)^2}$$

$$= \frac{2a_1 a_2}{a_1^2 + a_2^2} = \frac{2a_1/a_2}{1 + (a_1^2/a_2^2)}$$

$$\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{2\sqrt{\alpha}}{1 + \alpha}$$

Proved.

Q.4. Explain Temporal coherence.

Ans.

Temporal Coherence

It is the criteria of coherence that depends upon time. Since an ideal source of light emits a sinusoidal function of time. It has a constant amplitude of vibration at any point while its phase varies linearly with time. But in practice no source is ideal. So we can not related the phase at a point to another point in wave train. Therefore light from conventional sources is characterized by two important parameters.

1. **Coherence time :** It is the average time during which the wave remains sinusoidal and the phase of the wave packet can be predicted reliably.

2. **Coherence Length :** It is the length of wave packet over which it may be assumed to be sinusoidal and has predictable phase.

The average time interval for which the field remains sinusoidal is called temporal coherence.

Q.5. Explain spatial coherence.

Ans.

Spatial Coherence

If there is a definite phase relationship between the radiation field at different points in space, then there will be high coherence between the points, which is called "spatial coherence."

Let us consider a source S emitting waves and two points A and B on a line joining them with S as shown in fig. The phase relationship between these points depends on temporal coherence and on the distance AB. If AB is less then the coherence length L *i.e.*,

$$AB << L$$

Then there exists a definite phase relationship between A and B. If we now consider two equidistant point A and C from S, then the waves will reach A and C in exactly the same phase. Thus the points A and C have perfect spatial coherence.

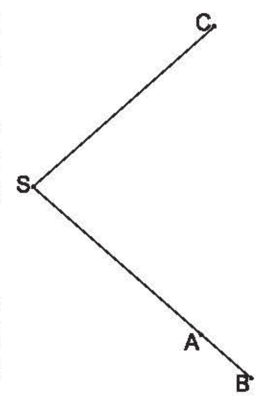


Fig.

Q.6. Discuss the conditions for interference of light.

Ans. Conditions for Sustained Interference :

1. The two interfering waves should be coherent *i.e.*, the phase difference between them must remain constant.
2. The two waves should have same frequency or wavelength.
3. If the interfering waves are polarised, they must be in the same state of polarisation.

Conditions for Good Contrast :

1. The separation between the light sources should be as small as possible.
2. The amplitudes of interfering waves should be equal or at least very nearly equal.
3. The distance of the screen from two sources should be large.
4. The two source must be narrow.

Q.7. How can coherent sources be produced in practice.?

Ans. If the two sources are derived from a single source by some device, then any phase change in one is simultaneously accompanied by the same phase change in the other. Thus, the phase difference between the two sources remains constant.

Some devices for creating coherent sources of light are :

(i) Young's Double mirror
(ii) Llyod's mirror
(iii) Fresnel's Double mirror
(iv) Fresnel's Biprism
(v) Michelson's Interferometer
(vi) The Biplates method.

Q.8. Describe Fresnel's biprism.

Ans.

Fresnel's Biprism

The biprism is a device to obtain two coherent sources to produced sustained interference. It is a combination of two prism (Biprism) of very small refracting angles, placed base to base. In practice, the biprism is made from a single plate by grinding and polishing, so that it is a single prism with one of its angles about 179° and the other two about 30′ (thirty minutes) each.

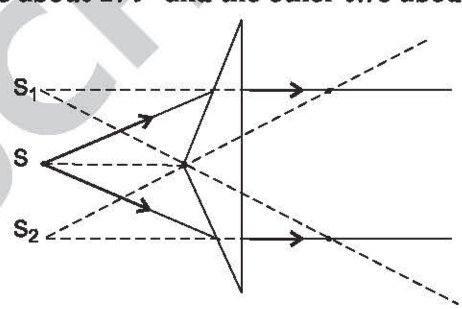


Fig. : Formation of two virtual sources from one source

Q.9. What is the difference between Biprism and Lloyd mirror fringes?

Ans. **Difference between Biprism and Lloyd's Mirror Fringes**

The following are the main points of difference between the Biprism and Lloyd's mirror fringes.

1. In Biprism the complete pattern of fringes is obtained while in Lloyd's mirror only a few fringes on one side of central fringe are visible. The central fringe itself being invisible.
2. In Biprism the central fringe is bright, while in Lloyd's mirror it is dark.
3. The central fringe in biprism is less sharp than that in Lloyd's mirror.

Q.10. A thin mica sheet $(\mu = 1.6)$ of 7 microns thickness introduced in the path of one of the interfering beams in a biprism arrangement shifts the central fringe to a position normally occupied by the 7th bright fringe from the centre. Find the wavelength of light used.

Sol. The shift is given by

$$x_0 = \frac{w}{\lambda}(\mu - 1)t$$

Here w is fringe width, μ-refractive index,
t –thickness of mica sheet, λ – wavelength

So that
$$\lambda = \frac{w}{x_0}(\mu - 1)t$$

Here
$$x_0 = 7w, \mu = 1.6 \text{ and } t = 7 \text{ microns} = 7\times10^{-6}\text{ m}$$

$$\lambda = \frac{w}{7w}(1.6 - 1)\times 7\times10^{-6}$$

$$= \frac{0.6\times7\times10^{-6}}{7}$$

$$\lambda = 6\times10^{-7}\text{ m} = 6000\text{Å}$$

Q.11. Explain, why different colours are exhibited by white light reflected from thin film.

Ans. When a thin film of oil on water, or a soap bubble, exposed to an extended source of white light, is observed under reflected light, brilliant colours are seen in the film or the bubble. These colours arises due to the interference of light waves reflected from the top and bottom surfaces of the film.

If the film is of uniform thickness everywhere and incident light is parallel, the path difference at each point of the film will be the same and the entire film will have uniform colouration.

Q.12. Account for the perfect blackness of the central spot in Newton's Ring system.

Ans. Newton's rings in reflected light are formed by interference between the ray (1) reflected directly from the upper surface of the air-film and the rays (2), (3) etc, which are obtained after internal reflections. These rays are shown in fig (1). Near about the point of contact, the thickness of the air film is almost zero and hence no path difference is introduced between the interfering rays. But the ray (2) reflected from the lower surface of the film suffers a phase change of π, while the ray (1) reflected from the upper does not suffer such change. Thus, the two interfering waves at the center are opposite in phase and destroy each other. Since the amplitude of (2) is less than ray (1) the destruction is not complete. But the sum of amplitudes of (2), (3) and (4) etc. which are all in phase is exactly equal to the amplitude of (1). Hence complete destructive interference is produced and the center of the ring system is perfectly black.

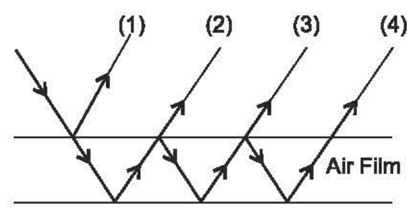


Fig.

Q.13. Write a short note on interference filter.

Ans. **Interference Filter**

An interference filter is an optical system that will transmit a very narrow range of wavelengths and thus provides a monochromatic beam of light. So when a parallel beam of white light is incident normally on a pair of plane parallel plates silvered on the inner surfaces, interference occurs for all the monochromatic components of such light. The transmitted beam, when dispersed by an auxillary spectroscope, shows a series of bright fringes in the spectrum, each formed by a wavelength some what different from the next. The maxima occur at a wavelength given by

$$2d = n\lambda \qquad \text{or} \qquad \lambda = \frac{2d}{n}$$

where d is the separation between the plates and n is a whole number. On either side of these maxima the intensity drops sharply. As d is reduced, the transmitted wavelength become widely space.

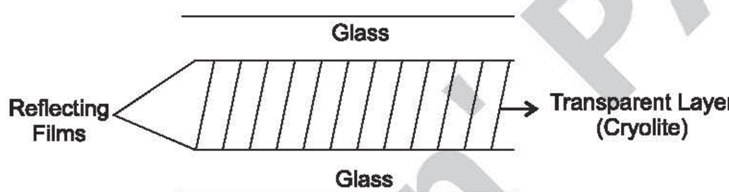


Fig. : Interference Filter

Q.15. What method is used in michelson interferometer to get coherent sources? can you observe circular fringes with white light in michelson interferometer?

Ans. Divison of amplitude method is used to coherent sources in michelson interferometer. The superposition of these beams leads to interference and produces interference fringes. If monochromatic light is replaced by white light and if the thickness of the film is small, a few curved and coloured localised fringes are obtained. "No circular fringes are observed with white light in michelson interferometer.

Q.16. Light of wavelength 6000 $\mathring{A}$ falls normally on a thin wedge shaped film of refractive index 1.39, forming fringes that are 2mm apart. Find the angle of wedge.

Sol. The spacing of fringes is given by

$$w = \frac{\lambda}{2\mu\theta}$$

where λ – wavelength, μ – refractive index

θ – angle of wedge (θ is small)

So,

$$w = \frac{\lambda}{2\mu\theta}$$

$$\theta = \frac{\lambda}{2\mu w} = \frac{6000\times10^{10}}{2\times1.39\times2\times10^{-3}}$$

$$\theta = 10.79\times10^{-5} \text{ radian}$$

Q.17. In an experiment with michelson interferometer, the reading for maximum intensity were found to be 0.6939 and 0.9884mm. If the mean wavelength for the two components of D-lines be 5893Å. Deduce the difference between the two wavelengths.

Sol. If x be the distance moved by the movable mirror between two consecutive positions of maximum indistinctness, we have

$$\Delta\lambda = \frac{\lambda_1 \times \lambda_2}{2x} = \frac{\lambda^2}{2x}$$

where λ is the average of λ_1 and λ_2

Here $$\lambda = 5893\ \text{Å} = 5893 \times 10^{-10}\ \text{m}$$

$$x = 0.9884 - 0.6939 = 0.2945\ \text{mm} = 2.945 \times 10^{-4}\ \text{m}$$

$\therefore$ $$\Delta\lambda = \frac{(5893 \times 10^{-10})^2}{2 \times 2.945 \times 10^{-4}} = 5.896 \times 10^{-10}$$

$$\Delta\lambda = 5.896\text{Å}.$$

Q.18. Differentiate between fringes of michelson interferometer and fabry perot interferometer.

Ans. The Fabry-Perot fringes differ from the michelson's fringes in two respects.

1. The Fabry-Perot fringes formed by multiple beam interference are much sharper than the michelson's fringes.
2. When the light consists of two or more close wavelengths then in F.P. interferometer each wavelength produced its own pattern, and the rings of one pattern are clearly separated from the corresponding rings of the other pattern. Hence the instrument is very suitable for the study of fine structure of spectral lines. In michelson's interferometer, separate patterns are not produced. The presence of two close wavelengths is judged by the alternate distinctness and indistinctness of the rings when the optical path difference is increased.

Q.19. In Newton's Rings experiment the diameter of 4th and 12th Rings are 0.4 cm and 0.7 cm respectively. Find the diameter of the 20th dark ring.

Sol. $D_{12} = 0.7$ cm $\quad D_4 = 0.4$ cm

we know that $$\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR}$$

If $$n + P = 12 \quad \text{and} \quad n = 4 \quad \text{then} \quad P = 8$$

$$\lambda = \frac{(0.7 \times 10^{-2})^2 - (0.4 \times 10^{-2})^2}{4 \times 8 \times R}$$

$$\lambda R = 1.031 \times 10^{-6}\ \text{m}$$

$$d_{20} = \sqrt{4R\lambda n} = \sqrt{4 \times 1.031 \times 10^{-6} \times 20}$$

$$d_{20} = 0.908 \times 10^{-2}\ \text{m}.$$

SECTION-C LONG ANSWER TYPE QUESTIONS

Q.1. Explain the formation of interference fringes by means of a Fresnel's biprism when a monochromatic source of light is used and derive an expression for the fringe-width. Describe biprism method for the determination of the wavelength of monochromatic light.
Are Biprism fringes localised or non-localised?

Ans.

Fresnel's Biprism

The biprism is a device to obtain two *coherent* sources to produce sustained interference. It is a combination of two prisms of very small refracting angles, placed base to base. In practice, the biprism is made from a single plate by grinding and polishing, so that it is a single prism with one of its angles about 179° and the other two about 30′ each.

Production of Fringes : The method of obtaining interference fringes is illustrated in Fig.1 S is a narrow vertical slit illuminated by *monochromatic* light. The light from S is allowed to fall symmetrically on the biprism P, placed at a small distance from S and having its refracting edge parallel to the slit. The light beams emerging from the upper and lower halves of the prism appear to start from two virtual images of S, namely S_1 and S_2, which act as *coherent* sources. The cones of light BS_1E and AS_2C, diverging from S_1 and S_2 are superposed and the interference fringes are formed in the overlapping region BC. These fringes are non-localised and may be obtained on a screen or seen through an eyepiece.

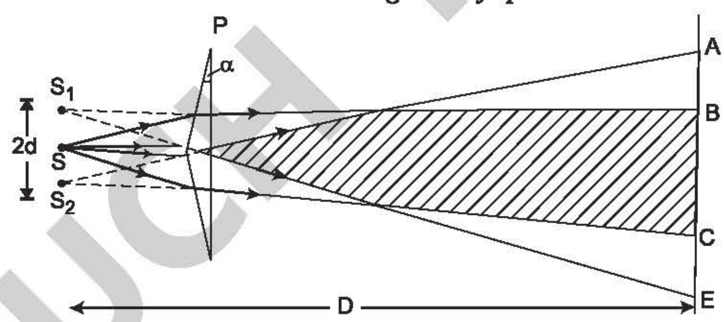


Fig.1

Theory of Fringes : Let S_1 and S_2 (Fig. 2) be the two virtual sources produced by the biprism, and XY the screen on which the fringes are obtained. The point O on the screen is equidistant from S_1 and S_2. Therefore the waves from S_1 and S_2 reach O in the same phase and reinforce each other. Hence the point O is the centre of a bright fringe.

The illumination at any other point P can be obtained by calculating the path difference $S_2P - S_1P$. Let us join S_2P and S_1P and draw perpendiculars S_1M_1 and S_2M_2 on the screen. Let $S_1S_2 = 2d$, $S_1M_1 = S_2M_2 = D$ and $OP = x$. Then

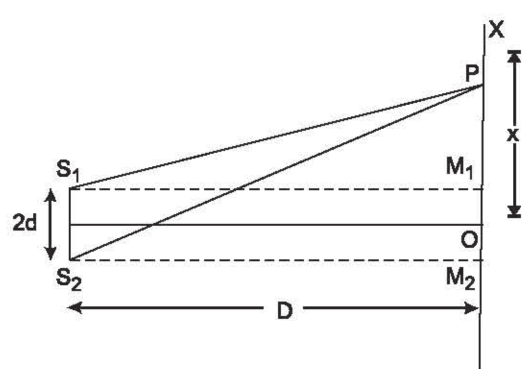


Fig.2

$$(S_2P)^2 = (S_2M_2)^2 + (PM_2)^2$$

$$= D^2 + (x+d)^2 = D^2\left[1 + \frac{(x+d)^2}{D^2}\right]$$

or
$$S_2P = D\left[1 + \frac{(x+d)^2}{D^2}\right]^{\frac{1}{2}}$$

$$= D\left[1 + \frac{1}{2}\frac{(x+d)^2}{D^2}\right], \text{ as } (x+d) << D$$

$$= D + \frac{1}{2}\frac{(x+d)^2}{D}$$

Similarly,
$$S_1P = D + \frac{1}{2}\frac{(x-d)^2}{D}$$

∴
$$S_2P - S_1P = \frac{1}{2}\frac{(x+d)^2}{D} - \frac{1}{2}\frac{(x-d)^2}{D} = \frac{2xd}{D}.$$

The resultant intensity at a point is maximum or minimum according as the path difference between the waves is an integral multiple of wavelength or an odd multiple of half-wavelength.

Thus, for P to be the centre of bright fringe, we must have

$$S_2P - S_1P = \frac{2xd}{D} = n\lambda, \qquad \text{where } n = 0, 1, 2...$$

or
$$x = \frac{D}{2d}(n\lambda)$$

For P to be the centre of a dark fringe, we must have

$$S_2P - S_1P = \frac{2xd}{d} = (2n+1)\frac{\lambda}{2}, \qquad \text{where } n = 0, 1, 2,...$$

or
$$x = \frac{D}{2d}(2n+1)\frac{\lambda}{2}$$

Let x_n and x_{n+1} denote the distances of the nth and $(n+1)$th bright fringes. Then, the distance between $(n+1)$th and nth bright fringes is given by

$$x_{n+1} - x_n = \frac{D}{2d}(n+1)\lambda - \frac{D}{2d}(n\lambda) = \frac{D}{2d}\lambda$$

This is independent of n, so that the distance between any two consecutive bright fringes is the same, $D\lambda/2d$. The same result holds for dark fringes. The distance $D\lambda/2d$ is called the 'fringe-width' denoted by W. Thus

$$W = \frac{D}{2d}\lambda$$

or
$$\lambda = W\left(\frac{2d}{D}\right)$$

It is clear from the expression that if W, D and $2d$ are measured, the wavelength of light λ can be obtained.

Experimental Arrangement and Adjustments : It consists of a good optical bench with three stands. The first carries an adjustable slit, the second a biprism and the third a micrometer eyepiece. Screws are provided to rotate the slit and the biprism in their own planes, and also to move the biprism and the eyepiece at right angles to the length of the optical bench. The slit is illuminated by monochromatic light (say, sodium light) whose wavelength is to be determined.

For obtaining sharp fringes the following adjustments are made in the given order :

(i) The optical bench is levelled in the horizontal plane.

(ii) The eyepiece is focussed on the cross-wires. One of the cross-wires is set vertical by mean of a plum-line.

(iii) The slit, the biprism and the eyepiece are adjusted to the same height.

(iv) The slit is made vertical by rotating it in its own plane.

(v) The biprism is moved at right angles to the optical bench until two equally-bright virtual slit images S_1 and S_2 are observed on looking through the biprism along the axis of the bench. The biprism is now rotated in its own plane until, on moving the eye across the bench one of the images crosses the edge of the biprism as a whole. This makes the edge of the biprism approximately parallel to the slit.

(vi) The eyepiece is moved at right angles to the length of the bench until the overlapping region comes in the field of view.

(vii) The slit is now made narrow. The fringes at once appear.

(viii) The biprism is slowly rotated in its own plane until the fringes are perfectly distinct. The edge of the biprism is now accurately parallel to the slit.

(ix) Finally, the line joining the slit and the edge of the biprism is made parallel to the length of the optical bench by removing the 'lateral shift'. For this, the cross-wire is set on a fringe in the centre and the eyepiece is moved along the bench away from the biprism. If the line joining the slit to the edge is not parallel to the bench, the fringe-system shifts laterally. The biprism is moved laterally until the cross-wire is again set on the same fringe. Now the eyepiece is moved toward the biprism when the fringe-system again shifts but in the opposite direction. Now the eyepiece is moved laterally until the cross-wire is again set on the same fringe. The process is repeated until the lateral shift completely disappears.

The following measurement are now made :

(i) **Measurement of Fringe-width *W* :** After obtaining the fringes, the vertical cross-wire of the eyepiece is set on a bright fringe on one side of the interference pattern. The reading of the micrometer screw is taken. Then the eyepiece is moved laterally so that the vertical cross-wire coincides with successive bright fringes and the corresponding readings are noted. From these readings the fringe-width W is found.

(ii) **Measurement of *D* :** To readings of the stands of the slit and the eyepiece on the scale of the optical bench are taken. The difference of the two readings gives D, which is corrected for the bench-error.

(iii) **Measurement of 2*d* :** The measure the distance $2d$ between S_1 and S_2, a convex lens of short focal length is mounted between the biprism P and the eyepiece E (Fig.3). By moving the lens along the length of the bench, two positions L_1 and L_2 are obtained such that the real images of S_1 and S_2 are obtained in the eyepiece. Let d_1 and d_2 respectively be the separations between the real images in the positions L_1 and L_2 respectively. If u and v respectively be the distances of the slits and the eyepiece from the lens in the position L_1, then from the magnification formula, we have

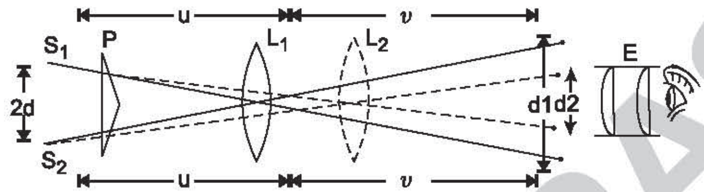


Fig.3

$$\frac{d_1}{2d} = \frac{v}{u} \quad ...(1)$$

As the two positions of the lens are conjugate, we have for the second position L_2

$$\frac{d_2}{2d} = \frac{u}{v} \quad ...(2)$$

Multiplying eq. (1) and (2), we get

$$\frac{d_1 d_2}{4d^2} = 1$$

or

$$2d = \sqrt{d_1 d_2}$$

Several sets of the readings of d_1 and d_2 are taken by moving the eyepiece into different positions and mean value of $2d$ is found.

Finally, the wavelength is determined by the formula

$$\lambda = W\left(\frac{2d}{D}\right).$$

Q.2. Calculate the displacement of the fringes when a thin transparent lamina is introduced in the path of one of the interfering beams in biprism. Show how this method is used for finding the thickness of a mica plate.

Ans.

Displacement of Fringes by the Introduction of a Thin Lamina

When a thin transparent plate, say of glass or mica, is introduced in the path of one of the two interfering light beams, the entire fringe-pattern is displaced to a point towards the beam in the path of which the plate is introduced. If the displacement be measured, the thickness of the plate can be obtained provided the refractive index of the plate and wavelength of light be known.

Let S_1 and S_2 (Fig.) be two *coherent* monochromatic sources giving light of wavelength λ. Let a thin plate of thickness t be introduced in the path of light from S_1. Let μ be the refractive index of the plate for the monochromatic light employed.

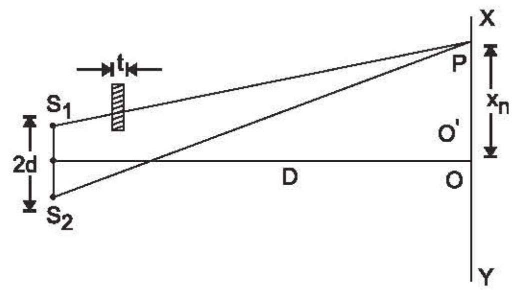


Fig.

Now, light from S_1 travel partly in air and partly in the plate. For the light path from S_1 to P, the distance travelled in air is (S_1P-t) and that in the plate is t. If v_0 and v_1 be the velocities of light in air and in the plate respectively, then the time taken for the journey from S_1 to P

$$=\frac{S_1P-t}{v_0}+\frac{t}{v_1}$$

$$=\frac{S_1P-t}{v_0}+\frac{\mu t}{v_0} \qquad \left[\because\ \mu=\frac{v_0}{v_1} \text{ or } \frac{1}{v_1}=\frac{\mu}{v_0}\right]$$

$$=\frac{S_1P+(\mu-1)t}{v_0}$$

It follows from this relation that the effective path in air from S_1 to P is $\{S_1P+(\mu-1)t\}$, that is, the air path S_1P is increased by an amount $(\mu-1)t$ as a result of the introduction of the plate.

Let O be the position of the central bright fringe in the absence of the plate, the optical paths S_1O and S_2O being equal. On introducing the plate, the two optical paths become unequal. Therefore, the central fringe is shifted to O', such that at O' the two optical paths become equal. A similar argument applies to all the fringes.

Now, at any point P, the effective path difference is

$$S_2P-[S_1P+(\mu-1)t]=S_2P-S_1P-(\mu-1)t$$

Let $S_1S_2=2d$, distance of screen from $S_1S_2=D$, and $OP=x_n$. Then

$$S_2P-S_1P=\frac{2d}{D}x_n$$

$\therefore$ effective path difference at

$$P=\frac{2d}{D}x_n-(\mu-1)t$$

If the point P is to be the centre of the n th bright fringe, the effective path difference should be equal to $n\lambda$. This is

$$\frac{2d}{D}x_n-(\mu-1)t=n\lambda$$

or

$$x_n=\frac{D}{2d}[n\lambda+(\mu-1)t] \qquad ...(1)$$

In the absence of the plate $(t=0)$, the distance of the n th maximum from O is $\frac{D}{2d}n\lambda$.

Therefore, the displacement of the nth bright fringe is given by

$$x_0 = \frac{D}{2d}[n\lambda + (\mu - 1)t] - \frac{D}{2d}n\lambda$$

or
$$x_0 = \frac{D}{2d}(\mu - 1)t \qquad ...(2)$$

This expression is independent of n, so that the displacement is the same for all the bright fringes. Similarly, it can be shown that the displacement of any dark fringe is also $\frac{D}{2d}(\mu - 1)t$.

Thus, the entire fringe-system is displaced through a distance $\frac{D}{2d}(\mu - 1)t$.

Determination of thickness of mica plate : The eq. (ii) shows that if the introduction of the given mica plate in one of the interfering beams produces a shift x_0, we have

$$x_0 = \frac{D}{2d}(\mu - 1)t$$

so that
$$t = \frac{x_0(2d)}{D(\mu - 1)}.$$

Q.3. (a) Discuss the formation of fringes obtained by a Lloyd's mirror. Give the theory and experimental arrangement of Lloyd's mirror to measure the wavelength of monochromatic light.

(b) What are achromatic fringes? Discuss an experimental arrangement for obtaining achromatic fringes. Draw necessary diagram.

Ans.

(a) Lloyd's Mirror

It is an arrangement to obtain two *coherent* sources of light to produce sustained interference. It consists of a plane mirror *MN* (Fig.1) polished on the *front* surface and blackened at the back (to avoid multiple reflections). S_1 is a narrow slit, illuminated by monochromatic light and placed with its length parallel to the surface of the mirror. Light from S_1 falls on the mirror at nearly *grazing* incidence, and the reflected beam appears to diverge from S_2, which is the virtual image of S_1. S_1 and S_2 act as coherent sources. The direct cone of light AS_1E and the reflected cone of light BS_2C are superposed and the interference fringes are obtained in the over-lapping region BC on the screen.

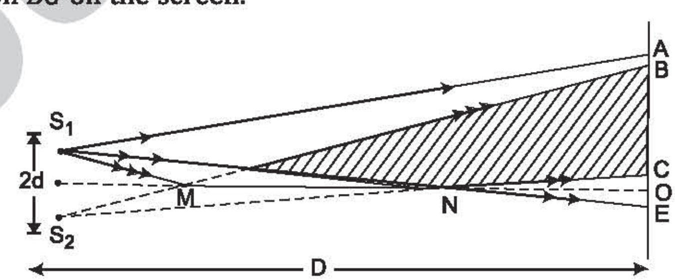


Fig.1

The central (zero-order) fringe which is expected to lie at O (the perpendicular bisector of S_1S_2) is not usually seen since only the direct light, and not the reflected light, reaches O. It can be seen by introducing a thin sheet of mica in the path of light from S_1 when the entire fringe-system is displaced in the upward direction.

With white light the central fringe is expected to be white but actually it is found 'dark'. This is because light suffers a phase change of π when reflected from the mirror. Therefore, the path difference between the interfering waves at the position of zero-order fringe becomes $\lambda/2$ (instead of zero) which is a condition for a minimum. Hence the fringe is dark.

Determination of Wavelength : Let $2d$ be the distance between the coherent sources S_1 and S_2 and D the distance of the screen from the sources. The fringe-width is then given by

$$W = \frac{D}{2d}\lambda$$

Thus, knowing W, D and $2d$, the wavelength λ can be determined.

To perform the experiment, the Llyod's mirror is mounted vertical on the upright of an optical bench and placed along the length of the bench. On an another upright is mounted a narrow vertical slit which is illuminated by the given light. The mirror is rotated about an axis parallel to the length of the bench until a set of distinct fringes is obtained in a micrometer eyepiece. The plane of the mirror is then exactly parallel to the length of the slit.

The fringe-width W is measured by means of micrometer eyepiece; the difference $2d$ by means of a convex lens using displacement method; and the distance D directly by a meter scale and corrected for the bench-error. λ is then calculated from the above formula.

(b) Achromatic Fringes and their Production by Lloyd's Mirror

A system of white and dark fringes, without any colours, obtained by white light are called 'achromatic-fringes'.

Ordinarily, with white light we obtain a central white fringe having on either side of it a few coloured fringes. This is because the fringe-width $W = (D\lambda/2d)$ is different for different wavelengths (colours). If, however, the fringe-width is made the same for all wavelength, then the maxima of each order for all wavelength coincide, resulting into achromatic fringes. That is, for achromatic fringes, we must have

$$\frac{D\lambda}{2d} = \text{constant}$$

or

$$\frac{\lambda}{2d} = \text{constant.}$$

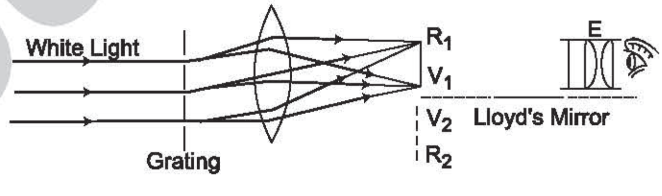


Fig.2

We can easily realise condition with a Lloyd's mirror by using a slit illuminated by a narrow spectrum of white light, as shown in Fig.2. The narrow spectrum R_1V_1 is produced by a prism, or preferably, by a plane diffraction grating. The Lloyd's mirror is placed with its surface close to the violet end of the spectrum and such that R_1V_1 is perpendicular to its plane. R_1V_1 and its virtual image R_2V_2 formed by the mirror act as coherent sources. They are equivalent to a number of pairs of sources of different colours. Thus, the pair R_1R_2 produces a set of red fringes, and the pair V_1V_2 a set of violet fringes. The intermediate pairs produce the sets of fringes of intermediate colours. The red and violet fringes will be of the same width if

$$\frac{\lambda}{2d} = \text{constant}$$

that is,
$$\frac{\lambda_r}{2d_r} = \frac{\lambda_v}{2d_v}$$

or
$$\frac{2d_r}{2d_v} = \frac{\lambda_r}{\lambda_v},$$

where $2d_r$ is the distance R_1R_2, and $2d_v$ the distance V_1V_2. Thus

$$\frac{R_1R_2}{V_1V_2} = \frac{\lambda_r}{\lambda_v}$$

Therefore, if the distance of the violet end V_1 from the surface of the mirror is so adjusted by displacing the mirror laterally that the above condition is satisfied, the red and the violet fringes will have the same width and will exactly be superposed on each other. Since in a grating spectrum *the dispersion is accurately proportional to the wavelength*, the condition $(\lambda 2d)$ = constant is simultaneously satisfied for all the wavelengths. Thus, when the above adjustment is made, fringes of all colours are superposed on one another. Hence achromatic fringes are observed in the eyepiece E placed in the over-lapping region.

Q.4. Discuss the formation of interference fringes due to a thin wedge-shaped film seen by normally reflected sodium light. What will happen if white light is substituted for the sodium light?

Ans.

Wedge-Shaped Film

Let us consider a thin wedge-shaped film of refractive index μ, bounded by two plane surfaces AB and CD (Fig.) inclined at an angle θ. Let the film be illuminated by sodium light from a slit held parallel to the edge of the wedge. Interference occurs between the rays reflected at the upper and the lower surface of the film.

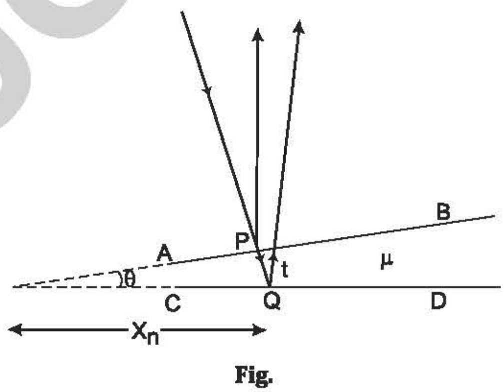


Fig.

When light is incident nearly *normally* at a point P on the film, the path difference between the rays reflected at the upper and the lower surfaces is $2\mu t$, where t is the thickness of the film at P. An additional path difference of $\lambda/2$ is introduced in the ray reflected from the upper surface. Hence the effective path difference between the two rays is

$$p = 2\mu t - \frac{\lambda}{2}$$

The condition for maximum intensity (bright fringe) is

$$p = 2\mu t - \frac{\lambda}{2} = n\lambda$$

or $$2\mu t = (2n+1)\frac{\lambda}{2}$$

and the condition for minimum intensity (dark fringe) is

$$p = 2\mu t - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

or $$2\mu t = n\lambda$$

Shape and Location : It is clear that for a bright or a dark fringe of a particular order, t must remain constant. Since in the case of wedge-shaped film, t remains constant along lines parallel to the thin edge of the wedge, the bright and the dark fringes are straight lines parallel to the thin edge of the wedge. These are called 'fringes of constant thickness', each fringe being the locus of points for which the film thickness t is a constant.

At the thin edge where $t = 0$, $p = \lambda/2$, a condition of minimum intensity. Therefore the edge of the film is dark.

When the light source is an extended one, then each point on the source gives rise to a pair of coherent waves from the wedge. These coherent pairs leave the wedge in different directions and their interference effects are same only at the wedge itself and then only if the wedge is thin. Interference fringes formed by *thin* wedge from *extended* source are thus localised at the wedge itself. These fringes can easily be seen by means of a low-power microscope focussed on the wedge.

Spacing between two Consecutive Fringes : For the nth dark fringe, we have

$$2\mu t = n\lambda$$

Let this fringe be obtained at a distance x_n (Fig.) from the thin edge. Then,

$$t = x_n \tan\theta = x_n\theta \qquad \text{(where } \theta \text{ is small)}$$

$$\therefore \qquad 2\mu x_n\theta = n\lambda \qquad ...(1)$$

Similarly, if the $(n+1)$st dark fringe is obtained at a distance x_{n+1} from the thin edge, then

$$2\mu x_{n+1}\theta = (n+1)\lambda \qquad ...(2)$$

Subtracting eq. (1) from (2), we get

$$2\mu\theta(x_{n+1} - x_n) = \lambda$$

Hence the spacing W is

$$W = x_{n+1} - x_n = \frac{\lambda}{2\mu\theta},$$

where θ is measured in radian.

Similarly, it can be shown that the spacing between two consecutive bright fringes (fringe-width) is $\frac{\lambda}{2\mu\theta}$.

Fringes in White Light : When the film is seen in white light, each colour (wavelength) produces its own interference fringes. The separation between consecutive fringes will be least for violet, and greatest for red. At the edge of the film $t=0$ and hence $p=\lambda/2$. Hence each wavelength gives minimum intensity at the direction of thickness increasing, we obtain a few coloured bands of mixed colour. For still greater thickness, the overlapping increases so much that uniform illumination is produced.

Q.5. (a) Describe and explain the formation of Newton's rings in reflected monochromatic light. Prove that in reflected light (i) diameters of bright rings are proportional to the square-roots of odd numbers, and (ii) diameters of dark rings are proportional to the square-roots of natural numbers.

(b) Account for the perfect blackness of the cental spot in Newton's ring-system. Can you obtain a bright centre?

Ans.

(a) Formation of Newton's Rings

When a plano-convex lens of *large* radius of curvature is placed with its convex surface in contact with a plane glass plate, an air-film is formed between the lower surface of the lens and the upper surface of the plate. The thickness of the film gradually increases from the point of contact outwards. If monochromatic light is allowed to fall normally on this film a system of alternate bright and dark concentric rings with their centre dark is formed in the air-film. These are called Newton's rings. They can be seen through a low-power microscope focussed on the film.

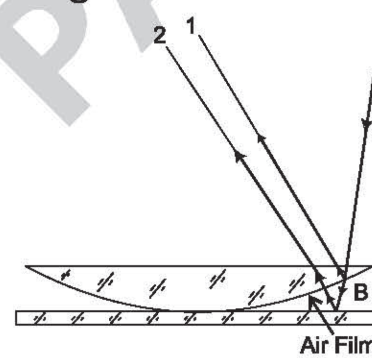


Fig.1

Newton's rings are formed as a result of interference between the light waves reflected from the upper and the lower surfaces of the air-film. In fig.1 they ray 1 and 2 are the interfering rays corresponding to an incident ray AB. As the rings are observed in reflected light, the effective path difference between the interfering rays is

$$p = 2\mu t \cos r + \frac{\lambda}{2},$$

where μ is the refractive index of the film (air), t the thickness of the film, at the point B and r the inclination of the ray. The factor $\lambda/2$ accounts for the phase change of π on reflection at the lower surface of the film (plate). Now $\mu=1$ (air-film) and $r=0$ (normal incidence).

$$\therefore \quad p = 2t + \frac{\lambda}{2}$$

At the point of contact of the lens and the plate, $t=0$, hence $p=\lambda/2$. This is the condition for minimum intensity. Hence the cental spot is dark.

The condition for maximum intensity (bright-fringe) is

$$p = n\lambda$$

or

$$2t + \frac{\lambda}{2} = n\lambda$$

or

$$2t = (2n-1)\frac{\lambda}{2} \text{ (Maxima)} \qquad ...(1)$$

The condition for minimum intensity (dark fringe) is

$$p = (2n+1)\frac{\lambda}{2}$$

or $$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

or $$2t = n\lambda. \text{ (Minima)} \quad ...(2)$$

It is clear that bright or dark fringe of any particular order n will occur for a constant value of t. since in the air-film, t remains constant along a circle with its centre at the point of contact, the fringes are in the form of concentric circles. Since each fringe is a locus of constant film thickness, these are known as the 'fringes of constant thickness'.

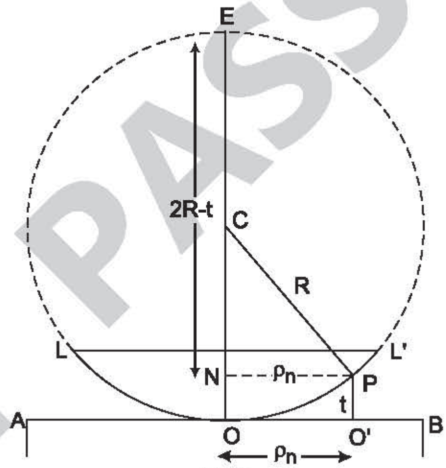


Fig.2

Diameters of Bright Rings : Let LOL' (Fig. 2) be the lens placed on the glass plate AB, the point of contact being O. Let R be the radius of curvature of the curved surface of the lens. Let ρ_n be the radius of a Newton's ring corresponding to a point P, where the film thickness is t. Draw perpendicular PN. Then, from the property of a circle, we have

$$PN^2 = ON \times NE$$

or $$\rho_n^2 = t \times (2R - t) = 2Rt - t^2$$

Since t is small compared to R, we can neglect t^2. Hence

$$\rho_n^2 = 2Rt \quad \text{or} \quad 2t = \frac{\rho_n^2}{R} \quad ...(3)$$

The condition for a bright ring is (see eq. 1)

$$2t = (2n-1)\frac{\lambda}{2}$$

But, from eq. (3), $2t = \rho_n^2 / R$

$$\therefore \quad \frac{\rho_n^2}{R} = (2n-1)\frac{\lambda}{2}$$

or $$\rho_n^2 = (2n-1)\frac{\lambda R}{2}$$

If D_n be the diameter of the n th bright ring, then $D_n = 2\rho_n$ or $\rho_n = D_n/2$. Substituting in the last expression,

$$D_n^2 = 2(2n-1)\lambda R$$

or $$D_n = \sqrt{2\lambda R}\sqrt{2n-1}$$

or $$D_n \propto \sqrt{2n-1}$$

As n is an integer, $(2n-1)$ is an odd number. Thus, ***the diameters of bright rings are proportional to the square-roots of the odd natural numbers.***

The diameters of the first few rings are in the ratio

$$1:\sqrt{3}:\sqrt{5}:\sqrt{7} \ldots\ldots$$

$$=1:1.732, 2.236, 2.646, \ldots\ldots$$

The separations between successive rings are in the ratio

$$0.732,\ 0.504,\ 0.410,\ \ldots\ldots$$

that is , ***the separation decreases as the order increases.***

Diameters of Dark Rings : The condition for a dark ring is (see eq. ii)

$$2t = n\lambda$$

But $2t = \rho_n^2 / R$

$$\therefore \qquad \frac{\rho_n^2}{R} = n\lambda$$

If D_n be the diameter of the nth dark ring, $\rho_n = D_n/2$

$$\therefore \qquad \frac{D_n^2}{4R} = n\lambda$$

$$\text{or} \qquad D_n = \sqrt{4nR\lambda}$$

$$\text{or} \qquad D_n = \sqrt{4R\lambda}\,\sqrt{n}$$

$$\text{or} \qquad D_n \propto \sqrt{n}$$

Thus, ***the diameters of dark rings are proportional to the square-roots of natural numbers.***

(b) Perfect Blackness of Central Spot

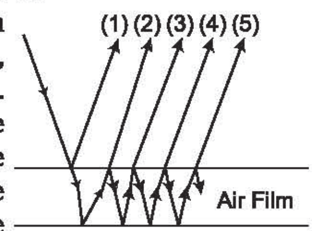


Fig.3

Newton's rings in reflected light are formed by interference between the ray (1) reflected directly from the upper surface of the air-film, and the rays, (2), (3), etc. which are obtained after one, three, five, etc. internal reflections. These rays are shown in Fig. 3. Near about the point of contact, the thickness of the air-film is almost zero and hence no path difference is introduced between the interfering rays. But the ray (2) reflected from the lower surface of the film suffers a phase change of π, while the ray (1) reflected from the upper does not suffer such change. Thus, the two interfering waves at the centre are opposite in phase and destroy each other. The destruction is however, not complete, since the amplitude of (2) is less than that of (1). But the sum of the amplitudes of (2), (3), (4) etc., which are all in phase, is exactly equal to the amplitude of (1), as shown by the Stokes' treatment. Hence complete destructive interference is produced and the centre of the ring-system is 'perfectly' black.

Bright Centre : If Newton's rings are obtained by using a crown-glass lens placed on a flint-glass plate with a small quantity of oil of sassafras between them, the centre of the ring-system is 'bright'. This is because the oil of sassafras is optically denser than the crown glass, but rarer than the flint glass. Therefore, the reflections at both the upper and the lower surfaces of the film take place under similar conditions, that is, in going from a rarer to a denser medium. Thus, there is a phase change of π at both reflections. Hence the relative phase difference between the interfering rays at the point of contact is zero and the central spot appears bright.

Q.6. Describe the construction and working of a Michelson's interferometer and explain the use of the compensating plate.

Describe how the interferometer may be used to obtain (i) circular fringes, (ii) straight fringes, (iii) white-light fringes.

Ans.

Michelson's Interferometer

It is an excellent device to obtain interference fringes of various shapes which have a number of application in optics.

Construction : Its main optical part are two plane mirrors M_1 and M_2 (Fig.1) and two similar optically-plane, parallel glass plates P_1, P_2. The plane mirrors M_1 and M_2 are silvered on their front surfaces and are mounted vertically on two arms at right angle to each other. Their planes can be slightly tilted about vertical and horizontal axes by adjusting screws at their backs. The mirror M_1 is mounted on a carriage provided with a very accurate and fine screw and can be moved in the direction of the arrows. The plates P_1 and P_2 are mounted exactly parallel to each other and inclined at 45° to M_1 and M_2. The surface of P_1 towards P_2 is *partially* silvered.

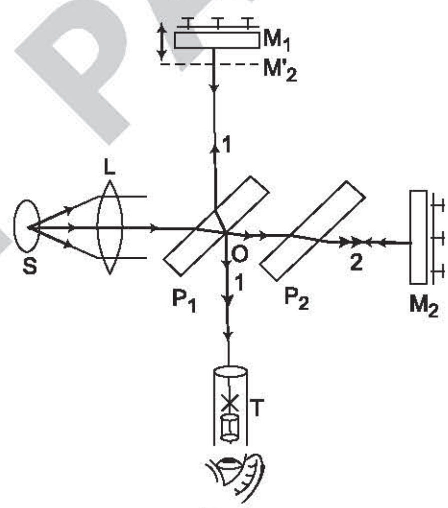


Fig.1

Working : Light from an extended monochromatic source S, rendered nearly parallel by a lens L, falls on P_1. A ray of light incident on the partially-silvered surface of P_1 is partly reflected and partly transmitted. The reflected ray 1 and the transmitted ray 2, travel to M_1 and M_2 respectively. After reflection at M_1 and M_2, the two rays re-combine at the partially-silvered surface and enter a short-focus telescope T. Since the rays entering the telescope are derived from the same incident ray, they are coherent and hence in a position to interfere. The interference fringes can be seen in the telescope.

Function of the Plate P_2 : After partial reflection and transmission at O, the ray 1 travels through the glass plate P_1 twice, while ray 2 does not do so even once. Thus, in the absence of P_2, the paths of rays 1 and 2 in glass are not equal. To equalise these paths a glass plate P_2, which has the same thickness as P_1, is placed parallel to P_1. P_2 is called the 'compensating plate'.

Form of Fringes : The form of the fringes depends on the inclination of M_1 and M_2. Let M_2' be the image of M_2 formed by reflection at the semi-silvered surface of P_1 so that $OM_2' = OM_2$. The interference fringes may be regarded to be formed by light reflected from the surfaces of M_1 and M_2'. Thus, **the arrangement is equivalent to an air-film enclosed between the reflecting surfaces M_1 and M_2'.**

Circular Fringes : When M_2 is exactly perpendicular to M_1, the film $M_1 M_2'$ is of uniform thickness and we obtain circular fringes localised at infinity, as explained below :

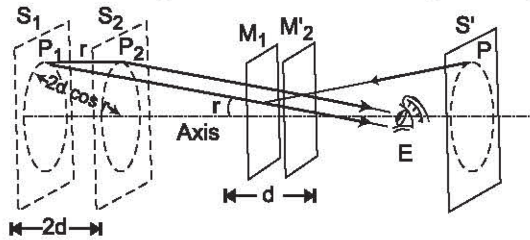


Fig.2

In Fig.2., M_1 and M_2' are the parallel reflecting surfaces. The actual source has been replaced by its virtual image S' formed by reflection in the partially-silvered surface. S' forms two virtual images S_1 and S_2 in M_1 and M_2'. Light from a point such as P on the extended source appears to an eye E to come from the corresponding coherent point P_1 and P_2 and S_1 and S_2. If d is the separation between M_1 and M_2', then $2d$ is the separation between S_1 and S_2. The path difference between the rays entering the eye is, clearly, $2d \cos r$.

If $\quad 2d \cos r = n\lambda \quad$ P appears bright

and if $\quad 2d \cos r = (2n+1)\frac{\lambda}{2} \quad$ P appears dark

The focus of points on the source which subtend the same angle r at the axis is a circle passing through P with its centre on the axis. Thus, a series of bright and dark circular fringes is seen. The order of the fringes decreases as r increases that is, as we move away from the centre. Since the interfering rays are parallel, these fringes are formed at infinity.

Straight Fringes : When M_2 is not perpendicular to M_1, the air-film between M_1 and M_2' is wedge-shaped. Since light is incident on the film at different angles, curved fringes with convexity towards the thin edge of the wedge are obtained (Fig.3). If the thickness of the film is very small, the fringes are practically straight. The fringe corresponding to $t = 0$ is perfectly straight. These fringes are formed near the film and are visible upto path differences comparatively much smaller than that in case of circular fringes.

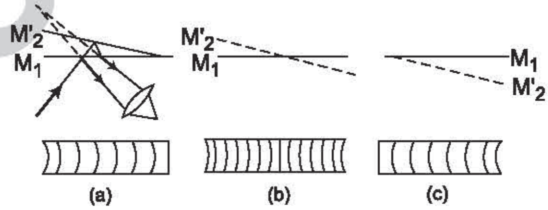


Fig.3

White-light Fringes : If, in the last case, monochromatic light is replaced by white light and if the thickness of the film is small, a few curved and coloured localised fringes are obtained, the fringe of zero thickness being again perfectly achromatic and straight. For large thickness of the film, uniform illumination is obtained.

Adjustment of the Michelson's Interferometer

(i) For Localised Fringes : The distances of the mirrors M_1 and M_2 from the silvered surface of P_1 are first made as nearly equal as possible by moving the movable mirror M_1. A pin-hole is placed between the lens and the plate P_1.

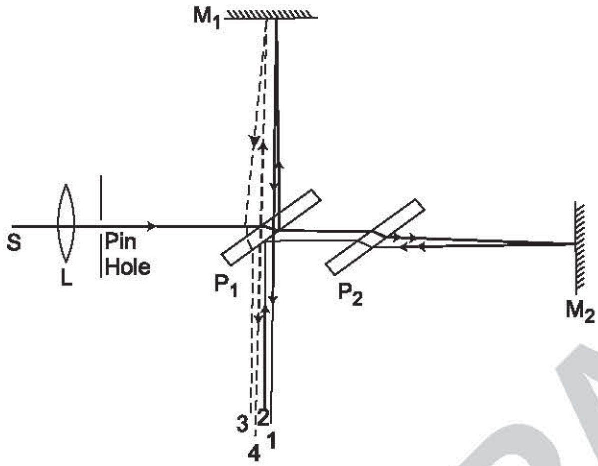


Fig.4

If M_1 is not perpendicular to M_2, four images of the pin-hole are obtained, two by reflection at the semi-silvered surface of P_1 and the other two by reflection at the other surface of P_1 (Fig.4). The former pair is naturally much brighter than the latter. The small screws at the back of the mirror M_2 are then adjusted until the two bright image appear to coincide. The pin-hole is now removed. If the coincidence of the images was apparent, the air-film between M_1 and M_2' would be wedge-shaped and localised fringes would appear.

(ii) For White-Light Localised Fringes : First, the localised fringes with monochromatic light are obtained. The mirror M_1 is then moved until the fringes become straight. Monochromatic light is replaced by white light. M_1 is further moved in the same direction until the central achromatic fringe is obtained in the field of view.

(iii) For Circular Fringes : After localised fringes are obtained, the screws of M_2 are adjusted so that the spacing between these fringes increases. This happens when the angle of the wedge decreases. If this adjustment be continued, at one stage the angle of the wedge will become zero and the film will be of constant thickness. At this stage circular fringes will appear. Finer adjustment is made until on moving the eye sideways or up and down, the fringes do not expand or contract.

Q.7. (a) Explain the principle of Fabry-Perot interferometer. Obtain an expression for the intensity distribution in the transmitted light and discuss the sharpness of the fringes obtain.

(b) Compare the fringes of Fabry-Perot interferometer with those of Michelson's interferometer.

Ans.

(a) Fabry-Perot Interferometer

It is a high resolving power instrument which makes use of the 'fringes of constant inclination' produced by the transmitted light after multiple reflections in an air-film between two parallel, highly-reflecting glass plates.

It consists of two optically-plane glass A and B (Fig.1) with their inner surfaces silvered, and placed accurately parallel to each other. Screws are provided to secure parallelism if disturbed. The plates themselves are slightly prismatic to avoid interference among the rays reflected at the outer unsilvered surfaces.

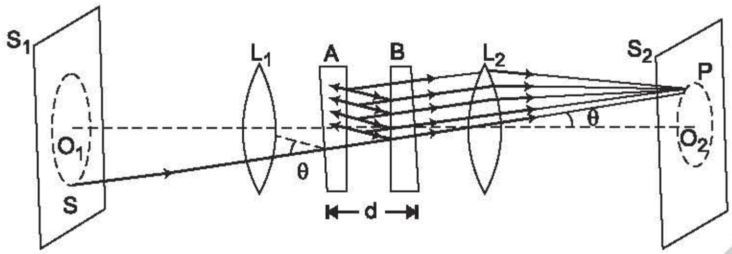


Fig.1

S_1 is a broad source of monochromatic light and L_1 a convex lens which makes the rays parallel. An incident ray suffers a large number of internal reflections successively at the two silvered surfaces, as shown. At each reflection a small fractional part of the light is transmitted also. Thus, each incident ray produces a group of coherent and parallel transmitted rays with a constant path difference between any two successive rays. A second convex lens L_2 brings these rays together at a point P in its focal plane where they interfere. Hence the rays from all points of the source produced in interference pattern on a screen S_2 placed in the focal plane of L_2. The phenomenon is called 'multiple-beam interference'.

Formation of the Fringes : Let d be the separation between the two silvered surfaces, and θ the inclination of a particular ray with the normal to the plates. Then the path difference between any two successive transmitted rays corresponding to the incident ray is $2d\cos\theta$. The condition for these rays to produce maximum intensity is given by

$$2d\cos\theta = n\lambda,$$

where n is an integer, called the order of interference, and λ the wavelength of light. The locus of points in the source which give rays of a constant inclination θ is a circle. Hence, with an extended source, the interference pattern consists of a system of bright concentric rings on a dark background, each ring corresponding to a particular value of θ.

Intensity Distribution : Let T and R be the fractions of the incident light intensity which are respectively transmitted and reflected at each silvered surface.

Then, the fractional transmitted and reflected amplitudes will be $\sqrt{T}$ and $\sqrt{R}$. The amplitudes of the successive rays transmitted through the pair of plates will be

$$aT, aTR, aTR^2 ......$$

where a is the incident amplitude.

The phase difference δ between any two rays reaching a point on the screen is given by

$$\delta = \frac{2\pi}{\lambda} \times \text{path diff.} = \frac{2\pi}{\lambda}(2d\cos\theta) \quad ...(1)$$

If the incident wave is represented by $y = ae^{i\omega t}$, then the waves reaching a point on the screen will be

$$y_1 = aTe^{i\omega t}$$

$$y_2 = aTRe^{i(\omega t - \delta)}$$

$$y_3 = aTR^2e^{i(\omega t - 2\delta)}, \text{ and so on.}$$

By the principle of superposition, the resultant amplitude is given by

$$A = aT + aTRe^{-i\delta} + aTR^2 e^{-2i\delta} + aTR^3e^{-3i\delta} + \ldots$$

$$= a\,T\,(1 + Re^{-i\delta} + R^2 e^{-2i\delta} + R^3 e^{-3i\delta} + \ldots)$$

$$= a\,T\left(\frac{1}{1 - R\,e^{-i\delta}}\right)$$

The complex conjugate of A is therefore

$$\bar{A} = a\,T\left(\frac{1}{1 - R\,e^{+i\delta}}\right)$$

Hence the resultant intensity I is given by

$$I = A\bar{A} = \frac{a^2T^2}{(1 - R\,e^{-i\delta})(1 - R\,e^{+i\delta})}$$

$$= \frac{a^2T^2}{1 + R^2 - R\,(e^{i\delta} + e^{-i\delta})} = \frac{a^2T^2}{1 + R^2 - 2\,R\cos\delta}$$

$$= \frac{a^2T^2}{(1 - R)^2 + 2R\,(1 - \cos\delta)} = \frac{a^2T^2}{(1 - R)^2 + 4R\sin^2\frac{\delta}{2}}$$

$$= \frac{a^2T^2}{(1 - R)^2}\left[\frac{1}{1 + \frac{4R}{(1 - R)^2}\sin^2\frac{\delta}{2}}\right] \qquad \ldots(2)$$

The intensity will be a maximum when $\sin^2\frac{\delta}{2} = 0$, that is $\delta = 2n\pi$, where $n = 0, 1, 2, \ldots$. Thus

$$I_{\max} = \frac{a^2T^2}{(1 - R)^2} \qquad \ldots(3)$$

Similarly, the intensity will be a minimum when $\sin^2\frac{\delta}{2} = 1$, that is, $\delta = (2n + 1)\,\pi$ where $n = 0, 1, 2, \ldots$ Thus,

$$I_{\min} = \frac{a^2T^2}{(1 - R)^2}\,\frac{1}{1 + \frac{4R}{(1 - R)^2}} = \frac{a^2T^2}{(1 + R)^2} \qquad \ldots(4)$$

The expression (2) can now be written as

$$I = \frac{I_{\max}}{1 + \frac{4R}{(1 - R)^2}\sin^2\frac{\delta}{2}} \qquad \ldots(5)$$

This is the intensity expression for the F.P. fringes.

Sharpness of the Fringes : If we plot I against δ for different values of R (the reflectivity of the plates), a set of curves is obtained (Fig.2). They show that larger the value of R, the more rapid is the fall of intensity on either side of a maximum. That is ***higher the reflectivity of the plates, sharper are the interference bright fringes.***

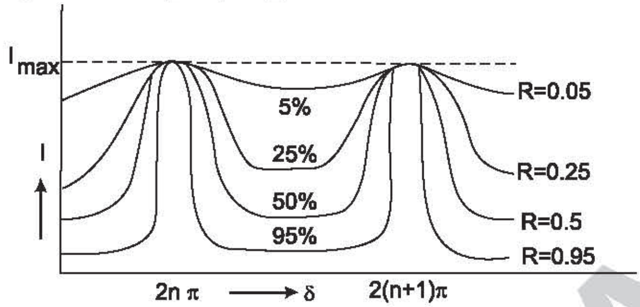


Fig. 2

Further, from eq. (iii) and (iv), the visibility V of the fringes is given by

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2R}{1+R^2}$$

V reaches 0.8 when $R = 0.5$ (50%) and approaches unity as R approaches 100%. Thus, ***higher the reflectivity, greater is the contrast in the fringes.***

Thus, in a Fabry-Perot interferometer, we obtain a system of sharp bright rings on a wide dark background.

(b) Comparison with a Michelson Interferometer

The Fabry-Perot fringes differ from the Michelson's fringes in two respects :

(i) The Fabry-Perot fringes formed by multiple-beam interference are much sharper than the Michelson's fringes.

A measurement of the sharpness of a fringe is the "half-width" which is the width of the $I-\delta$ curve at the place where $I = \frac{1}{2} I_{max}$ (Fig.3). Substituting this condition in the intensity expression

$$I = \frac{I_{max}}{1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}, \text{ we get}$$

$$\frac{1}{2} = \frac{1}{1 + \frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}$$

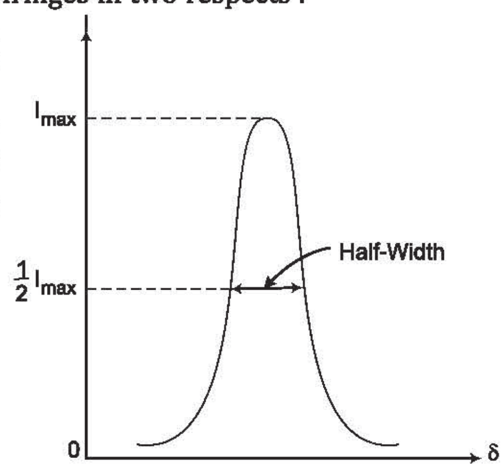


Fig. 3

or $$\frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2} = 1 \quad \text{or} \quad \delta = 2\sin^{-1}\left(\frac{1-R}{2\sqrt{R}}\right)$$

This represents the angular distance from the point of maximum intensity to the point where the intensity has fallen to half its maximum value, twice of which is the angular half-width. It approaches 0 as the reflectivity R approaches 1. For $R = 0.8$, we have

$$\delta = 2\sin^{-1}\left(\frac{1-0.8}{2\sqrt{0.8}}\right) = 2\sin^{-1}\frac{1}{9} = \frac{2}{9} = 0.22 \text{ radian}$$

In Michelson's interferometer, the fringes are formed by the interference of two beams, and the intensity distribution is given by

$$I = I_{max} \cos^2 \frac{\delta}{2}$$

For $\frac{I}{I_{max}} = \frac{1}{2}$, we have $\delta = 2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{2} = 1.57$ radians

which is about 7 tines the half width of F. P. fringes.

Q.8. Describe Newton's rings method for measuring the wavelength of monochromatic light and give the necessary theory. What will happen if a little water is introduced between the lens and the plate?

Ans. Experimental Arrangement for obtaining Newton's Rings

The experimental arrangement is shown in Fig.1. A plano-convex lens L of large radius of curvature is placed on a plane glass plate P such that the curved surface of the lens touches the glass plate. Light from a monochromatic source (sodium lamp) is allowed to fall on a glass plate G which is held inclined at an angle of 45° with the vertical. The light reflected from G falls normally on the air-film enclosed between L and P. Interference occurs between the rays reflected from the upper and the lower surfaces of this film. The interference rings are seen by a low-power microscope M focussed on the air-film where the rings are formed.

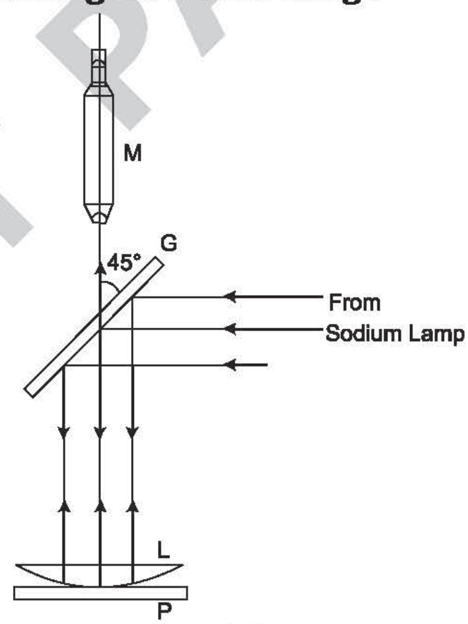


Fig.1

Theory : The effective path-difference between the interfering rays is $2\mu t \cos(r + \theta) + \frac{\lambda}{2}$, where μ is the refractive index of the film, r the angle of refraction into the film, θ the angle of film at any point and λ the wavelength of light used.

As the radius of curvature of the lens L is large, the angle θ of the film at any point is negligible, and for normal incidence of light, $r = 0$. Hence $\cos(r + \theta) = 1$. Also $\mu = 1$ for the air-film. Hence the effective path difference is simply $2t + \frac{\lambda}{2}$.

The condition for the n th bright fringe is

$$2t + \frac{\lambda}{2} = n\lambda \quad \text{or} \quad 2t = (2n-1)\frac{\lambda}{2} \qquad ...(1)$$

Let R be the radius of curvature of the curved surface of the lens. If ρ_n be the radius of n th bright ring at which the film thickness is t, then if follows from the property of a circle that

$$2t = \frac{\rho_n^2}{R}$$

Substituting this value of $2t$ in equation (1), we get

$$\rho_n^2 = \frac{(2n-1)\lambda R}{2}$$

If D_n is the diameter of the n th bright ring, $\rho_n = D_n/2$

$$\therefore \qquad D_n^2 = 2(2n-1)\lambda R \qquad ...(2)$$

If D_{n+p} is the diameter of $(n+P)$th bright ring, then

$$D_{n+p}^2 = 2[2(n+p)-1]\lambda R \qquad ...(3)$$

Subtracting equation (2) from (3), we get

$$D_{n+p}^2 - D_n^2 = 2[2(n+p)-1]\lambda R - 2(2n-1)\lambda R = 4p\lambda R$$

$$\therefore \qquad \lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \qquad ...(4)$$

On measuring the diameters of the rings and the radius of curvature R, the wavelength λ can be calculated with the help of the formula (4). If λ is known, then the radius of curvature R of the lens may be determined.

Method : The apparatus is arranged to produce Newton's rings. The eyepiece of the microscope is focussed on the crosswire and then the microscope is focussed to see the rings. The microscope is adjusted such that the intersection of the cross-wires coincides with the centre of the ring-system and one of the cross-wires is perpendicular to the direction of travel of the microscope. The microscope is then moved to about 24 th ring from centre. It is now moved back by slow motion screw and the cross-wire is set at the edges of the bright rings (say after every 2 or 4 rings). The reading of the microscope is taken each time. The observations are taken to about 4 th ring and then the microscope is moved to the other side of the central spot and the readings upto the same number of ring (24 th) are taken. The diameters of the different rings are then calculated.

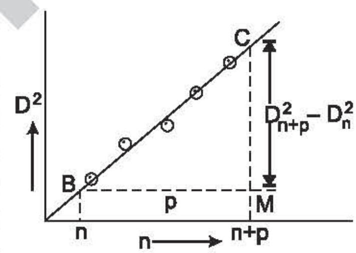


Fig.2

A graph is plotted between D^2 (diameter)^2 and n (number of ring). This graph is a straight line (Fig.2). The slope of this line gives

$$\frac{D_{n+p}^2 - D_n^2}{P}$$

Next, the radius of curvature R of the curved surface of the lens touching the glass plate is determined either by a spherometer or by Boys' optical method.

Thus, knowing $\dfrac{D_{n+p}^2 - D_n^2}{p}$ and R in equation (iv), λ can be calculated.

When a little water is introduced between the lens and the plate, the rings 'contract' according to the relation

$$\frac{\text{diameter of a ring in water-film}}{\text{diameter of same ring in air-film}} = \frac{1}{\sqrt{(\mu)}},$$

where μ is the refractive index of water.

UNIT-VI

Diffraction

SECTION-A VERY SHORT ANSWER TYPE QUESTIONS

Q.1. What is diffraction?

Ans. The bending of waves around the corners of small obstacles and apertures is called diffraction. It is of two types :

1. Fresnel's Diffraction
2. Fraunhofer Diffraction.

Q.2. Is Young's double-slit experiment an interference experiment or a diffraction experiment?

Ans. It is both a diffraction and an interference experiment, each narrow slit produces its diffraction pattern having a broad central maximum covering almost entire field of view. The interference pattern is observed against a background of almost uniform intensity of the broad central diffraction maxima of the two slits.

Q.3. What is a zone plate?

Ans. It is specially designed diffraction screen such that light is obstructed from alternate half period zones.

Q.4. Write the formula for radii and focal length of zone plate.

Ans. The radii of circles are proportional to the square root of natural number

$$r_n \propto \sqrt{n}$$

The focal length of zone plate is given by

$$f_n = \frac{r_n^2}{n\lambda}$$

Q.5. Draw the intensity pattern from a single slit (Fraunhofer diffraction).

Ans. The intensity distribution pattern due to a single slit is as shown in fig. 1.

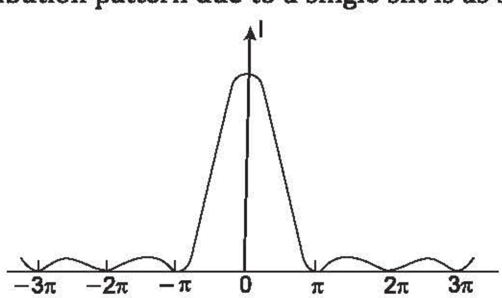


Fig. 1

Q.6. What is a plane transmission diffraction grating? Write grating formula.

Ans. It is an arrangement equivalent to a large number of parallel slits of equal widths and separated from one another by equal opaque strips.

The grating formula is given by

$$(e+d)\sin\theta = \pm n\lambda$$

The $\pm$ sign shows that there are two principle maxima for each order lying on either side of the zero order maximum.

$(e+d)$ is grating element.

n = order of spectrum, λ = wavelength, θ = diffraction angle.

Q.7. Define the limit of resolution of a telescope.

Ans. The limit of resolution is defined as the angle subtended at the objective by the two point-objects which are just resolved when seen through the telescope. The smaller the value of this angle, the higher is said to be the resolving power of the telescope.

Limit of resolution of a telescope is given by $= 1.22\dfrac{\lambda}{d}$.

Q.8. What do you mean by resolving power of a microscope?

Ans. The resolving power of a microscope represents its ability to form distinctly seperate images of two objects lying close together.

"It is measured by the smallest distance between two point objects whose images are just resolved by the objective of the microscope."

The smaller is the distance the higher is said to be resolving power.

SECTION-B SHORT ANSWER TYPE QUESTIONS

Q.1. Differentiate between interference and diffraction.

Ans.

Difference between Interference and Diffraction

The difference between interference and diffraction are as follows :

1. Interference occurs due to the interaction of light coming from two different wave front originating from the same source, while diffraction is the result of interaction between light coming from different parts of the same wavefront.
2. In an interference pattern regions of minimum intensity are perfectly black, while in diffraction points of minimum intensity are not perfectly black.
3. The interference fringes are usually of the same width but the diffraction fringes are never of the same width.
4. In interference all bright bands are of uniform intensity while in diffraction they are not of same intensity.
5. The interference fringes are equally spaced (not always) while diffraction fringes are never equally spaced.

Q.2. Distinguish between Fresnel and Fraunhofer classes of diffraction.

Ans.

Difference between Fresnel and Fraunhofer Classes of Diffraction

Diffraction Phenomenon are divided into two groups :

1. **Fresnel Diffraction :** In this class of diffraction, the source of light on the screen on which diffraction pattern is observed, or usually both are at finite distances from the diffraction obstacle or aperture. In this case no lenses are used and the incident wavefront is either spherical or cylindrical.
2. **Fraunhofer Diffraction :** In this type of diffraction, the source of light and the screen are effectively at infinite distances from the diffracting obstacle or the aperture. This is achieved by placing the source and screen in the focal planes of two lenses. The incident wavefront is plane.

Q.3. Compare the working of a zone plate with that of convergence lens.

Ans.

Comparison between the Action of Zone Plate and Convex (Convergence) Lens

Resemblance : 1. Focal length of both depends upon wavelength, hence both show chromatic aberration.

2. The relation between the conjugate distances is similar for both *i.e.*, each one produces real image of an object on the other side of the object.

Difference : 1. The zone plate have multiple focal lengths in comparison to convex lens and forms a series of point images of decreasing intensity.

2. For a convex lens, all the rays reaching an image point have the same optical path, but for the zone plate the path difference between the rays from two successive transparent zones is λ.
3. For a zone plate the focal length for red colour is less than that for violet *i.e.*, $f_r < f_v$, while in case of lens $f_r > f_v$. Hence the order of colours in the chromatic effects are opposite in two cases.

Q.4. Consider a zone plate with radii $r_n = 0.1\sqrt{n}$ cm. For $\lambda = 5\times10^{-15}$ cm. Calculate the positions of foci.

Sol. The foci of zone plate is given by

$$f_n = \frac{r_n^2}{n\lambda}, \frac{r_n^2}{3n\lambda}, \frac{r_n^2}{5n\lambda} \ldots \text{etc}$$

$$f_n = \frac{(0.1\sqrt{n})^2}{n\lambda}, \frac{(0.1\sqrt{n})^2}{3n\lambda}, \frac{(0.1\sqrt{n})^2}{5n\lambda} \ldots$$

$$= \frac{0.01\,n}{n\lambda}, \frac{0.01\,n}{3n\lambda}, \frac{0.01\,n}{5n\lambda}, \ldots$$

$$= \frac{0.01}{\lambda}, \frac{0.01}{3\lambda}, \frac{0.01}{5\lambda}, \ldots$$

$$= \frac{0.01}{5\times10^{-15}}\text{ cm}, \frac{0.01}{3\times5\times10^{-15}}\text{ cm}, \frac{0.01}{5\times5\times10^{-15}}\text{ cm}$$

$$= 2\times10^{12}\text{ cm}, 0.66\times10^{12}\text{ cm}, 0.4\times10^{12}\text{ cm}$$

$f_1 = 2\times10^{12}$ cm, $f_2 = 0.66\times10^{12}$ cm, $f_3 = 0.4\times10^{12}$ cm.

Q.5. Differentiate between spectrum produced by prism and grating.

Ans. Difference between Prismatic and Grating Spectra

	Prism Spectrum	Grating Spectrum
1.	The prism spectrum is formed by dispersion, which is due to the different velocities of light of different wavelengths through the prism.	The grating spectrum is formed by diffraction, the angle if diffraction for the maximum intensity being different for different wavelengths.
2.	The prism forms only one spectrum.	The grating forms a number of spectra of different orders on each side of central image.
3.	The prism spectrum is bright, since all the incident light is distributed only in a single spectrum.	The grating spectra are much fainter because most of the intensity goes to the zero order maximum and the rest is distributed among the spectra of different orders.
4.	Starting from the position of the direct image of the slit, the spectral colours are in the order from red to violet.	Starting from the zero-order maximum, the spectral colours are in the order from violet to red.
5.	The prism spectrum depends on the material of the prism.	The grating spectra are independent of the material of the grating.
6.	The resolving power of a prism is small. Hence usually the prism spectrum does not show the fine structure of spectral lines.	The resolving power of the grating is large. Hence grating spectrum shows the fine structure of spectral lines.

Q.6. Distinguish between dispersive and resolving power of a grating.

Ans. Dispersive Power of Grating : The dispersive power of a diffraction grating is defined as the rate of change of the angle of diffraction with the wavelength of light. It is expressed as $d\theta/d\lambda$.

The dispersive power is given by

$$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$$

Resolving Power of Grating : The resolving power of the grating expresses the degree of closeness which the spectral lines can have and yet be distinguished as two. It is measured by $\lambda / d\lambda$, where $d\lambda$ is the smallest wavelength difference that can be just resolved at wavelength λ. Its value is given by

$$\frac{\lambda}{d\lambda} = Nn$$

Where N is the total number of rulings on the grating. This greater is the width of the rules surface, higher is the resolving power.

Q.7. Explain Rayleigh criterion for just resolution.

Ans. Rayleigh Criterion for just Resolution

Lort Rayleigh proposed the following criterion for resolution which has been universally adopted :

"Two spectral lines of equal intensities are just resolved by an optical instrument when the principal maxima of diffraction pattern due to one falls on the first minimum of the diffraction pattern of the other.

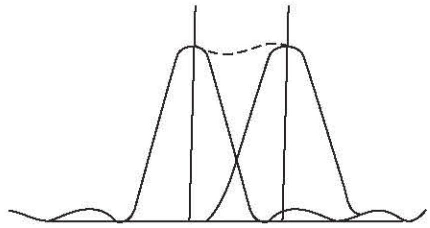


Fig. 1

Fig. 1 shows the intensity curves of two patterns such that the principal maxima of one coincides with the first minimum of the other. The eye will see the combined effect of the two, which is shown by the resultant dotted curve. The curve shows a dip in the middle indicating the presence of two different spectral lines. The lines are said to be just resolved.

If an optical instrument just resolve two spectral lines of wavelengths λ and $\lambda + d\lambda$, then $\lambda / d\lambda$ is taken as the measure of resolving power of the instrument.

Q.8. A plane transmission grating has 40,000 lines in all, with grating element 12.5×10^{-5} cm. Calculate the maximum resolving power for which it can be used in the range of wavelength 5000 Å.

Sol. The maximum order that can be observed with a grating of element $(e+d)$ for a wavelength λ is given by

$$n_{max} = \frac{e+d}{\lambda} \qquad [(e+d)\sin\theta = n\lambda]$$

Here, $(e+d) = 12.5 \times 10^{-5}$ cm and $\lambda = 5000$ Å $= 5 \times 10^{-5}$ cm

$$\therefore \quad n_{max} = \frac{12.5 \times 10^{-5} \text{ cm}}{5 \times 10^{-5} \text{ cm}}$$

Hence, the second order is the highest which is observed with this grating. The maximum resolving power with this grater

$$R = Nn = 40000 \times 2 = 80000$$

Q.9. A parallel beam of light is normally incident on a plane transmission grating having 4250 lines per cm and a second order spectral line is observed at an angle of 30°. Calculate the wavelength of light.

Sol. When light falls normally on a grating, the directions θ of the maximum intensity in the diffracted light are given by

$$(e+d)\sin\theta = n\lambda \qquad n = 0, \pm 1, \pm 2, \ldots$$

Where $(e+d)$ is the grating element and n is the order of maximum

$$\therefore \quad \lambda = \frac{(e+d)\sin\theta}{n}$$

There are 4250 lines per cm, therefore

$$(e+d) = \frac{1}{4250} \text{ cm}$$

$\theta = 30°$ and $n = 2$

$\therefore$
$$\lambda = \frac{1}{4250}\text{ cm}\,\frac{\sin 30°}{2} = 5.882 \times 10^{-5}\text{ cm}$$
$$\lambda = 5882\text{ Å}$$

SECTION-C LONG ANSWER TYPE QUESTIONS

Q.1. What are Fresnel's half period zones? Prove that the area of a half-period zone on a plane wavefront is independent of the order of the zone, and that the amplitude due to a large wavefront at a point infront it is just half that due to the first half-period zone acting alone.

Ans.

Fresnel's Half-Period Zones

Light waves travelling in space set the hypothetical ether particles in vibration. The continuous locus of the either particles in the same phase of vibration is called the 'wavefront'. According to Huygens' principle, each point on wavefront sends out secondary wavelets. Fresnel assumed that these wavelets interfere and produce a resultant intensity of light at any point. In order to calculate the resultant intensity at a point due to a wavefront, Fresnel divided the wavefront into a number of zones called 'Fresnel's half-period zones', as under :

Construction of Half-period Zones : Let *ABCD* (Fig. 1) be a plane wavefront of monochromatic light of wavelength λ, travelling from left to right.

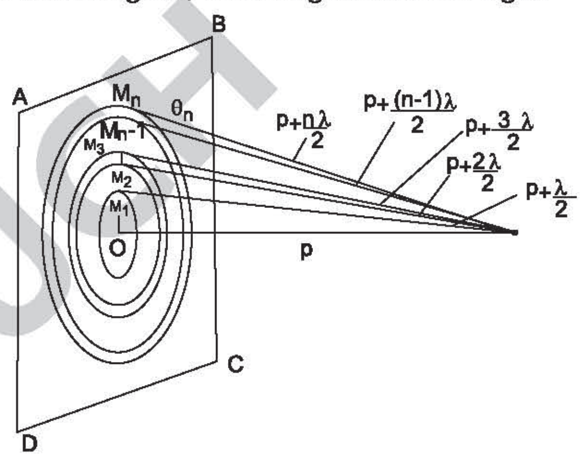


Fig. 1

Let P be an external point at which the effect of the entire wavefront is to be found. Let us draw from the point P a perpendicular PO to the wavefront. Let $PO = p$. The point O is called the 'pole' of the wavefront corresponding to P. Let us draw, with P as centre and radii $p + \lambda/2, p + 2\lambda/2$, $p + 3\lambda/2$, etc., a series of spheres. The sections of these spheres by the plane wavefront are concentric circles about the common centre O. The area of the first (innermost) circle is called the 'first half-period zone'; the annular area between the first circle and the second circle is called the 'second half-period zone', and so on. Thus, the annular area between $(n-1)$th circle and the nth circle is the nth half-period zone.

Area of a Zone : The area of the nth zone (see Fig. 1)

$$= \pi OM_n^2 - \pi OM_{n-1}^2$$
$$= \pi [(PM_n^2 - PO^2) - (PM_{n-1}^2 - PO^2)]$$
$$= \pi \left[\left\{ \left(p + \frac{n\lambda}{2} \right)^2 - p^2 \right\} - \left\{ \left(p + (n-1)\frac{\lambda}{2} \right)^2 - p^2 \right\} \right]$$
$$= \pi \left[pn\lambda + \frac{n^2\lambda^2}{4} - p(n-1)\lambda - (n-1)^2 \frac{\lambda^2}{4} \right]$$
$$= \pi \left[p\lambda + \frac{\lambda^2}{4} \{n^2 - (n-1)^2\} \right]$$
$$= \pi \left[p\lambda + \frac{\lambda^2}{4}(2n-1) \right] \qquad ...(1)$$
$$\approx \pi p\lambda,$$

because $p >> \lambda$, and so the λ^2 term can be ignored as compared to $p\lambda$ in above. That is, the area of each zone ($\pi p\lambda$) is approximately the same, free from the order n of the zone. (Strictly, it increases slightly with n).

The average distance of the nth zone from P (see Fig. 1)

$$= \frac{\left\{ p + \frac{n\lambda}{2} \right\} + \left\{ p + (n-1)\frac{\lambda}{2} \right\}}{2}$$
$$= p + (2n-1)\frac{\lambda}{4}. \qquad ...(2)$$

Amplitude due to a Zone : The amplitude of the disturbance at P due to a zone is :

(i) directly proportional to the area of the zone (as the number of point-sources in a zone are proportional to its area),

(ii) inversely proportional to the average distance of the zone from P, and

(iii) directly proportional to the obliquity factor $(1 + \cos\theta_n)$, where θ_n is the angle between the normal to the zone and the line joining the zone to P.

Therefore, using eq. (1) and (2), the amplitude due to the to the nth zone is

$$R_n \propto \frac{n\left[p\lambda + \frac{\lambda^2}{4}(2n-1) \right]}{p + (2n-1)\frac{\lambda}{4}} (1 + \cos\theta_n)$$
$$\propto \pi\lambda (1 + \cos\theta_n).$$

Now, as order n of the zone increases, θ_n increases and $\cos\theta_n$ decreases. Therefore, **the amplitude of the wave at P due to a zone decreases as the order of the zone increases.**

Resultant Amplitude due to all the Zones (whole Wavefront *ABCD*) : Let $R_1, R_2, R_3, \ldots R_n$ be the amplitudes due to the first, second, third ...*n*th zones respectively. They are in descending order of magnitude. Further, since the average distance of *P* from two consecutive zones differ by $\lambda/2$, the waves from two consecutive zones reach *P* in the opposite phase. Hence if the amplitude due to the first zone is positive, that due to second zone is negative, that due to the third is positive, and so on. Hence the resultant amplitude at *P* due to the entire wavefront is given by

$$R = R_1 - R_2 + R_3 - R_4 + \ldots (-1)^{n-1} R_n \qquad \text{...(3)}$$

Now, the successive terms $R_1, R_2, R_3 \ldots$ gradually decrease in magnitude. Thus, R_2 is slightly smaller than R_1 but slightly greater than R_3. Hence to a close approximation, we may put

$$R_2 = \frac{R_1 + R_3}{2} \qquad \text{...(4)}$$

Similarly, $$R_4 = \frac{R_3 + R_5}{2} \qquad \text{...(5)}$$

and so on. The eq. (3) can be written as

$$R = \frac{R_1}{2} + \left(\frac{R_1}{2} - R_2 + \frac{R_3}{2}\right) + \left(\frac{R_3}{2} - R_4 + \frac{R_5}{2}\right) + \ldots.$$

the last term being $\frac{R_n}{2}$ or $\left(\frac{R_{n-1}}{2} - R_n\right)$ according as n is odd or even.

In view of the relations (4), (5)... etc. the value of each bracket in the above expression is zero. Hence,

$$R = \frac{R_1}{2} + \frac{R_n}{2} \qquad \text{(when } n \text{ is odd)}$$

and $$R = \frac{R_1}{2} + \frac{R_{n-1}}{2} - R_n \qquad \text{(when } n \text{ is even)}$$

In practice, 1 is very large so that, on account of obliquity, we may wite

$$R_{n-1} = R_n = 0$$

$$\therefore \quad R = \frac{R_1}{2}$$

Thus the amplitude due to the complete wavefront at a point infrant *R* it is just half that due to the first half period zone acting alone.

Q.2. What is a zone plate? Give its theory. Show that a zone plate has multiple foci. What is meant by 'phase reversal zone plate'?

Ans.

Zone Plate and its Construction

A zone plate is a special diffracting screen designed to obstruct the light from the alternate half-period zones. It is constructed by drawing a series of concentric circles on a sheet of white paper with radii proportional to the square-root of the natural numbers. The alternate zones are painted black. A highly reduced photograph of this drawing is then taken on a plane glass plate. The negative thus obtained is the zone plate (Fig.). This plate behaves like a convex lens.

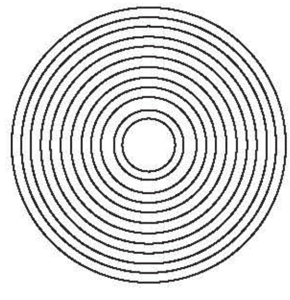


Fig. 1

Theory of the Zone Plate : Let O (Fig. 2) be a luminous point-object emitting spherical waves of wavelength λ. Let AB be an imaginary transparent plate perpendicular to the plane of the paper. Let OP be a perpendicular to AB and produced to I. Let us find the intensity of light at the point I on a screen.

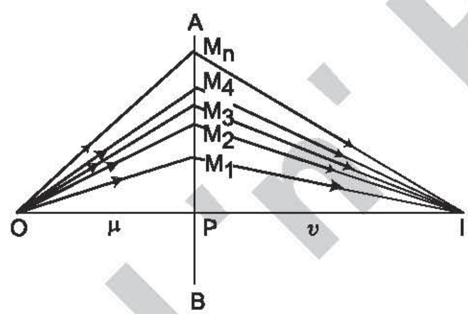


Fig. 2

Let concentric circles be drawn on the plate with P as centre and radii equal to $PM_1 = r_1$, $PM_2 = r_2$, ... $PM_n = r_n$, such that

$$OM_1I - OPI = \frac{\lambda}{2}$$

$$OM_2I - OPI = \frac{2\lambda}{2}$$

....

....

$$OM_nI - OPI = \frac{n\lambda}{2} \qquad ...(1)$$

Then, for the point I, the area of the first circle is the first half-period zone, the area between the second and the first circle is the second half-period zone, and so on, the area between the nth and thc $(n-1)$th circle being the nth zone. (This is because the path of the waves reaching I through two consecutive zones differ by $\lambda/2$).

Let us now calculate the radius r_n, of n th circle. Let $OP = u$, $P = v$, then

$$OM_n = [u^2 + r_n^2]^{1/2}$$

$$= u\left[1 + \frac{r_n^2}{u^2}\right]^{1/2}$$

$$= u\left[1 + \frac{r_n^2}{2u^2}\right] \text{ to a first approximation}$$

$$= u + \frac{r_n^2}{2u} \qquad ...(2)$$

Similarly,
$$M_n I = v + \frac{r_n^2}{2v} \qquad ...(3)$$

$\therefore$
$$OM_n I - OPI = OM_n + M_n I - OPI$$
$$= u + \frac{r_n^2}{2u} + v + \frac{r_n^2}{2v} - (u + v)$$
$$= \frac{r_n^2}{2}\left(\frac{1}{u} + \frac{1}{v}\right) \qquad ...(4)$$

Comparing eq. (1) and (4), we get

$$\frac{r_n^2}{2}\left[\frac{1}{u} + \frac{1}{v}\right] = \frac{n\lambda}{2} \qquad ...(5)$$

or
$$r_n^2 = \frac{uv}{u+v} n\lambda \qquad ...(6)$$

or
$$r_n = \sqrt{\frac{u\,v\,\lambda}{u+v}}\,\sqrt{n}$$

or
$$r_n \propto \sqrt{n},$$

that is, **the radii of circles are proportional to the square-roots of natural numbers**. Thus, if the alternate zones be made opaque, the plate will serve as a zone plate.

Focal Length : The relation between u and v, the respective distances of the object and the image, is given by eq. (5) which gives

$$\frac{1}{v} + \frac{1}{u} = \frac{n\lambda}{r_n^2}$$

Comparing it with the lens formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$, we get

$$f = \frac{r_n^2}{n\lambda},$$

which gives the focal length of the zone plate. Since the wavelength λ appears in this expression, a zone plate has severe chromatic aberration.

Multiple Foci of Zone Plate : The zone plate has a number of foci. The eq. (5) shows that for an object at infinity ($u = \infty$), the radius of the nth circle will be given by

$$r_n^2 = vn\lambda$$

so that the area of the nth zone is

$$\pi r_n^2 - \pi r_{n-1}^2 = \pi v n\lambda - \pi v\,(n-1)\,\lambda$$
$$= \pi v\lambda.$$

In this case the image I will be formed at a distance $v = \frac{r_n^2}{n\lambda}$ (equal to the focal length).

Now, consider a point I_3 at a distance $v_3 = \frac{r_n^2}{3n\lambda}$ from the zone plate. It we imagine the zone plate to be divided into half-period elements, the area of each half-period element will be one-third the area of each zone. Hence each zone on the zone on the plate will contain three half-period elements corresponding to I_3.

The resultant amplitude at I_3 will be, therefore,

$$S = (s_1 - s_2 + s_3) + (s_7 - s_8 + s_9) + (s_{13} - s_{14} + s_{15}) + \ldots$$

$$= \left\{\left(\frac{s_1}{2} - s_2 + \frac{s_3}{2}\right) + \left(\frac{s_1}{2} + \frac{s_3}{2}\right)\right\} + \left\{\left(\frac{s_7}{2} - s_8 + \frac{s_9}{2}\right) + \left(\frac{s_7}{2} + \frac{s_9}{2}\right)\right\} + \ldots$$

$$= \frac{s_1 + s_3}{2} + \frac{s_7 + s_9}{2} + \frac{s_{13} + s_{15}}{2} + \ldots.$$

$$= \frac{1}{2}[s_1 + s_3 + s_7 + s_9 + s_{13} + s_{15} + \ldots.],$$

where s_1, s_2, s_3 etc., are roughly one-third of R_1, R_2, R_3 etc. Hence the point I_3 receives sufficient intensity, though less than that at I. Hence I_3 is another image of O. Similarly, it can be shown that points I_5, I_7, I_9 etc., distant $\frac{r_n^2}{5n\lambda}, \frac{r_n^2}{7n\lambda}, \frac{r_n^2}{9n\lambda}$ from the zone plate are images O, but of successively diminishing intensity. Hence a zone plate has multiple foci, the focal lengths being

$$\frac{r_n^2}{n\lambda}, \frac{r_n^2}{3n\lambda}, \frac{r_n^2}{5n\lambda}, \ldots\ldots \text{ etc.}$$

Phase Reversal Zone Plate : Wood, in 1898, instead of blocking the alternate zones of the zone plate, introduced an additional optical path of $\lambda/2$ between the wave from successive zones. Then, the amplitudes from successive zones helped each other. Such plates are called 'phase reversal plates.' They are prepared in the following way :

A chemically cleaned glass plate is coated with a thin layer of gelatine solution and dried. It is then sensitised by immersing it in a weak solution of potassium dichromate for a few seconds. It is now again dried in the dark, and after placing in contact with an ordinary zone plate, it is exposed to sunlight. Light passing through transparent zones acts on gelatine and makes it insoluble in water while the gelatine in contact with opaque zones remains soluble in water. By immersing the glass plate in water the gelatine of the unexposed parts is dissolved to such a depth that the optical paths from successive zones on the glass. Plate have an additional optical path of $\lambda/2$. Such 'Phase reversal zone plates' produce four times more intense image than an ordinary zone plate.

Q.3. Describe Fraunhofer's diffraction due to a single slit and deduce the positions of maxima and minima and find the relative intensities of successive maxima. Plot the intensity distribution curve. What will happen if the width of the slit is made equal to the wavelength of light?

Ans. Fraunhofer's Diffraction at a Single Slit

Let a parallel beam of monochromatic light of wavelength λ be incident normally upon a narrow slit of width $AB = e$ (Fig. 1) placed perpendicular to the plane of paper. Let the diffracted light be focussed by a convex lens L on a screen XY placed in the focal plane of the lens. The diffraction pattern obtained on the screen consists of a central bright band, having alternate dark and weak bright bands of decreasing intensity on both sides.

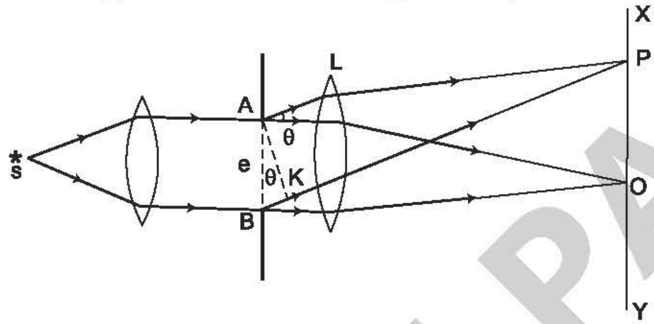


Fig. 1

Explanation : In terms of wave theory, a plane wavefront is incident normally on the slit AB. According to Huygens' principle, each point in AB sends out secondary wavelets in all directions. The rays proceeding in the same direction as the incident rays are focussed at O; while those diffracted through an angle θ are focussed at P. Let us find out the resultant intensity at P.

Let AK be perpendicular to BK. As the optical paths from the plane AK of the point P are equal, the path difference between the wavelets from A and B in the direction θ is

$$BK = AB \sin\theta = e \sin\theta$$

The corresponding phase difference is

$$\frac{2\pi}{\lambda} \times \text{path difference} = \frac{2\pi}{\lambda}(e \sin\theta)$$

Let the width AB of the slit be divided into n equal parts. The amplitude of vibration at P due to the waves from each part will be the same, say equal to a. The phase difference between the waves from any two consecutive parts is $\frac{1}{n}\left(\frac{2\pi}{\lambda} e \sin\theta\right) = d$ (say). Hence the resultant amplitude at P is given by

$$R = \frac{a \sin(nd/2)}{\sin(d/2)} = \frac{a \sin\left(\frac{\pi e \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e \sin\theta}{n\lambda}\right)}$$

Let us put $\frac{\pi e \sin\theta}{\lambda} = \alpha$. Then

$$R = \frac{a \sin\alpha}{\sin\alpha/n} = \frac{a \sin\alpha}{\alpha/n} \qquad [\because \alpha/n \text{ is small}]$$

$$= \frac{na \sin\alpha}{\alpha}$$

As $n \to \infty, a \to 0$ but the product na remains finite. Let $na = A$. Then

$$R = \frac{A \sin \alpha}{\alpha}$$

Therefore, the resultant intensity at P, being proportioal to the square of the amplitude, is

$$I = R^2 = A^2 \left(\frac{\sin \alpha}{\alpha}\right)^2 \qquad ...(1)$$

the constant of proportionality being taken as unity for simplicity.

Directions of Minima : It is clear from eq. (1) that the intensity is a minimum (zero) when

$$\frac{\sin \alpha}{\alpha} = 0$$

or $\quad \sin \alpha = 0$, (but $\alpha \neq 0$)

or $\quad \alpha = \pm m\pi$,

where m has an integral value 1, 2, 3 except zero.

But $\alpha = \dfrac{\pi e \sin \theta}{\lambda}$. Therefore, the last expression becomes

$$\frac{\pi e \sin \theta}{\lambda} = \pm m\pi$$

or $\quad e \sin \theta = \pm m\lambda \qquad ...(2)$

This equation gives the direction of the first, second, third ...... minima by putting $m = 1, 2, 3, \ldots$

Directions of Maxima : To find the directions of maximum intensity, let us differentiate eq. (1) with respect to α and equate to zero, that is,

$$\frac{dI}{d\alpha} = 0$$

or $\quad \dfrac{d}{d\alpha}\left[A^2 \left(\dfrac{\sin \alpha}{\alpha}\right)^2\right] = 0$

or $\quad A^2 \left(\dfrac{2 \sin \alpha}{\alpha}\right) \dfrac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} = 0$

or $\quad \dfrac{\alpha \cos \alpha - \sin \alpha}{\alpha^2} = 0$

or $\quad \alpha \cos \alpha - \sin \alpha = 0$

or $\quad \alpha = \tan \alpha$.

This equation is solved graphically by plotting the curves.

$$y = \alpha$$

and $\quad y = \tan \alpha \qquad ...(3)$

The first of these is a straight line through origin making an angle of 45°, while the second is a discontinuous curve having a number of branches (Fig.2). The points of intersection of the two give the values of a satisfying eq. (3). These values are approximately given out as

$$\alpha = 0, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$$

or more exactly as

$$\alpha = 0, 1.430\pi, 2.462\pi, 3.471\pi \ldots$$

Substituting the approximate values of in eq. (1), we get the intensities of various maxima. Thus, the intensity of the central (principal) maximum is

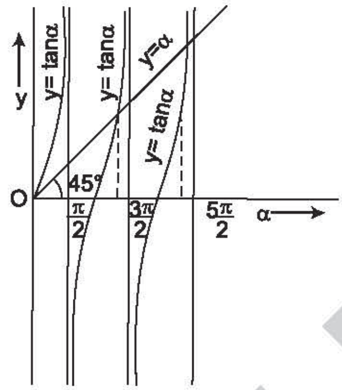


Fig.2

$$I_0 = A^2\left(\frac{\sin 0}{0}\right)^2 = A^2$$

Similarly, the intensity of the first subsidiary maximum is

$$I_1 \approx A^2\left\{\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right\}^2 \approx \frac{4}{9\pi r^2}A^2 \approx \frac{A^2}{22},$$

that of the second subsidiary maximum is

$$I_2 \approx A^2\left\{\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right\}^2 \approx \frac{4}{25\pi^2}A^2 \cong \frac{A^2}{61}$$

and so on. Thus, the intensities of the successive maxima are in the ratio

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : \ldots.$$

or

$$1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} : \ldots.$$

Clearly, most of the incident light is concentrated in the principal maximum which occurs in the direction given by

$$\alpha = 0$$

or

$$\frac{\pi e \sin\theta}{\lambda} = 0$$

or

$$\theta = 0,$$

that is, in the same direction as the incident light.

Thus, the diffraction pattern consists of a bright principal maximum in the direction of the incident light, having alternately minima and weak subsidiary maxima of rapidly decreasing intensity on either side of it. The minima lie at $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi$ .... The subsidiary maxima do not fall exactly mid-way between two minima, but are displaced toward the centre of the pattern by an amount which decreases with increasing order. The intensity distribution in the pattern is shown in Fig.3.

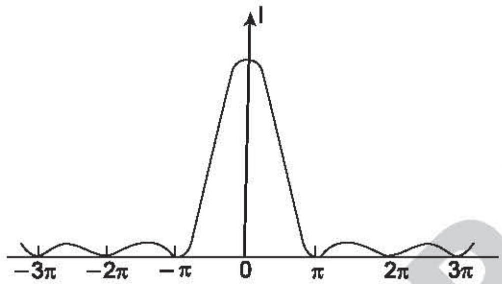


Fig.3

What happens if slit is made narrower : The 'first' minimurn on either side of the central maximum occurs in the direction θ given by

$$e \sin\theta = \pm\lambda$$

When the slit is narrowed (*i.e.*, e is reduced, the angle θ increases which means that the central maximum becomes wider. When the slit-width is as small as wavelength ($e = \lambda$), the first minimum occurs at $\theta = 90°$, which means that the central maximum fills the whole space.

Q.4. Explain Fraunhofer's diffraction due to a double slit. How does its intensity distribution curve differ from the curve obtained due to a single slit? What is the effect of (a) increasing the slit-width, (b) increasing the slit separation and (c) increasing the wavelength of light. What are missing orders?

Ans.

Fraunhofer's Diffraction at a Double-Slit

Let a parallel beam of monochromatic light of wavelength λ be incident normally upon two parallel slits *AB* and *CD* (Fig.1.), each of width e, separated by opaque space of width *d*. The distance between the corresponding points of the two slits is $(e + d)$. Let the diffracted light be focussed by a convex lens *L* on a screen *XY* placed in the focal plane of the lens. The pattern obtained on the screen is the diffraction pattern due to a single slit on which a system of interfernce fringes is superposed.

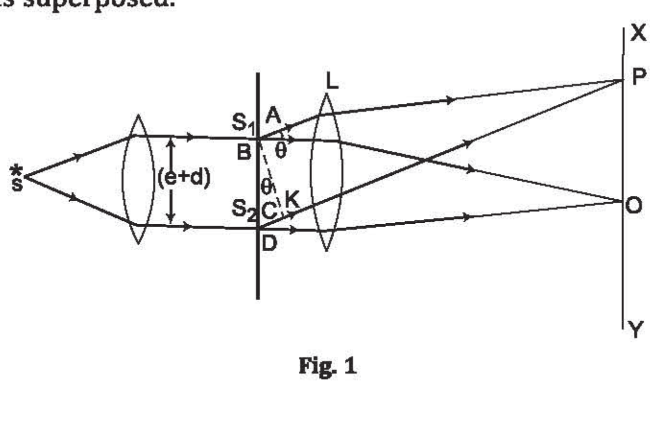


Fig. 1

Explanation : By Huygens' principle, every point in the slits AB and CD sends out secondary wavelets in all directions. From the theory of diffraction at a single slit, the resultant amplitude due to wavelets diffracted from each slit in a direction θ is

$$\frac{A\sin\alpha}{\alpha} \quad ...(1)$$

where A is a constant and $\alpha = \frac{\pi e \sin\theta}{\lambda}$.

We can, therefore, consider the two slits as equivalent to two coherent sources placed at the middle points S_1 and S_2 of the slits, and each sending a wavelet of amplitude $\frac{A\sin\alpha}{\alpha}$ in a direction θ. Consequently, the resultant amplitude at a point P on the screen will be the result of interference between two waves of same amplitude $\frac{A\sin\alpha}{\alpha}$, and having a phase difference δ (say).

Let us drop S_1K perpendicular to S_2K. The path difference between the wavelets from S_1 and S_2 the direction θ is

$$S_2K = (e+d)\sin\theta$$

Hence the phase difference between them is

$$\delta = \frac{2\pi}{\lambda} \times \text{path difference} = \frac{2\pi}{\lambda}(e+d)\sin\theta.$$

The resultant amplitude R at P can be determined by the vector amplitude diagram (Fig.), which gives

$$OB^2 = OA^2 + AB^2 + 2(OA)(AB)\cos BAC$$

or

$$R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 + \left(\frac{A\sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A\sin\alpha}{\alpha}\right)\left(\frac{A\sin\alpha}{\alpha}\right)\cos\delta$$

$$= \left(\frac{A\sin\alpha}{\alpha}\right)^2 (2+2\cos\delta)$$

$$= \frac{A^2\sin^2\alpha}{\alpha^2} 4\cos^2\frac{\delta}{2} = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta,$$

where

$$\beta = \frac{\delta}{2} = \frac{\pi}{\lambda}(e+d)\sin\theta.$$

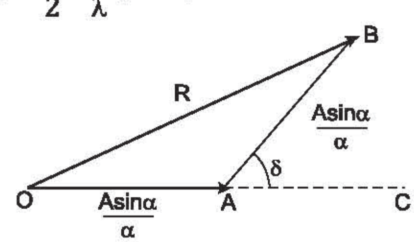


Fig. 2

Therefore, the resultant intensity at P is

$$I = R^2 = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta \quad ...(2)$$

Thus, the intensity in the resultant pattern depends on two factors : (1) $\sin^2\alpha/\alpha^2$ which gives diffraction pattern due to each individual slit and (2) $\cos^2\beta$ which gives interference pattern due to diffracted light waves from the two slits.

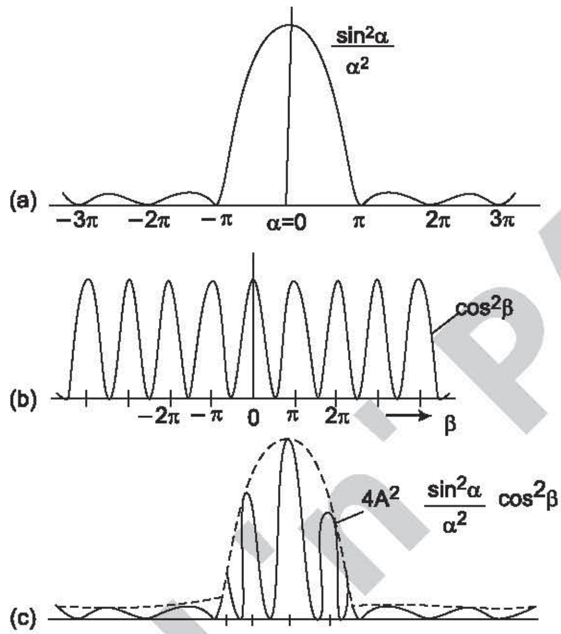


Fig.3

The diffraction term $\frac{\sin^2\alpha}{\alpha^2}$ gives a central maximum in the direction $\theta = 0$, having alternately minima and subsidiary maxima of decreasing intensity on either side (Fig.3a). The minima are obtained in the directions given by

$$\sin\alpha = 0$$

or $$\alpha = \pm m\pi$$

or $$\frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

or $$e\sin\theta = \pm m\lambda, \qquad ...(3)$$

where $m = 1, 2, 3,...$ (but not zero).

The interference term $\cos^2\beta$ gives a set of equidistant dark and bright fringes, as in Young' double-slit interference experiment (Fig.3b). The bright fringes (maxima) are obtained in the directions given by

$$\cos^2\beta = 1$$

or $$\beta = \pm n\pi$$

or $$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

or $$(e+d)\sin\theta = \pm n\lambda, \qquad ...(4)$$

where $n = 0, 1, 2,...$ The various maxima corresponding to $n = 0, 1, 2,......$ are the zero-order, first-order, second-order......maxima.

The intensity distribution in the resultant diffraction pattern is shown in Fig.3c, which is a plot of the product of the constant term $4A^2$, diffraction term $\frac{\sin^2 \alpha}{\alpha^2}$ and the interference term $\cos^2 \beta$. The entire pattern may be considered as a consisting of interfernce fringes due to light from both slits, the intensities of these fringes being governed by diffraction occurring at the individual slits.

(a) **Effect of increasing the slit-width :** If we increase the slit-width l, the envelope of the fringe-pattern changes so that its central peak is sharper. The fringe-spacing, which depends on slit-separation, does not change. Hence less interference maxima now fall within the central diffraction maximum.

(b) **Effect of increasing the distance between slits :** If the slit-width l is kept constant and the separation d between them is increased, the fringes become closer together, the envelope of the pattern remaining unchanged. Thus, more interference maxima fall within the central envelope.

Absent Orders : For certain values of d, certain interference maxima become absent from the pattern. Suppose for some value of θ, the following conditions are simultaneously satisfied :

$$(e+d)\sin\theta = \pm n\lambda \qquad \text{(interference maxima) and}$$

and $$e\sin\theta = \pm m\lambda. \qquad \text{(diffraction minima)}$$

According to the first condition, there should be an interference maximum in the direction θ, but according to the second condition there is no diffracted light in this direction. Therefore, the interference maximum will be absent in this direction.

From the above two eq. we get

$$\frac{e+d}{e} = \frac{n}{m} \qquad ...(5)$$

If $d = e$, then $$n = 2m$$
$$= 2, 4, 6, ... \qquad (\text{since } m = 1, 2, 3, ...)$$

that is, the 2nd, 4th, 6th .... order interference maxima will be absent, that is, they will coincide with 1st, 2 nd, 3 rd... order diffraction minima. Thus, the central diffraction maximum will have three interference (the zero-order and two first-order) maxima.

If $d = 2e$, then $$n = 3m \qquad \text{[from eq. (5)]}$$
$$= 3, 6, 9, ... \qquad (\text{since } m = 1, 2, 3, ...),$$

that is, the 3rd, 6th, 9th...order interference maxima will coincide with 1st, 2nd, 3rd,...order diffraction minima. Thus, the central diffraction maximum will have five interference maxima.

If $d = 3e$, then $n = 4m = 4, 8, 12, ...$

that is, 4th, 8th, 12th,...order interference maxima will coincide with 1st, 2nd, 3rd...order diffraction minima. Thus, the central diffraction maximum will have seven interference maxima.

If $d = 4e$, then the central diffraction maximum will contain nine interference maxima (Fig.4).

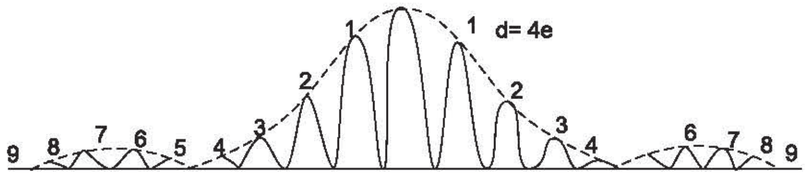


Fig.4

Thus, as d increases to e, $2e$, $3e$, $4e$,... the number of interference maxima within the central diffraction maximum increases.

(c) **Effect of increasing wavelength :** On increasing à, the envelope becomes broader, and the fringes move further apart.

Q.5. Give the construction and theory of a plane diffraction grating of the transmission type, and explain the formation of spectra by it.

Ans. **Plane Transmission Diffraction Grating**

A diffraction grating is an arrangement equivalent to a large number of parallel slits of equal widths and separated from one another by equal opaque spaces. It is made by ruling a large number of fine, equidistant and parallel lines on an optically-plane glass plate with a diamond point. The rulings scatter the light and are effectively opaque while the unruled parts transmit light and act as slits.

Theory : Let AB (Fig.1.) be the section of a plane transmission grating, the lengths of the slits being perpendicular to the plane of the paper. Let e be the width of each slit and d the width of each opaque space between the slits. Then $(e+d)$ is called the 'grating element'. The points in two consecutive slits separated by the distance $(e+d)$ are called the "corresponding points".

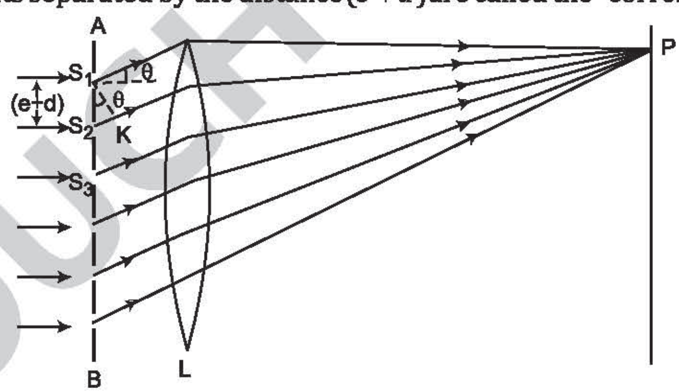


Fig.1.

Let a parallel beam of monochromatic light of wavelength λ be incident normally on the grating. By Huygens' principle, all the points in each slit send out secondary wavelets in all directions. By the theory of Fraunhofer diffraction at a single slit, the wavelets from all points in a slit diffracted in a direction are equivalent to a single wave of amplitude $\frac{A \sin \alpha}{\alpha}$ starting from the middle point of the slit, where $\alpha = \frac{\pi}{\lambda} e \sin \theta$.

Thus, if N be the total number of slits in the grating, the diffracted rays from all the slits are equivalent to N parallel rays, one each from the middle points S_1, S_2, S_3, ... of the slits.

Let S_1K be perpendicular to S_2K. Then the path difference between the rays from the slits S_1 and S_2 is

$$S_2K = S_1S_2 \sin\theta = (e+d)\sin\theta.$$

The corresponding phase difference is

$$\frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta \qquad \text{(say)}$$

Hence the resultant amplitude in the direction θ is

$$R = \frac{A\sin\alpha}{\alpha}\frac{\sin N\beta}{\sin\beta}$$

The resultant intensityis therefore given by

$$I = R^2 = \frac{A^2\sin^2\alpha}{\alpha^2}\frac{\sin^2 N\beta}{\sin^2\beta} \qquad \text{...(1)}$$

The first factor $\frac{A^2\sin^2\alpha}{\alpha^2}$ gives a diffraction pattern due to a single slit, while the second factor $\frac{\sin^2 N\beta}{\sin^2\beta}$ gives the interference pattern due to N slits. Let us consider the intensity distribution due to the second factor.

Principal Maxima : When $\sin\beta = 0$, that is,

$$\beta = \pm n\pi, \qquad \text{where } n = 0, 1, 2, 3, \ldots$$

we have $\sin N\beta = 0$, and thus $\frac{\sin N\beta}{\sin\beta} = \frac{0}{0}$, that is, indeterminate. Let us find its value by the usual method of differentiating the numerator and the denominator. Thus

$$\text{limit}_{\beta\to\pm n\pi}\frac{\sin N\beta}{\sin\beta} = \text{limit}_{\beta\to\pm n\pi}\frac{N\cos N\beta}{\cos\beta}$$

$$= \frac{N\cos N(\pm n\pi)}{\cos(\pm n\pi)} = \pm N.$$

The intensity is then [from eq. (1)]

$$I = \frac{A^2\sin^2\alpha}{\alpha^2}N^2,$$

which is a maximum. These maxima are most intense and are called 'principal maxima'. They are obtained in the directions given by

$$\beta = \pm n\pi$$

or $$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

or $$(e+d)\sin\theta = \pm n\lambda, \qquad \text{...(2)}$$

where $n = 0, 1, 2, \ldots$ For $n = 0$, we get the 'zero-order maximum'. For $n = \pm 1, \pm 2, \pm 3, \ldots$ we obtain the first, second, third... order principal maxima respectively. The $\pm$ sign show that there are two principal maxima for each order lying on either side of the zero-order maximum.

The positions of principal maxima given by eq. (2) are the same as given in a two-slit diffraction pattern. Thus, in a diffraction grating the positions of principal maxima do not alter whether the number of slits is 2 or N ($N > 2$), provided the grating element is the same.

Minima : When $\sin N\beta = 0$, but $\sin \beta \neq 0$, then

$$\frac{\sin N\beta}{\sin \beta} = 0$$

and hence [from eq. (1)]

$$I = 0,$$

which is a minimum. These minima are obtained in the directions given by

$$\sin N\beta = 0$$

or $$N\beta = \pm m\pi$$

or $$N \frac{\pi}{\lambda}(e + d)\sin\theta = \pm m\pi$$

or $$N(e + d)\sin\theta = \pm m\lambda \qquad ...(3)$$

where m takes all integral values except 0, N, $2N$,.....nN, because these values of m make $\sin \beta = 0$, which gives principal maxima.

It is clear from above that $m = 0$ gives a principal maximum, $m = 1, 2, 3, ...(N - 1)$ give minima and then $m = N$ gives again a principal maximum. Thus, **there are $(N - 1)$ minima between two consecutive principal maxima.**

Secondary Maxima : As there are $(N - 1)$ minima between two consecutive principal maxima, there must be $(N - 2)$ other maxima between two principal maxima. These are called 'secondary maxima.' Their positions are obtained by differentiating eq. (1) with respect to β and equating it to zero. Thus

$$\frac{dI}{d\beta} = \frac{A^2 \sin^2 \alpha}{\alpha^2} 2\left[\frac{\sin N\beta}{\sin \beta}\right] \frac{N \cos N\beta \sin \beta - \sin N\beta \cos \beta}{\sin^2 \beta} = 0$$

or $$N \cos N\beta \sin \beta - \sin N\beta \cos \beta = 0$$

or $$\tan N\beta = N \tan \beta \qquad ...(4)$$

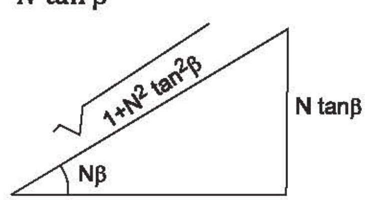


Fig.2.

To find the value of $\frac{\sin^2 N\beta}{\sin^2 \beta}$ under the condition (4), we make use of the triangle shown in Fig.2. This gives

$$\sin N\beta = \frac{N \tan \beta}{\sqrt{(1 + N^2 \tan^2 \beta)}}$$

$$\therefore \quad \frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{N^2 \tan^2 \beta}{(1+N^2 \tan^2 \beta)\sin^2 \beta} = \frac{N^2}{(1+N^2 \tan^2 \beta)\cos^2 \beta}$$

$$= \frac{N^2}{\cos^2 \beta + N^2 \sin^2 \beta} = \frac{N^2}{1+(N^2-1)\sin^2 \beta}.$$

This shows that the intensity of the secondary maxima is proportional to $\frac{N^2}{1+(N^2-1)\sin^2 \beta}$, whereas the intensity of principal maxima is proportional to N^2. Therefore

$$\frac{\text{intensity of secondary maxima}}{\text{intensity of principal maxima}} = \frac{1}{1+(N^2-1)\sin^2 \beta}.$$

Hence, greater the value of N, the weaker are secondary maxima. In an actual grating N is very large. Hence these secondary maxima are not visible in the grating spectrum.

The grating spectrum is graphically presented in Fig.3.

Formation of Multiple Spectra by Grating : When a beam of light of wavelength λ falls normally on a grating, the principal maxima are formed in the directions given by

$$(e+d)\sin\theta = \pm n\lambda; \qquad n = 0, 1, 2, 3, \ldots.$$

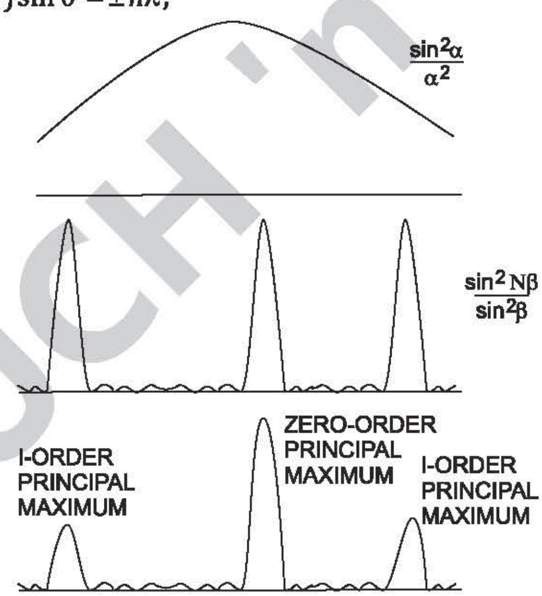


Fig.3.

This equation shows that for a given order n, the angle of diffraction varies with the wavelength. The longer the wavelength, the greater is the angle of diffraction. Hence, if the incident light be white, then cach order will contain principal maxima of different wavelengths in different directions. The principal maxima of all wavelengths corresponding to $n = 1$ will form the first-order spectra, and so on. The principal maxima of all wavelengths corresponding to $n = 0$ will, however, be along the same direction $\theta = 0$. Hence the zero-order maximum will be white; having on either side of it the first-order spectra, the second-order spectra, and so on.

Q.6. Explain and derive an expression for the resolving power of a microscope.

Ans.

Resolving Power of a Microscope

The resolving power of a microscope represents its ability to form distinctly separate images of two objects lying close together. **It is measured by the smallest distance between two point-objects whose images are just resolved by the objective of the microscope**. The smaller is the distance, the higher is said to be the resolving power.

Let O and O' (Fig.1.) be two point-objects whose images are just resolved by the objective AB of a microscope. Let i be the semi-vertex angle of the cone of rays received by the objective from O.

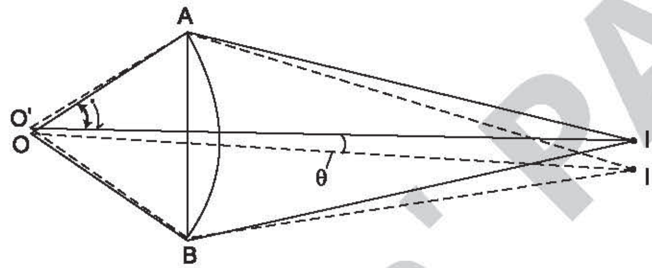


Fig.1.

The boundary of the objective acts as a circular aperture. Hence the images of O and O' formed by the objective are actually Fraunhofer diffraction patterns. Each pattern consists of a central bright disc surrounded by a series of alternate dark and bright rings. The centres of the dises lie at I and I', the geometrical images of O and O' respectively.

According to Rayleigh's criterion, O and O' will be just resolved when the centre I of the disc due to O falls on the first dark ring due to O', and vice-versa. This means that for just resolution, the waves from O', after diffraction by the objective, must form the first dark ring passing through I. Airy has shown that this will happen when the path difference between the extreme rays, $O'BI - O'AI$ is given by

$$O'BI - O'AI = 1.22\lambda$$

Since the paths AI and BI are equal, the above condition becomes

$$O'B - O'A = 1.22\lambda \qquad ...(1)$$

Now, let s be the distance between O and O' (Fig.2). As O and O' are very close together, we can take $O'A$ to be parallel to OA and $O'B$ parallel to OB. From the figure 2, we have

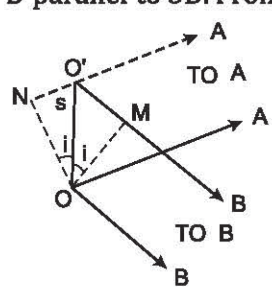


Fig.2.

$$O'B - OB = O'M = s \sin i$$

and $$OA - O'A = O'N = s \sin i.$$

Adding these equations and remembering that $OA = OB$, we get

$$O'B - O'A = 2s \sin i$$

Substituting this value of $O'B - O'A$ in eq. (1), we get

$$2s \sin i = 2.22\,\lambda$$

or $$s = \frac{1.22\,\lambda}{2 \sin i}$$

If the space between the object and the objective is filled with an oil of refractive index μ, then

$$s = \frac{1.22\,\lambda}{2\mu \sin i}$$

because the path difference $O'B - O'A$ is then multiplied by μ.

The quantity $\mu \sin i$ is called the numerical aperature $(N.A)$ of the objective. Hence,

$$s = \frac{1.22\,\lambda}{2\,N.A.}$$

This expression gives the linear distance between the two objects just resolved and is, therefore, a measure of the resolving power of the microscope.

The above expression holds when the object-points O and O' are *self-luminous*. When the objects are not self-luminous, but are illuminated with external light of wavelength λ, then the light starting from the two objects becomes, atleast partially coherent. Abbe has shown that in such a case the resolving power is obtained very approximately by omitting the factor 1.22. That is

$$s = \frac{\lambda}{2\,N.A.}$$

To obtain better resolution with microscopes we often use ultraviolet light which has a smaller λ. The electron-beams which behave like waves under some circunstances, have wavelengths 10^5 times shorter than visible light. This has led to the development of electron microscopes which have extremely high resolving power.

●

UNIT-VII

Polarisation

SECTION-A VERY SHORT ANSWER TYPE QUESTIONS

Q.1. What do you mean by polarisation of light?

Ans. Light is an electromagnetic wave in which electric and magnetic field vectors vibrating in mutually perpendicular directions. Since ordinary light has electric vector in all direction and if by some means we confined the electric vector only in one direction, then we say that light is polarised and this phenomena is called polarisation of light. This is the phenomenon which prove that light is a transverse wave.

Q.2. Define plane polarised light.

Ans. The plane polarised light wave is a wave in which the electric vector is everywhere confined to a single plane.

Q.3. Name the methods to produce linearly polarised light.

Ans. Linearly polarised light may be produced from unpolarised light using of the following five optical phenomena :

1. Reflection.
2. Refraction.
3. Scattering.
4. Dichroism (selective absorption).
5. Double refraction (sirefringence).

Q.4. Define Brewster law.

Ans. In 1892, Brewster proved that the tangent of the angle at which polarisation is obtained by reflection is numerically equal to the refractive index of the medium. If Q_p is the angle and μ is the refractive index of the medium (Fig.) then,

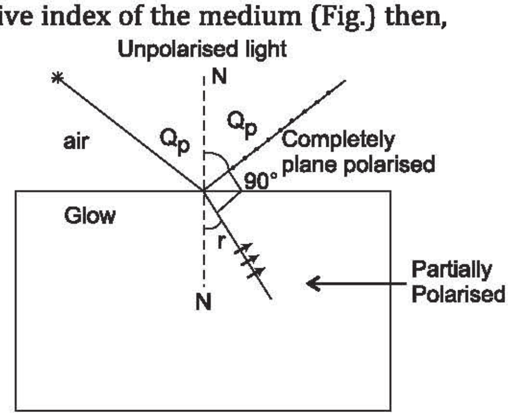


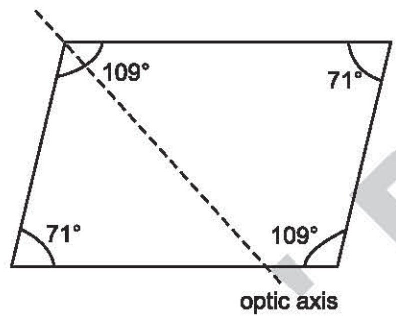
Fig.

$$\mu = \tan \theta_P$$

This is known as Brester law.

Q.5. Define optic axis.

Ans. A line passing through any one of the blunt corners and making equal angles with the three faces which meet there is the direction of 'optic axis' of the crystal. Optic axis is a direction and not a line. Hence, any line parallel to the one described above represents the optic axis.



Q.6. What is Nicol's prism?

Ans. It is an optical device made from a calcite crystal for producing and analysing plane polarised light.

Q.7. Motor car wind screens and head lights are fitted with polaroids. Why?

Ans. This is to prevent the car driver from the dazzling light of another car coming from the opposite direction. When two cars approaches each other from opposite directions in the night, the vibration planes of polaroids in one car becomes at right angles to those of the polaroids in the other car. Hence light from the head lights of one car is completely cut off by wind screen of other car.

Q.8. Define specific rotation.

Ans. The angle through which the plane of polarisation is rotated depends upon :

1. thickness of the substance.
2. concentration of solution.
3. wavelength of light.
4. the temperature.

Specific rotation for a given wavelength of light at a given temperature is defined as the rotation produced by one decimeter length of the solution containing 1 gm of optically active material per cc of solution

$$S = \frac{\theta}{l \times C}$$

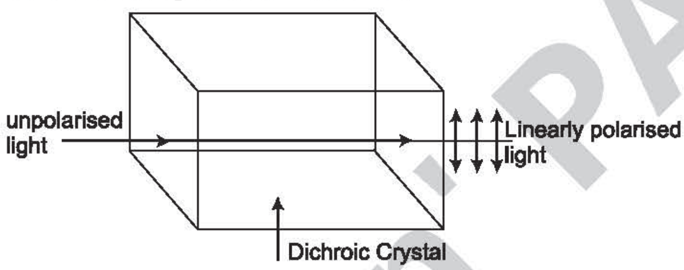
θ = rotation in degrees, l = length in decimeters, C = concentration in gm/cc.

SECTION-B SHORT ANSWER TYPE QUESTIONS

Q.1. Explain principle of dichroism.

Ans. **Principle of Dichroism**

In 1815 biot discovered that certain mineral crystals absorb light selectively. When natural light passes through a crystal such as 'tourmaline', it is split in to two components, which are polarized in mutually perpendicular planes. The crystal strongly absorbs light that is polarised in a direction parallel to a particular plane in the crystal but freely transmit the light components polarized in a perpendicular direction. This difference in the absorption for the rays is known as selective absorption or dictroism.



Q.2. Explain birefringence (or double refraction).

Ans. **Birefringence (Double Refraction)**

If a beam of unpolarized light is allowed to pass through a calcite or quartz crystal, it is split up in to two refracted rays. One of the two ray obey simple law of refraction called the ordinary ray (*O*-ray). The other ray behave in an extraordinary way and called the extraordinary ray (*E*-ray).

The crystals which having this type of property are called a 'doubly refracting crystal' and they exhibit the property 'Birefringence'. The principle of Birefringence can be seen in Fig.

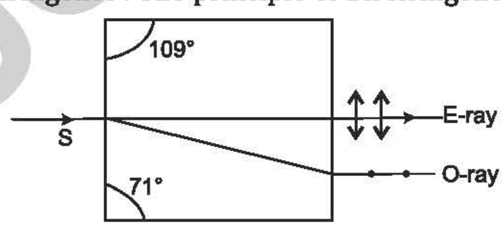


Fig.

Both the rays (*O* and *E*) are plane polarised, but their plane of polarisation are right angle to each other.

Q.3. What do you mean by retardation plates? Explain.

Ans. **Retardation Plates**

It is a plate cut from a doubly refracting uniaxial crystal with its optic axis parallel to the refracting faces. It can introduce a given phase difference between θ and *E* rays, which travel through it.

The phase difference introduced by the retardation plate is given by

$$\delta = \frac{2\pi}{\lambda}(\mu_O - \mu_E)t$$

where λ is the wavelength of light, t is the thickness of the plate, μ_0 and μ_E are the refractive index of the material for the O-ray and E-ray respectively.

Two main types of retardation plates are :

1. Quarter wave plate and
2. Half wave plate.

Q.4. Write a short note on quarter wave plate.

Ans.

Quarter Wave Plate

A doubly-refracting crystal plate having a thickness such as to produce a path difference of $\frac{\lambda}{4}$ between the ordinary (O-ray) and extraordinary ray (E-ray) is called a quarter wave plate. As light is incident normal to the optic axis, the E and O components travel in the same direction but with different velocities.

In calcite (negative crystal) the velocity of E-ray is greater than the velocity of O-ray. Therefore refractive index seen by O-ray is greater than refractive index for E-ray. If 't' is the thickness of the crystal plate, then the path difference between two rays

$$\delta = t(\mu_0 - \mu_E)$$

If the plate is to act as a quarter wave plate (QWP), this path difference should be equal to $\lambda/4$ *i.e.*,

$$\frac{\lambda}{4} = (\mu_0 - \mu_E)t$$

or $$t = \frac{\lambda}{4(\mu_0 - \mu_E)} \quad \text{for negative crystals } (\mu_0 > \mu_E)$$

and $$t = \frac{\lambda}{4(\mu_E - \mu_0)} \quad \text{for positive crystals } (\mu_E > \mu_0)$$

The QWP is used for producing circulary and elliptically polarised light. It is used for analysing all kinds of polarised light.

Q.5. Write short note on half wave plate (HWP).

Ans.

Half Wave Plate

There plates are made up of birefringente crystal of calcite or quartz with its refracting faces cut parallel to the optic axis. The thickness 't' of this plate is $\delta\theta$ chosen that it introduces a phase difference of π or path difference of $\lambda/2$ between the O-ray and the E-ray when light is incident normally on the face of the crystal. The thickness of the crystal is given by

$$t = \frac{\lambda}{2(\mu_0 - \mu_E)} \quad \text{for negative crystal}$$

and $$t = \frac{\lambda}{2(\mu_E - \mu_0)} \quad \text{for positive crystal}$$

Here μ_0 and μ_E are refractive indices seen by ordinary and extraordinary ray.

Q.6. If a Quarter Wave Plate (*QWP*) and a half wave plate (*HWP*) be given to you, how would you proceed to distinguish them from each other?

Ans. The *QWP* and *HWP* can be distinguish by analysing the emergent light by Nicol Prism, when plane polarised light falls normally on both of them.

In case of *QWP*, the light emerging from the plate may be elliptically-polarized, circularly-polarized or plane polarised depending upon the orientation of plate. Hence on examining through the rotating Nicol we shall observe.

(i) Variation in intensity with non zero minimum, when the emergent light is elliptically polarised.

(ii) No variation in intensity when the emergent light is circularly polarised.

(iii) Variation in intensity with zero minimum when the emergent light is plane polarized.

In case of HWP, the plate produces a phase difference of π between the two components, the state of polarisation of the emergent light is the same as that of the incident light. Therefore, the light emerging from the HWP will be plane polarised for all orientations of the plate. Hence rotation of the Nicol prism will always give variation in intensity with zero minimum.

SECTION-C LONG ANSWER TYPE QUESTIONS

Q.1. Describe the construction of a Nicol's prism. Explain how it can be used as a polariser and as an analyser. Would a similar prism prepared from quartz serve a similar purpose?

Ans.

Nicol's Prism

It is an optical device made from a calcite crystal for producing and analysing plane-polarised light.

Construction : A calcite crystal *ABCD* (Fig.1.) about three times as long as it is wide is taken. Its end faces *AB* and *CD* are ground such that the angles in the principal section become 68° and 112° instead of 71° and 109°. The crystal is then cut apart along the plane *A' D* perpendicular to both the principal section and the end faces *A' B* and *CD'*. The two cut surfaces are ground and polished optically flat. They are then cemented together by canada balsam which is a transparent liquid of refractive index 1.55 for sodium light. The crystal is then enclosed in a tube blackened inside.

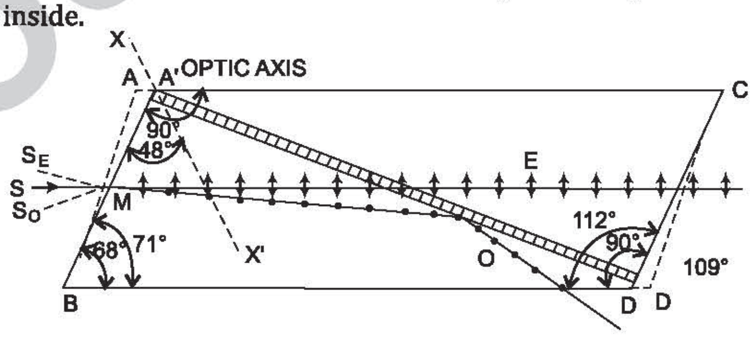


Fig.1.

Action : When a ray *SM* of unpolarised light nearly parallel to *BD'* is incident on the face *A' B*, it is split up into two refracted rays, the *O*-ray and the *E*-ray. Both the rays are plane-polarised. The *O*-ray has vibrations perpendicular to the principal section.

Now, the refractive index of canada balsam (1.55) is less than the refractive index of calcite for the *O*-ray (1.658), but greater than the refractive index of calcite for the *E*-ray (1.486). Therefore, when the *O*-ray reaches the layer of the canada balsam, it is passing from an optically denser to a rarer medium. Since the length of the crystal is large, the angle of incidence of the *O*-ray at the calcite-balsam surface becomes greater than the critical angle (69°) for the *O*-ray. Hence the *O*-ray is totally reflected at the calcite-balsam surface and is absorbed by the tube containing the crystal. The *E*-ray, however, on reaching the calcite-balsam surface passes from a rarer to a denser medium and is transmitted.

Since the *E*-ray is plane-polarised, the light emerging from the Nicol is plane-polarised with vibrations parallel to the principal section. These vibrations are parallel to the shorter diagonal of the end face of the crystal.

Limitation : The Nicol prism works only when the incident beam is slightly convergent or slightly divergent. If the incident ray makes angle much smaller than *SMB* with the face *A′ B*, the *O*-ray will strike the calcite-balsam surface at an angle less than the critical angle (69°). Therefore, the *O*-ray will also be transmitted and the light emerging from the Nicol will not be plane-polarised.

If the incident ray makes an angle much greater than *SMB*, the *E*-ray will become more and more parallel to the optic axis *xx′* so that its refractive index will increase and become greater than that of the balsam. Then the *E*-ray will also be totally reflected from the calcite-balsam surface and no light will emerge from the Nicol. Hence to obtain plane-polarised light, the incident beam should not be too wide. With the dimensions chosen; the semi-vertical angles of the cone of incident light $S_E MS_O$ So should not exceed 14°. It is in order to make the ray *SM* lie within the range of 14° that the end faces are grounded to modify the angles.

Uses : The Nicol prism can be used both as a 'polariser' and as an 'analyser'. When an unpolarised ray of light is incident on a Nicol prism *P* (Fig.2a), the ray emerging from *P* is plane-polarised with vibrations in the principal section of *P*. If this ray falls on a second prism *A*, whose principal section is parallel to that to *P*, its vibrations will be in the principal section of *A*. Hence the ray will behave as *E*-ray in the prism *A* and will be completely transmitted. The intensity of the emergent light will be a maximum.

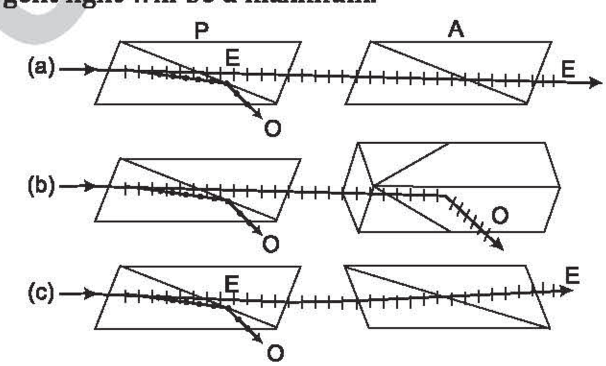


Fig.2.

Now if the Nicol *A* be rotated such that its principal section becomes perpendicular to that of *P* (Fig.2b), the vibrations in the plane-polarised ray incident on *A* will be perpendicular to the principal section of *A*. Hence the ray will behave as *O*-ray inside *A* and will be lost by total reflection at the calcite-balsam surface. Therefore, no light will emerge from A. In this position the two Nicols are said to be 'crossed'.

If the Nicol A be further rotated to have its principal section again parallel to that of P (Fig.2c), the intensity of emergent light will again be a maximum.

The prism P is called the 'polariser' and prism A is called the 'analyser'.

These facts can be used for analysing plane-polarised light. If the given light on viewing through a rotating Nicol shows variation in intensity with minimum intensity zero, the given light is plane-polarised.

A similar prism prepared from quartz would not serve a similar purpose although it is doubly-refracting. To use quartz for producting plane-polarised light, we have to use either a Rochon or a Wollaston prism, which are known as 'double-image prisms'.

Q.2. What is meant by optical rotation (or rotatory polarisation)? Give an outline of Fresnel's theory of optical rotation. Discuss the dependence of rotation on λ and the experimental evidence in support of the theory.

Ans.

Optical Rotation (Rotatory Polarisation)

When plane-polarised light passes through certain substances, the plane of polarisation of the light is rotated about the direction of propagation of light through a certain angle. This phenomenon is called 'optical rotation' or 'rotatory polarisation'. The substances which rotate the plane of polarisation are said to be "optically-active", and the property is called "optical activity".

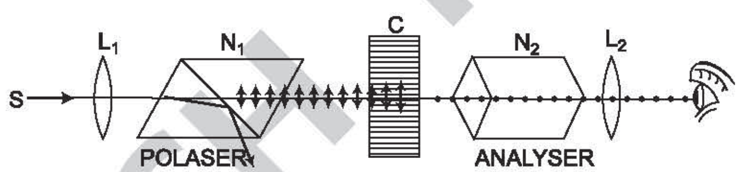


Fig.1.

If plane-polarised light emerging from a Nicol N_1 is examined through another Nicol N_2, it is completely cut off when the principal section of N_2 is perpendicular to that of N_1. But if a quartz plate C (Fig.1.) cut with optic axis perpendicular to its face be placed between N_1 and N_2, some light begins to pass through N_2. The light is, however, again completely cut off if N_2 is rotated through a certain angle. This shows that the light emerging from the quartz plate is still plane-polarised, but its plane of polarisation has been rotated by the plate through a certain angle. Thus, quartz is optically-active. Many liquids and organic substances in solution (as cane-sugar) are found to be optically-active.

There are two types of optically-active substances. Those which rotate the plane of polarisation clockwise (looking against the direction of light) are called 'dextro-rotatory' or 'right-handed', while those which rotate anti-colckwise are called 'leavo-rotatory' or 'left-handed'. Quartz occurs in both forms. The rotation produced by a plate 1 mm thick is called the 'specific rotation'. For sodium light, the specific rotation of quartz is 21.7°.

Biot, in 1815, studied the phenomenon in detail and gave the following laws :

(i) The angle of rotation of the plane of polarisation, for a given wavelength, is directly proportional to the length of the optically-active substance traversed.

(ii) For solutions and vapour, the angle of rotation for a given path-length is proportional to the concentration of the solution or vapour.

(iii) The rotation produced by a number of optically-active substances is equal to the algebraic sum of individual rotations. The anti-colckwise and clockwise rotations are taken with opposite signs.

(iv) The angle of rotation is approximately inversely proportional to the square of wavelength. More accurately, for quartz we have

$$\text{angle of rotation} = A + \frac{B}{\lambda^2}$$

Thus, if white plane-polarised light having vibration in the direction *AA* (Fig.2.) be incident normally on a quartz plate, the different colours are rotated through different angles, as shown. The field of view, therefore, appears coloured. This phenomenon is called 'rotatory dispersion.'

Fresnel's Theory of Optical Rotation : It is based on the principle of dynamics that a linear vibration may be described as the resultant of two opposite circular motions of the same frequency. Fresnel made the following assumptions :

(i) The incident plane-polarised light on entering a substance is broken up into 40 circularly-polarised waves, one clockwise and the other anti-clockwise.

(ii) In an optically-inactive substance the two waves travel with the same velocity, but in an optically-active substance they travel with different velocities. (in dextro-rotatory substance the clockwise wave travels faster, while in leavo rotatory substance the anti-clockwise wave travels faster). Hence a phase difference is developed between them as they traverse the substance.

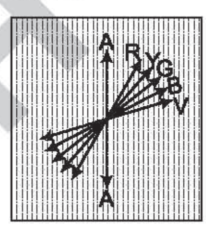


Fig.2.

(iii) On emergence, the two circular components recombine to form plane-polarised light whose plane of polarisation is rotated with respect to that of the incident light by an angle depending on the phase difference between them.

Suppose that plane-polarised light is incident normally on a quartz plate cut perpendicular to the optic axis. Let the first face of the plate be in the *x, y* plane. Let the vibrations in the incident light be represented by

$$x = a\cos\omega t \qquad \text{...(1)}$$

These vibrations, just on entering the crystal, are broken up into two equal and opposite circular motions (Fig.3a) which are represented by

$$\left.\begin{aligned} x_1 &= \frac{a}{2}\sin\omega t \\ y_1 &= \frac{a}{2}\cos\omega t \end{aligned}\right\} \begin{array}{c}\text{clockwise} \\ \text{circular-motion}\end{array} \qquad \text{...(2)}$$

and
$$\left.\begin{aligned} x_2 &= -\frac{a}{2}\sin\omega t \\ y_2 &= \frac{a}{2}\cos\omega t \end{aligned}\right\} \begin{array}{l}\text{anti-clockwise}\\ \text{circular-motion}\end{array} \qquad ...(3)$$

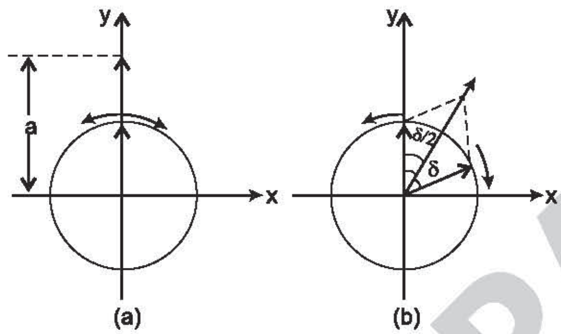


Fig.3.

These circular components are propagated through the plate with different velocities. Therefore, when they emerge from the plate, there is a phase difference (say) between them. Suppose that the clockwise component advances infront of the other (Fig.3b). The emergent circular component can then be represented by

$$\left.\begin{aligned} x_1 &= \frac{a}{2}\sin(\omega t+\delta) \\ y_1 &= \frac{a}{2}\cos(\omega t+\delta) \end{aligned}\right\} \text{ and } \left.\begin{aligned} x_2 &= -\frac{a}{2}\sin\omega t \\ y_2 &= \frac{a}{2}\cos\omega t \end{aligned}\right\}$$

The resultant displacements along the two axes are

$$x = x_1 + x_2 = \frac{a}{2}[\sin(\omega t+\delta) - \sin\omega t]$$

$$= a\cos\left(\omega t + \frac{\delta}{2}\right)\sin\frac{\delta}{2} \qquad ...(4)$$

and
$$y = y_1 + y_2 = \frac{a}{2}[\cos(\omega t+\delta) + \cos\omega t]$$

$$= a\cos\left(\omega t + \frac{\delta}{2}\right)\cos\frac{\delta}{2} \qquad ...(5)$$

Dividing eq. (4) by eq. (5), we have

$$\frac{x}{y} = \tan\frac{\delta}{2}$$

This represents a straight line inclined at angle $\delta/2$ to the y-axis (Fig.3b). Hence the light emerging from the plate is plane-polarised, with vibrations inclined at angle $\delta/2$ to the y-axis, that is, to the vibrations in the incident light. Thus, the quartz plate has rotated the plane of vibration by $\delta/2$.

If μ_A and μ_C are the refractive indices of quartz in the direction of the optic axis for anti-clockwise and clockwise circularly-polarised light respectively and d is the thickness of the crystal plate, then the phase difference δ is given by

$$\delta = \frac{2\pi}{\lambda}(\mu_A - \mu_C)d$$

Hence the rotation of the plane of vibration (or, of the plane of polarisation) is

$$\theta = \frac{\delta}{2} = \frac{\pi d}{\lambda}(\mu_A - \mu_C).$$

Experimental Verification of Fresnel's Theory : To verify Fresnel's theory, a beam of plane-polarised light is made to fall normally on a rectangular block *ABFG* (Fig.4). The block is made of alternate prisms of right-handed and left-handed quartz, all having their optic axes perpendicular to the end faces *AB* and *FG*. If the Fresnel's hypothesis is correct, this plane-polarised light will break up into two opposite circularly-polarised waves (Left-handed and Right-handed) which travel the first prism with different velocities but in the same direction. Upon passing through the first oblique boundary *BC*, the *R*-wave which was faster in the first prism (*R*-prism) becomes the slower in the second. The opposite is true for the *L*-wave. Hence the second prism (*L*-prism) is a denser medium for the *R*-wave and rarer for the *L*-wave. Hence, in the second prism the *R*-wave bends towards the base (downward), and the *L*-wave away from the base (upward). At the second boundary *CD*, the velocities are again interchanged so that the *R*-wave bends away from the base (downward) and the *L*-wave towards the base (upward). The net result is that the angular separation increases at each successive boundary. By using a number of prisms it is possible to produce a detectable separation between the two circularly polarised waves.

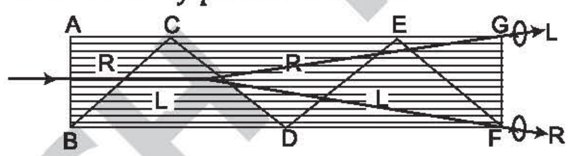


Fig.4.

When the two waves are analysed by a $\frac{\lambda}{4}$-plate and a Nicol prism, they are found to be circularly-polarised in opposite directions, thus verificying Fresnel's hypothesis.

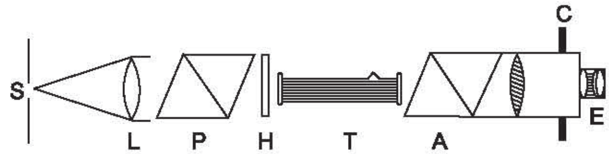
Q.3. Describe the construction and working of a laurent half shade polarimeter. Explain how would you we it to determine the specific rotation of sugar solution.

Ans.

Laurent's Half-Shade Polarimeter

It is an instrument used for finding the optical rotation of certain solutions. When used for finding the optical rotation of sugar, it is called a saccharimeter. If the specific rotation of sugar is known, the concentration of the sugar solution can be determined.

Construction : The optical parts of a polarimeter are shown in Fig.1. Monochromatic light from a source, usually a sodium lamp, after passing through a narrow slit *S* is rendered into a parallel beam by the lens *L*. This fight is rendered plane polarised by the Nicol *P* and after passing through the half-shade device *H*, a glass tube *T* containing the solution is made to fall on the analysing Nicol *A*. The light is viewed through a Galilean telescope *E*. The analysing Nicol *A* can be rotated about the axis of the tube and its rotation can be measured with the help of a vernier scale on the graduated circular scale *C* divided in degrees.



The position of the analyser is adjusted so that the field of view is completely dark. The tube *T* is filled with the required solution and is placed in position. The field now becomes illuminated. Darkness can again be achieved by rotating the analysing Nicol *A* through a certain angle which gives the optical rotation for the solution. It is found that when *A* is rotated the total darkness of the field of view is attained rather gradually and hence it is difficult to find the exact position correctly for which complete darkness is achieved. Laurent devised an ingenious method to achieve this. The arrangement is known as Lauren's half-shade device.

Laurent's half-shade device : It consists of a semi-circular plate *ADB* of glass cemented to a semi-circular plate *ACB* of quartz. The quartz plate is cut with its optic axis parallel to the line of separation *AOB*. The thickness of the quartz plate is such that it introduces a phase difference of π between the *O* and *E*-vibrations. In other words, it is half wave plate. The thickness of the glass plate is such that it absorbs the same amount of light as is done by the quartz half wave plate.

Suppose light after passing through the polariser *P* is incident normally on the half-shade plate and has vibrations along *OP*. On passing through the glass half the vibrations will remain along *OP*, but on passing through the quartz half these will be split up into *E* and *O* components The vibrations of the *O*-component are along *OD* and those of *E*-component along *OA* (Fig.2).

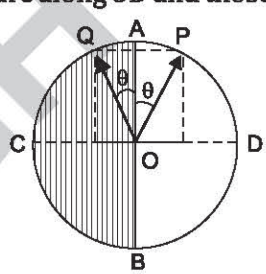


Fig.2.

On passing through the quartz plate a phase difference of π is introduced between these two vibrations. The *O*-vibrations will advance in phase by π and will occur along *OC* instead of *OD* on emergence. The resultant vibration on emerging from the quartz plate will, there fore, be along *OQ*, such that

$$\angle POA = \angle QOA$$

If the analysing Nicol is fixed with its principal plane parallel to *OP*, the plane polarised light through glass half will pass and hence it will appear brighter than the quartz half from which light is partially obstructed.

If the principal plane of the Nicol is parallel to *OQ*, the quartz half will appear brighter than the glass half due to the same reason.

When the principal plane of the analysing Nicol is parallel AOB, the two halves will appear **equally bright**. It is because the vibrations emerging out of the two halves are eually inclined to its principal plane and hence two components will have equal intensity.

When the principal plane of the analyser is at right angle to AOB, again the components of OP and OQ are equal. The two halves are again equally illuminated, but as the intensity of the components passing through is small as compared to that in the previous case, the two halves are said to be **equally dark**.

The eye can easily detect a slight change when the two halves are equally dark. The readings are, therefore, taken for this position.

To find the strength of sugar solution : Fill the polarimeter tube with water and find the reading on the circular scale corresponding to equally dark position of the half shade device.

Now fill the tube completely with the given sugar solution and again find the reading on the circular scale for equally dark positions of the half shade device. The diflerence between the scale readings gives the optical rotation θ produced by the given length l in decimetres of the sugar solution. If S is the specific rotation of sugar for the same wavelength and at the same temperature, then

$$\text{Concentration } C = \frac{\theta}{l \times S} \text{ gm/cc} \qquad ...(1)$$

If the strength of sugar solution is known in gm/cc, then specific rotation $S' = \frac{\theta}{lC}$ ...(2)

Q.4. Explain the working of Biquartz Polarimeter.

Ans.

Biquartz Polarimeter

It is a simple and accurate instrument, much more sensitive than the half shade one, for finding the angle of rotation produced by an optically active substance. It consists of a condensing lens, a polarizing Nicol, a biquartz plate, a tube to contain the active solution, analysing Nicol and a telescope arranged in order in the same way as in half shade polarimeter (Fig.1.) with the following two differences :

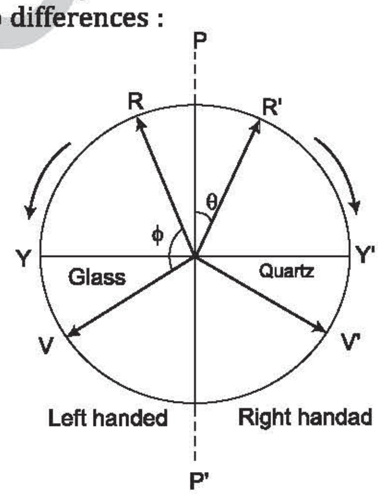


Fig.1.

(1) Half shade device (Laurent's plate) is replaced by a biquartz plate.

(2) Sodium light is replaced by white light.

Action of the biquartz plate : It consists of two semi-circular plates *PYP'*, one of left handed quartz and the other of right handed quartz, both cut perpendicular to the optic axis and joined together along the diameter *PP'* so as to form a composite circular plate as shown in Fig.1. The thickness of each plate is such that it rotates the plane of polarisation for yellow light by 90°. This thickness is 3.75 mm for quartz.

When white light rendered plane polarised with a polariser travels through the biquartz normally, it is travelling along the optic axis (since the plates have been cut perpendicular to the optic axis) and therefore, the phenomenon of rotatory dispersion takes place. Different colours suffer different rotations. For longer wavelengths the rotations is less, for shorter one it is more. Hence if the principal plane of the polariser is parallel to *PP'*, the red and violet regions will be rotated in the two halves through the angle θ and ϕ respectively where $\phi > \theta$. The intermediate wavelengths will be rotated through angles lying between ϕ and θ. For yellow light the rotation is 90° and hence *YY'* is a straight line.

If the principal plane of the analyser is parallel to *PP'* (Fig.2a), the yellow light will not be transmitted through the analyser while the red and blue (greyish-violet) will be present in the same proportion in each half. Thus the two halves will appear equally grey-violet coloured. This colour is called the sensitive tint or tint of passage.

If the analyser is now rotated through a very small angle in the clockwise direction (Fig.2b), the longer wavelengths (predominantly red from the right-half will be transmitted through it to a greater degree than the shorter ones predominantly violet and blue) and thus causing its colour to change from grey to red (pink). Just reverse is the case for the left half where shorter wavelengths are transmitted to a greater degree than the longer ones and thus cause its colour to change from grey to blue. If on the other hand, the analyser is rotated in the anti-clockwise direction (Fig.2c), the colour of left half changes from grey to red while that of the right half from grey to blue. Thus there is a marked change brought about in the appearance of the biquartz plate on slight rotation of the analyser. Hence the position of sensitive tint is very sensitive and is, therefore, employed for the accurate measurement of the angle of rotation.

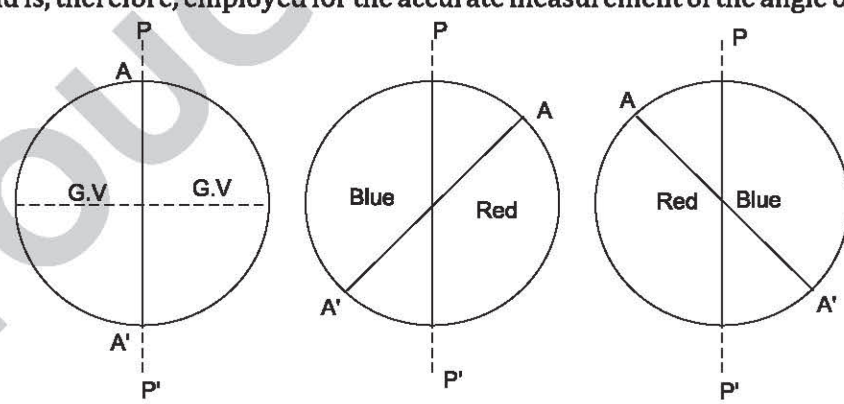


Fig.2.

Determinetation of specific rotation : For this purpose, the biquartz is kept in position occupied by the half shade (Fig.). The tube is filled with solvent and the position of sensitive tint is obtained by rotating the analyser. This reading of the analyser is noted. The process is repeated with optically active solution in the tube. The difference between the two readings of the analyser gives the optical rotation θ (in degrees) produced by the solution.

In actual experiment, value of θ is determined for different concentration of the solution and a graph is then plotted between θ and C which comes out to be a straight line as shown in Fig.3. From graph, value θ/C is obtained and then specific rotation is calculated by the formula

$$S = \frac{\theta}{lC}$$

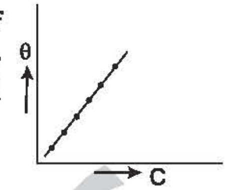


Fig.3.

where l is the length of the solution (tube) in decimeter and C is the concentration in gm/cc.

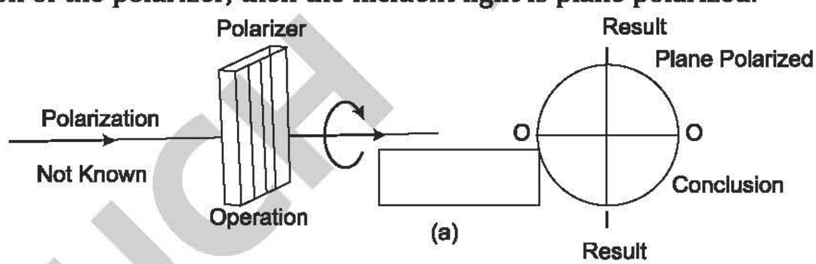
Q.5. How would you analyze plane, circularly and elliptically polarised light by a $\lambda/4$ plate?

Ans.

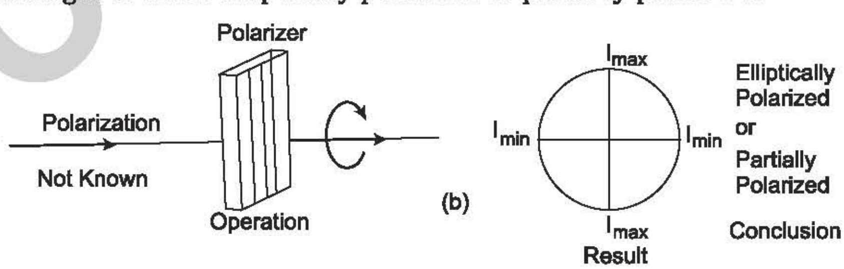
Analysis of Polarized Light

In practice light may exhibit any one of the three types of polarization, or may be unpolarized or a mixed type. The unaided eye cannot distinguish the different types of polarization. However, using a polarizer and a quarter wave plate, the actual type of polarization of a light beam can be ascertained. The following steps are used in the analysis of the type of polarization.

1. The light of unknown polarization is allowed to fall normally on a polarizer. The polarizer is slowly rotated through a full circle and the intensity of the transmitted light is observed. If the intensity of the transmitted light is extinguished twice in one full rotation of the polarizer, then the incident light is plane polarized.

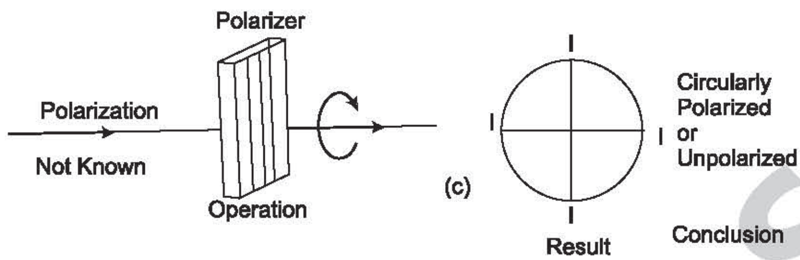


2. If the intensity of the transmitted light varies between a maximum and a minimum value but does not become extinguished in any position of the polarizer, then the incident light is either elliptically polarized or partially polarized.



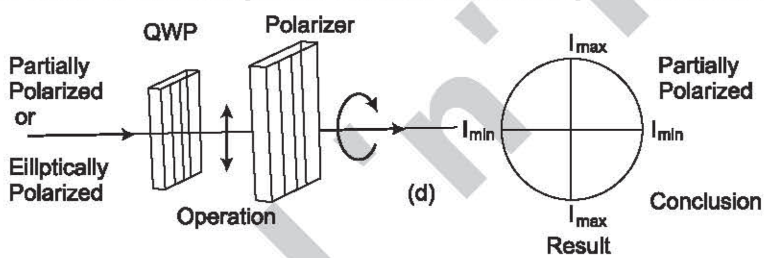
3. If the intensity of the transmitted light remains constant on rotation of the polarizer, then the incident light is either circularly polarized or unpolarized.

   To distinguish between elliptically polarized and partially polarized or between the circularly polarized and unpolarized light, we take the help of a quarter wave plate. The light is first made to be incident on the quarter wave plate and then it passes through the polarizer.

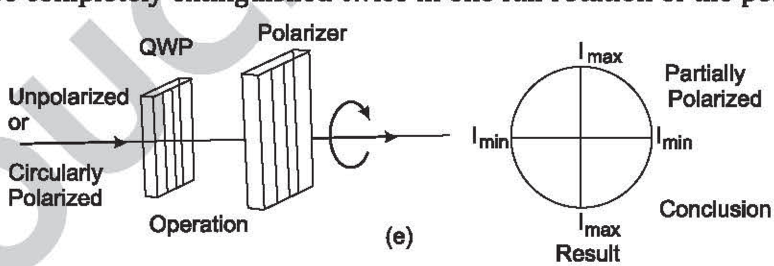


4. If the incident light is elliptically polarized, the quarter wave plate converts it into a plane polarized beam. When this linearly polarized light passes through the polarizer, it would be extinguished twice in one full rotation of the polarizer.

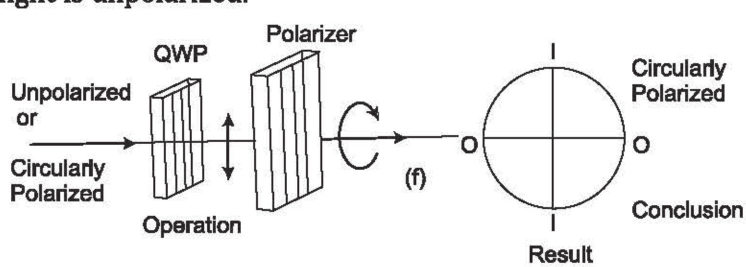
   On the other hand, if the transmitted light intensity varies between a maximum and a minimum without becoming zero, then the incident light is partially polarized.



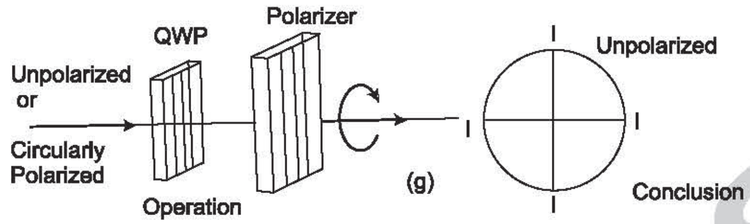
5. If the incident light is circularly polarized, the quarter wave plate converts it into plane polarized light. When this linearly plane polarized light passes through the polarizer, it would be completely extinguished twice in one full rotation of the polarizer.



On the other hand, if the intensity of the transmitted light stays constant, then the incident light is unpolarized.



Analysis of polarized light. A polarizer and a quarter wave plate help in determines the type of the polarization of light.



Q.6. What is a Babinet compensator? Explain is construction and working. How will you analyse elliptically polarised light with the help of Babinel comensator.

Ans.

Babinet Compensator

A compensator is an optical device whose function are componsate a path difference. It is used in conjunction with a polariser and analyzer combination to investigate optically polarized light. The compensator helps in comparising the axis of the ellipse and the ratio of their bughts. Elliptically polarized light may be considered as a result of two coherent plane polarized waves coming in mutually orthogonal directions and with an initial path difference of $\lambda/4$. When the elliptically polarized light passes through the device such that it introduces a further path difference of $\lambda/4$, the total path difference between the perpendicular waves become $\lambda/2$ and the vibrations recombine after emerging from the device to produce plane polarized light. From the analysis of this plane polarized light, the information regarding the incident elliptically polarized light can be obtained.

Construction

The Babinet compensator is made of two wedge-shaped quartz sections, ABC and ADC, having equal acute angles. The wedges are placed against each other such that they form a small rectangular block as shown in Fig.1 (a) One of the quartz wedges is fixed and the other can be displaced along their plane of contact with the help of a micrometer screw arrangement. Thus, the combination acts as a plate of variable thickness.

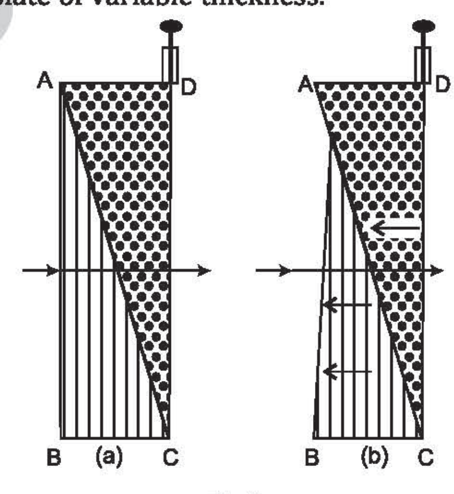


Fig.1.

The optic axis of the first section is parallel to its refracting edge *AB* and the optic axis of the second section is in a direction perpendicular to the edge. The two optic axes are perpendicular to each other and also perpendicular to the incident beam.

Production of Polarized Light

Let plane polarized light be incident normally on the face *AB* of the compensator. It splits into *e*-ray and *o*-ray parallel and perpendicular to *AB* respectively. The *e*-ray travels slower than *o*-ray in the first section, since quartz is a positive uniaxial crystal. When these rays enter the second section, the *e*-ray becomes *o*-ray since the optic axis in the second section is in a direction normal to that in the first prism. Similarly, *o*-ray becomes *e*-ray. Thus, the two rays exchange their velocities in passing from one section to the other section. The net effect is that one section cancels the effect of the other.

If d_1 is the thickness of the first section and μ_c and μ_e are the refractive indices of quartz for *e* and *o*-ray respectively, the path difference between the *e*- and *o*-rays in the first section will be

$$\Delta_1 = [\mu_e - \mu_o] d_1$$

As the principal planes of the two sections are at right angles, the *e*- and *o*-rays change their roles in going from the first section to the second section. The velocities of *e*-ray and *o*-ray interchange and if the thickness of the second section is d_2, then the path difference between the rays in the second prism will be

$$\Delta_2 = [\mu_o - \mu_e] d_2$$

As the compensator is thin, the separation of the rays is negligible. The net path difference between the two rays after emerging from the crystal will be

$$\Delta = \Delta_1 + \Delta_2$$

$$\Delta = [\mu_e - \mu_o) d_1 + (\mu_o - \mu_e) d_2$$

$$= (\mu_e - \mu_0)(d_1 - d_2) \quad \text{...(1)}$$

The net phase difference is $\delta = \frac{2\pi}{\lambda}(\mu_e - \mu_0)(d_1 - d_2)$ ...(2)

For a ray passing through the centre of the compensator where $d_1 = d_2$ the net path difference and hence the phase difference is zero. It means that the effect of one wedge is exactly cancelled by the other. This is true for all wavelengths and the incident wave is transmitted as such. Plane polarized light incident on the compensator will emerge as plane polarized light with its plane of vibration parallel to that of the incident light.

Any desired thickness difference $(d_2 - d_1)$ can be achieved at the centre of the compensator by moving the second section relative to the first section. Thus, any desired value of phase difference can be obtained between the *e* and *o*-rays. Therefore, the light emerging will be either plane or circular or elliptically polarized light, depending on the phase difference.

Thus, the compensator has the same effect as that of a wave plate of varying thickness. The advantage of compensator is that it can be arranged to suit any wavelength where as a quarter wave plate is designed to suit only one particular wavelength.

Analysis of Elliptically Polarized Light

Using the compensator, one can determine the characteristics of elliptically polarized light.

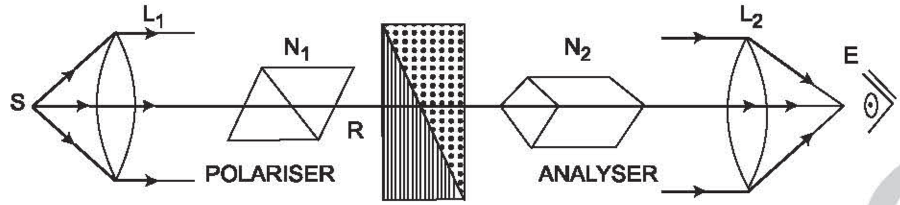


Fig.2.

Let the compensator be placed between crossed polarizer N_1 and analyser N_2, as shown in Fig.2. Let the transmission axis of polarizer be oriented at 45° with respect to the optic axis of wedge ABC of the compensator. At midpoint R the light emergent from the compensator is plane polarized in the same plane as transmitted by N_1 and therefore it will be extinugished by the analyser N_2. Similarly, at distances from the midpoint for which the retradation is $1\lambda, 2\lambda, 3\lambda, ..., m\lambda$, and the emergent light is plane polarized in the same plane as transmitted by N_1 and hence will be extinguished by the analyser. So the field of view is crossed by a series of equidistant parallel dark bands.

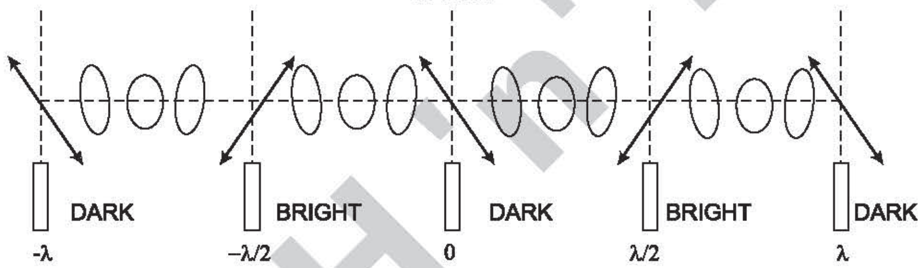


Fig.3.

At positions between them, where the path difference corresponds to an odd multiple of $\lambda/2$, *i.e.*, $\lambda/2$, $3\lambda/2$, $5\lambda/2$, ... $(2m+1)\lambda/2$, the transmitted light is plane polarized. The analyser transmits the light completely and those regions will be bright. In all other cases, the emerging light is elliptically polarized with varying parameters of the ellipse, as shown in Fig. 3. If white light is used, the central band will be dark while others will be coloured.

By using white light source, the compensator is adjusted such that the central dark band is under cross wire and the micrometer reading is noted. The micrometer screw is turned through an angle such that the compensator introduces a phase difference of $\pi/2$ at cross wire. Then elliptically polarized light is made to be incident on the compensator. The central dark band undergoes a shift with respect to the cross wire. The compensator is rotated through an angle α in its own plane until the central dark band is on the cross wire. The axes of the incident elliptically polarized light are parallel to the optic axes of the wedges of the compensator.

Phase difference : The elliptical vibration can be regarded as made of two mutually perpendicular linear vibrations, which are having a phase difference, δ. δ can be determined as follows.

First the compensator is illuminated with white plane-polarised light and the micrometer is adjusted to bring the central dark band on the cross-wires. The white light is then replaced by elliptically polarised light. The central band shifts to a point where the original phase difference δ between the two component vibrations of elliptical polarised light is exactly balanced by the phase difference introduced by the compensator. This phase difference is determined by rotating the screw until the central dark band is again on the cross-wires. If this rotation is ϕ, then

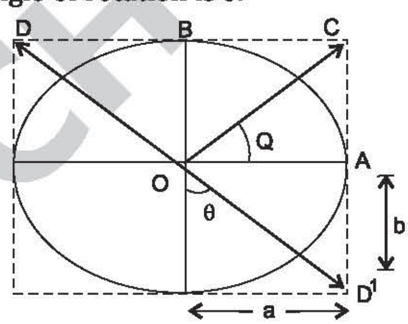
$$\frac{\delta}{2\pi} = \frac{\phi}{\alpha} \text{ or } \delta = \frac{2\pi\phi}{\alpha} \qquad ...(1)$$

Position of axes : The position of the major and minor axes of the given elliptical vibration can be found as follows. The compensator is illuminated with white polarised light and the micrometer screw is adjusted to bring the central dark band on the cross-wires. The screw is then turned through an angle $\alpha/4$ so that the compensator introduces a phase diiference of $90°$. The central dark band now is not on the cross wires. The elliptically polarised light is made incident on the compensator. Then the compensator is rotated in its own plane until the central dark band again comes on to the cross-wires. The axes of the incident light are parallel to the optic axes of the wedges of the compensator.

Ratio of the axes : Referring to the Fig.4, *OA* and *OB* represent the optic axes of the two wedges of the compensator. *OC* is the direction of the principal section of the analyser. *DD'* is the direction of vibration of light emerging from the compensator at the cross-wires. The tangent of the angle θ that the principal section of the analyser makes with the optic axes of the wedge gives the ratio of the axes. thus,

$$\tan\theta = a/b$$

θ is determined by rotating the analyser until the bands disappear and the field becomes uniformly illuminated. The angle of rotation is θ.



Advantages : A quarter wave plate produces a fixed phase difference between *o*-ray and *e*-ray and can be used only for monochromatic light of one particular wavelength. In case of a compensator, the phase difference between the rays can be varied continuously and hence a compensator, made of a combination of wedges, can be used for light of any given wavelength.

Q.7. What do you mean by dichroism? How will you achieve polarisation by dichroic crystal?

Ans.

Dichroism

Dichroism (two coloured) is the change in colour evident as the mineral is rotated under plane polarized light. The primary cause of dichroism in minerals is due to absorption of particular wavelengths of light. This selective absorption of certain wavelengths of light causes the transmitted light to appear colored. This color is a function of the thickness and the particular chemical and crystallographic nature of the mineral. Dichroism is defined as the property of certain materials, in which light in different polarization states travelling through it experiences a different absorption coefficient. In other words, it is the property of some crystals of absorbing one of the two components more strongly than the other due to having different absorption coefficients. A number of crystalline materials absorb more light in one incident plane than another, so that light progressing through the material become more and more polarized artisotropy in absorption is called dichroism. There are several naturally occurring dichroin materials, and many more are artificially manufactured Polaroid is the trade name for the most commonly used dichroic material. It selectively absorbs light from one plane, typically transmifting less than 1% through a sheet of Polaroid. It may transmit more than 80% of light in the perpendicular plane. The word Polaroid usually refers to H-sheet Polaroid, which is a sheet of iodine-impregnated polyvinyl alcohol, as discussed in article 7.5.

Principle of Dichroism

Dichroism is a property of certain materials in which light in different polarization state travelling through it experience a different absorption coefficients for two different polarization States of the incident EM wave. This material shows a very high absorption coefficient for the vibrations of electric field which are perpendicular to the plane of paper but it shows a very less absorption coefficient to the electric field vibrations which are in the plane of the paper. This means that the perpendicular vibrations will be strongly absorbed by this material where as the vibrations which are in the plane of the paper will be easily transmitted through it. This is how we obtain polarization in these dichroic materials.

Polarisation by Dichroic Crystal

The different methods are used to obtain polarized light. These are polarisation by reflection and polarization by refraction. The third method we are going to use to obtain polarised light is by absorption. The term that we will use over here is selective absorption.

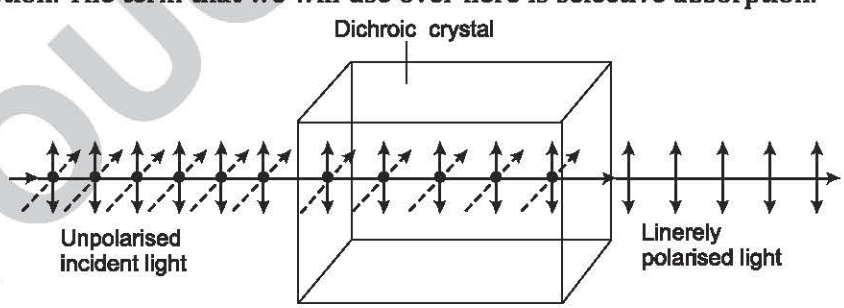


Fig. : Plane-polarised light transmitted by a dichroic crystal. (i) Horizontal component vibrations completely absorbed (ii) Vertical vibrations partially absorbed (1%)

We know that the EM wave consists of electric field as well as magnetic field but when we are talking about polarization we always talk about only the electric field at any point in space and at any instant of time can be divided into two orthogonal components. When such unpolarised light falls of dichroic material, say tourmaline crystal, both components experience an absorption coefficient, one absorption coefficient for vibrations in the plane of paper (relatively small attenuation) and another for absorption coefficient for vibrations in perpendicular plane of paper (relatively strong attenuation). However, it should be noted that there is a vast difference in these two absorption coefficients, Absorption coefficient for vibrations in the plane of paper is relatively small that produces small attenuation, keeping the magnitude almost same in the transmitted light. But the absorption coefficient for vibrations in the perpendicular plane of paper is relatively strong that produces large attenuation. The magnitude of this component continuously goes on decreasing and is almost vanishes during its journey through the crystal (Fig.1). Thus, in the transmitted light, vibrations in perpendicular plane of paper are almost absent and only vibrations in the plane of paper are present. Thus, we get a polarized light.

Here we have obtained polarization by selective absorption. The word selective is used because only magnitude of one orthogonal component (vibrations in the perpendicular plane of paper) is (almost) completely attenuated while other is allowed to transmit without any appreciable attenuation. Thus, we get a linearly polarized light by using the dichroic property of the tourmaline crystal. Here we usually use two terms namely transmission axis and absorption axis.

Transmission axis

Transmission axis is that axis along which the electric field component easily passes through the crystal. In above example, magnitude of one of the two orthogonal components, having Vibrations in the plane of paper almost remains the same (with negligible attenuation), while passing through the crystal.

Absorption axis

Absorption axis is that axis along which the electric field component is completely absorbed and gets completely attenuated while travelling through the crystal. In above example, magnitude of one of the two orthogonal components, having vibrations in the perpendicular plane of paper almost attenuated, while passing through the crystal.

The mineral tourmaline is the best known dichroic crystal of natural materials. Tourmaline refers to a class of boron silicates. A tourmaline crystal has a unique optic axis and any electric field vector which is perpendicular to that axis is strongly absorbed. In laboratory, an artificial dichroic device can be constructed based on dichroic property and is known as *Wire Grid Polariser*.

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UNIT-VIII

Lasers

SECTION-A VERY SHORT ANSWER TYPE QUESTIONS

Q.1. What do you mean by LASER?

Ans. The word 'LASER' is acronym of "Light amplification by stimulated emission of radiation". This laser light is different from the ordinary light. LASER was developed by Maiman in 1960, there was a standard statement about LASER that "LASER is a solution in search of a problem."

Q.2. What are the characteristics of a LASER beam?

Ans. The LASER Beam has the following characteristics :

1. Highly coherent and monochromatic
2. Highly directional
3. High intensity
4. Divergence.

Q.3. What do you mean by coherance?

Ans. The term coherence refers to the degree of co-relation between the phases at different points and different times in a beam of light or radiation.

Coherence is of two types : 1. Spatial coherence, 2. Temporal coherence.

Q.4. Explain population inversion.

Ans. The transition probability for stimulated emission depends upon the number of atoms in excited state and the energy density of the incident radiation.

To achieve laser action the higher energy states should have the large number of atoms as compare to ground state and when this situation is created we called this population inversion.

Q.5. Write down the relation between Einstein's coefficients.

Ans. The relation between Einstein's coefficients are

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

where, h = Planck's constant, ν = frequency, c = speed of light.

This is the formula for the ratio between the spontaneous emission and induced emission coefficients.

Q.6. What are the requirements for population inversion?

Ans. The requirements for population inversion and to produce laser action are :

1. An active medium
2. A pumping arrangement

Active medium is a medium of atoms which have a metastable energy state. Such atoms produce more stimulated emissions.

Pumping is a procedure adopted to achieve population inversion.

Q.7. What are metastable states?

Ans. When an atom achieves energy from external source and reaches to an excited state. It will remain ϕ in excited state for only 10^{-8} sec after that it will release energy and comes back to ground state. The average time for which an atom remains in an excited state is known as its "mean life".

Energy states having mean life of more than 10^{-3} sec are known as metastable states.

Q.8. Name the different types of LASERS.

Ans. Lasers are classified on the basis of the material used as active medium. They are divided into four categories :

1. Solid state lasers
2. Gas lasers
3. Semiconductor lasers
4. Liquid lasers

Q.9. Write the wavelength emitted by Ruby laser and He-Ne laser.

Ans. The wavelength of light by :

$$\text{Ruby Laser} = 6943\ \text{Å}$$
$$\text{He-Ne laser} = 6328\ \text{Å}.$$

Q.10. Give few applications of LASER.

Ans. The applications of LASER are :

1. LASER in scientific research : Raman spectroscopy, measurement of length, determination of velocity of light etc.
2. LASER in nuclear energy : Isotope separation, nuclear fusion, in nuclear fission etc.
3. LASER in medicine and surgery : Cosmetic surgery, eye surgery and refractive surgery, soft tissue surgery etc.
4. To make halograms, in compact discs (CD), optical data storage etc.

Q.11. Write the threshold condition for LASER action.

Ans. The threshold condition for LASER action is given by

$$\gamma_{th} = \alpha + \frac{1}{2L} \ln\left(\frac{1}{r_1 r_2}\right)$$

where, L = length between mirrors, α = loss per unit length, r_1 and r_2 = reflectivity of mirrors.

SECTION-B SHORT ANSWER TYPE QUESTIONS

Q.1. Explain spontaneous and stimulated emission.

Ans.

Spontaneous Emission

Let us consider two energy levels having energies E_2 and E_1 as shown in fig.(1). When an atom in an excited state E_2 falls to a ground state E_1 by spotaneously emitting a photon of frequency $\nu = \frac{E_2 - E_1}{h}$. The process is known as spontaneous emission.

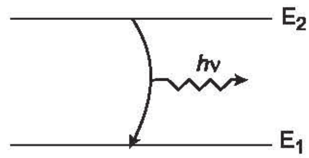

Fig.1.

Stimulated Emission

When an incident photon of frequence $\nu = \frac{E_2 - E_1}{h}$ is incident on the atom in excited state, then it stimulates the atom to move in to ground state and emits a photon of same frequency as incident photon. This process is known as stimulated emission.

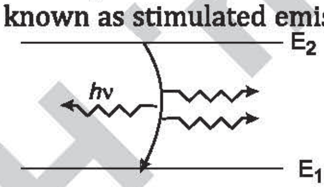

Fig.2.

Q.2. Describe the construction ruby laser with necessary diagram.

Ans.

Construction of Ruby Laser (A Pulsed Laser)

This is the first laser developed in 1960, and is a solid-state laser. It consists of a pink ruby cylindrical rod whose ends are optically flat and parallel (Fig.1). One end is fully silvered and the other is only partially silvered. Upon the rod is wound a coiled flash lamp filled with xenon gas.

This ruby laser makes use of the three level scheme of population inversion. It consists of the following these main parts :

1. **The Working Substance :** The working substance is a ruby crystal. Ruby is a crystal of aluminium oxide (Al_2O_3) doped with 0.05%. Chromium oxide (Cr_2O_3). Aluminium atoms in the crystal are replace by Cr^{+++} ions, These impurity (chromium) ions give a pink color to ruby and give rise to laser action.
2. **The Resonant Cavity :** Ruby crystal is grown in special furnaces and then shaped into rods of nearly 10 cm length and 0.8 cm in diameter. Flat end faces A and B of the rod are made parallel, plane and polished so that one of its ends is completely silvered to become fully reflecting while the other is only partially silvered (Fig.1). An intense laser beam emerges out from partially reflecting end B.

Ruby rod is surrounded by a cylindrical glass envelope through which liquid nitrogen or water is circulated to keep the rod cool.

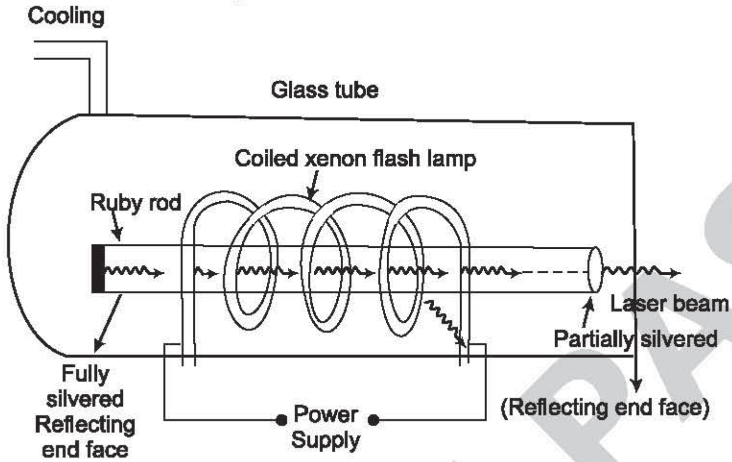

Fig.

3. **The optical Pumping System :** Optical pumping system is in the form of helical xenon flash tube, optical pumping is done by this flash tube. It is wound round the glass envelope surrounding the ruby rod. The flash tube is provided with a suitable power supply.

Q.3. Explain the workings of ruby laser.

Ans.

Workings of Ruby Laser

Working : The ruby rod is a crystal of aluminium oxide (Al_2O_3) doped with 0.05% chromium oxide (Cr_2O_3), so that some of the Al^{+++} ions are replaced by Cr^{+++} ions. These "impurity" chromium ions give pink colour to the ruby and give rise to the laser action.

In Fig.2 is shown a simplified version of the energy-level diagram of chromium ion. It consists of an upper short-lived energy level (rather energy band) E_3 above its ground-state energy level E_1, the energy difference $E_3 - E_1$ corresponding to a wavelength of about 5500 Å. There is an intermediate excited-state level E_2 which is metastable having a life-time of 3×10^{-3} sec (about 10^5 times greater than the life-time of E_3 which is $=10^{-8}$ sec).

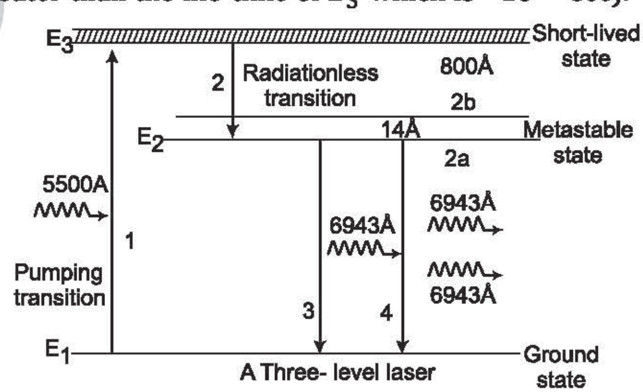

Fig.

Normally, most of the chromium ions are in the ground state E_1. When a flash of light (which lasts only for about a milli-second) falls upon the ruby rod, the 5500–Å radiation photons are absorbed by the chromium ions which are "pumped" (raised) to the excited state E_3. The transition 1 is the (optical) pumping transition. The excited ions given up, by collision, part of their energy to the crystal lattice and decay to the "metastable" state E_2. The corresponding transition 2 is thus a radiationless transition. Since the state E_2 has a much longer life-time, the number of ions is this state goes on increasing while, due to pumping, the number in the ground state E_1 goes on decreasing. Thus, population inversion is established between the metastable (excited state E_2) and the ground state E_1.

When an (excited) ion passes spontaneously from the metastable state to the ground state (transition 3), it emits a photon of wavelength 6943Å. This photon travels through the ruby rod and, if it is moving parallel to the axis of the crystal, is reflected back and forth by the silvered ends until it stimulates an excited ion and causes it to emit a fresh photon *in phase with the stimulating photon*. This "stimulated" transition 4 is the laser transition. (The photons emitted spontaneously which do not move axially escape through the sides of the crystal). The process is repeated again and again because the photons repeatedly move along the crystal, being reflected from its ends. The photons thus multiply. When the photon-beam becomes sufficiently intense, part of it emerges throught the partially-silvered end of the crystal.

There is a drawback in the three-level laser such as ruby. The laser requires high pumping power because the laser transition terminates at the ground state and more than one-half of the ground-state atoms must be pumped up to the higher state to achieve population inversion. Moreover, ions which happen to be in their ground state absorbs the 6943-Å photons from the beam as it builds up.

The ruby laser is a "pulsed" laser. The active medium (Cr^{+++} ions) is excited in pulses, and it emits laser light in pulses. While the Xenon pulse is of several millisecond duration; the laser pulse is much shorter, less than a millisecond duration. It means enhanced instantaneous power.

SECTION-C LONG ANSWER TYPE QUESTIONS

Q.1. What do you understand by spotaneous and induced emission? Calculate Einstein's *A* and *B* coefficients and discuss their physical significance.

Ans.

The Einstein's Coefficients

We consider two levels of an atomic system as shown in Fig. and let N_1 and N_2 be the number of atoms per unit volume present in the energy levels E_1 and E_2 respectively. The atomic system can interact with electromagnetic radiation in three distinct ways :

(a) An atom in the lower energy level E_1 can absorb the incident radiation at a frequency $\omega = (E_2 - E_1)/h$ and be excited to E_2; this excitation process requires the presence of radiation. The rate at which absorption takes place from level 1 to level 2 will be proportional to the number of atoms present in level E_1 and also to the energy density of the radiation at the frequency $\omega = (E_2 - E_1)/h$. Thus if $u(\omega)d\omega$ represents the radiation energy per unit volume between ω and $\omega + d\omega$, then we may write the number of atoms undergoing absorptions per unit time per unit volume from level 1 to level 2 as :

$$\Gamma_{12} = B_{12} u(\omega) N_1 \qquad ...(1)$$

where B_{12} is a constant of proportionality which depends on the energy levels E_1 and E_2. Notice here that $u(\omega)$ has the units of energy density per frequency interval.

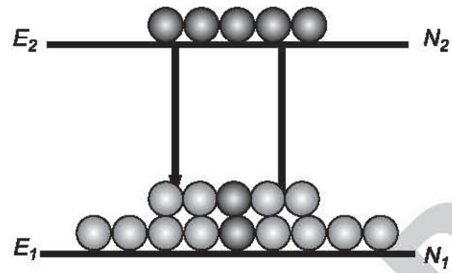

Fig. : Two states of an atom with energies E_1 and E_2 with corresponding population densities of N_1 and N_2 respectively.

(b) For the reverse process : the deexcitation of the atom from E_2 to E_1 : Einstein postulated that an atom can make a transition from E_2 to E_1 through two distinct processes, known as stimulated emission and spontaneous emission. In the case of stimulated emission, the radiation which is incident on the atom stimulates it to emit radiation and the rate of transition to the lower energy level is proportional to the energy density of radiation at the frequency ω. Thus, the number of stimulated emissions per unit time per unit volume will be

$$\Gamma_{21} = B_{21} u(\omega) N_2 \qquad ...(2)$$

where B_{21} is the coefficient of proportionality and depends on the energy levels.

(c) An atom which is in the upper energy level E_2, can also make a spontaneous emission; this rate will be proportional to N_2 only, and thus we have for the number atoms making spontaneous emissions per unit time per unit volume

$$U_{21} = A_{21} N_2 \qquad ...(3)$$

At thermal equilibrium between the atomic system and the radiation field, the number of upward transitions must be equal to the number of downward transitions. Hence, at thermal equilibrium

$$N_1 B_{12} u(\omega) = N_2 A_{21} + N_2 B_{21} u(\omega)$$

or

$$u(\omega) = \frac{A_{21}}{(N_1/N_2)B_{12} - B_{21}} \qquad ...(4)$$

Using Boltzmann's law, the ratio of the equilibrium populations of levels 1 and 2 at temperature T is

$$\frac{N_1}{N_2} = e^{(E_2 - E_1)/k_B T} = e^{h\omega/k_B T} \qquad ...(5)$$

where k_B ($=1.38 \times 10^{-23}$ J/K) is the Boltzmann's constant. Hence

$$u(\omega) = \frac{A_{21}}{B_{12} e^{h\omega/k_B T} - B_{21}} \qquad ...(6)$$

Now, according to Planck's law, the radiation energy density per unit frequency interval is given by (see appendix *D*)

$$u(\omega) = \frac{h\omega^3 n_0^3}{\pi^2 c^3} \frac{1}{e^{h\omega/k_B T} - 1} \quad ...(7)$$

where *e* is the velocity of light in free space and n_0 is the refractive index of the medium. Comparing Eq. (6) and (7), we obtain

$$B_{12} = B_{21} = B \quad ...(8)$$

and

$$\frac{A_{21}}{B_{21}} = \frac{h\omega^3 n_0{}^3}{\pi^2 c^3} \quad ...(9)$$

Thus the stimulated emission rate per atom is the same as the absorption rate per atom and the ratio of spontaneous to stimulated emission coefficients is given by Eq. (9). The coefficients *A* and *B* are referred to as the Einstein *A* and *B* coefficients. At the thermal equilibrium, the ratio of the number of spontaneous to stimulated emission is given by

$$R = \frac{A_{21} N_2}{B_{21} N_2 u(\omega)} = e^{h\omega/k_B T} - 1 \quad ...(10)$$

Thus at thermal equilibrium at a temperature *T*, for frequencies, $\omega >> k_B T/\hbar$, the number of spontaneous emissions far exceeds the number of stimulated emissions.

The ratio of Einsten's coefficient tells us whether which process *i.e.*, stimulated or spontaneous will take place.

Q.2. Explain population inversion in 3 and 4 level laser system. Give comparison.

Ans.

Population Inversion (3-and 4-Level Schemes)

The transition probability for induced emission depends upon

(i) The number of atoms in the excited state and

(ii) The energy density of the incident radiation $\rho(\nu)$.

The achieve a higher probability of stimulated emission we make use of the phenomenon of population inversion.

When we have a collection of large number of atoms, say N_0, in thermal equilibrium, the distribution of atoms in different energy states is given by Maxwell-Boltzmann statistics. If *T K* is the temperature of collection, N_1 the number of atoms in the energy state E_1 and N_2 that in the energy state E_2, then

$$N_1 = N_0 e^{-E_1/kT} \quad \text{and} \quad N_2 = N_0 e^{-E_2/kT}$$

or

$$N_2 = N_1 e^{-(E_2 - E_1)kT} = N_1 e^{-h\nu/kT} \quad \text{where } h\nu = (E_2 - E_1) \quad ...(1)$$

As $E_2 > E_1$, $N_2 < N_1$. Thus, in the normal distribution, the number of atom in the higher energy state is less than the number of atoms in the lower energy state. In other words, the population of atoms in higher energy levels is less than that in the lower energy levels.

When radiation of *matching* frequency $\nu = \frac{(E_2 - E_1)}{h}$ is incident on such a collection, the atoms are excited due to *stimulated absorption.* The excited atoms return to the normal state by any of the two processes *i.e.*, spontaneous emission or stimulated emission. In order to achieve higher probability of stimulated emission, two conditions must be satisfied :

(i) The higher energy state should have a longer mean life *i.e.*, it should be a metastable state.

(ii) The number of atoms in the higher energy state E_2 must be greater than that in E_1.

The establishment of situation in which the number of atoms in the higher energy level is greater than in the lower energy level is called **population inversion.**

It should be noted that population inversion is a *non-equilibrium condition.*

Requirements for population inversion and laser action : For the purpose of population inversion and to produce laser action we require (i) *an active medium* and (ii) *a pumping arrangement.*

(i) **Active medium :** An active medium (material) is a medium the atoms of which have a *metastable energy state.* Such atoms produce more stimulated emission than spontaneous emission and when excited soon reach the state of population inversion leading to laser action.

(ii) **Pumping :** The procedure adopted to achieve population inversion is called *pumping.* For achieving and maintaining the condition of population inversion, we have to raise continuously the atoms from the lower energy level to the upper energy level. This require energy to be supplied to the system.

There are a number of techniques used for producing population inversion *e.g.* optical pumping, electrical discharge, direct conversion etc. However, optical pumping is very important and convenient method.

Optical pumping : The procedure adopted to achieve population inversion is called *pumping.* In *optical pumping*, the active material is illuminated with light of suitable frequency $\nu = \frac{E_2 - E_1}{h}$. As a result, an atom in the lower energy state E_1 absorbs the incident photon of energy $h\nu$ and is raised to the highest energy level E_2. As the excited atoms lose their energy by spontaneous emission and drop to the lower energy level in a very short time, the process fails to produce necessary population inversion.

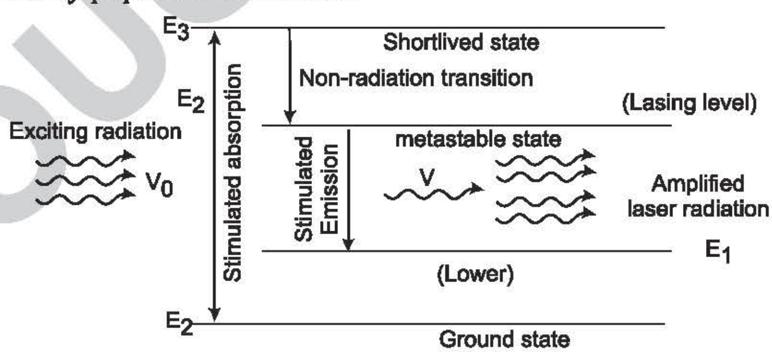

Fig.1 : Three level scheme

In practice, therefore, a population inversion is brought about by *(i) a three level scheme or (ii) a four level scheme.*

(i) **Three level scheme :** Consider an atom with energy levels E_1, E_2 and E_3 with increasing values of energy. E_1 is the ground state, E_3 is a short lived state and E_2 is an intermediate metastable state. Further, the transition from E_3 to E_1 is forbidden but transition from E_3 to

E_2 is allowed. When these atoms are irradiated with an exciting radiation of the *right* frequency $\nu_0 = \frac{E_3 - E_1}{h}$, the atoms are excited to the E_3 state by the process of *stimulated absorption*.

Some excited atoms quickly drop to the intermediate level E_2 by spontaneous emission or by a *non-radiative* process, thereby converting their excess energy into vibrational kinetic energy of the atoms forming the substance (Fig.1). As E_2 is a metastable state, the stoms remain in this excited state for comparatively longer time of 10^{-3} sec. as compared to 10^{-8} sec. for the short lived state E_3 and for this time, the population of the state, E_2 is more than that of state E_1 thus resulting in population inversion of the collection of atoms.

If an atom in the state E_2 decays by spontaneous emission or stimulated emission, it emits a radiation of frequency $\nu = \frac{E_2 - E_1}{h}$. The photon may produce stimulated emission from another atom, thereby giving two coherent photons, moving in the same direction. These two photons produce two more photons and so on, producing an amplified beam of photons.

(ii) **Four level scheme :** In *four level scheme* there is an additional *very shortlived* energy state E_1' between the ground state (level) E_1 and *metastable* state E_2. When exciting radiation of right frequency $\nu_0 = \frac{E_3 - E_1}{h}$ is incident on the lasing medium the atoms in the ground state E_1 are excited to the level E_3 by the process of *stimulated absorption*. The atoms stay at the E_3 level for a very short time of about 10^{-8} sec. and quickly drop down to the metastable state E_2. Spontaneous transition from the metastable state E_2 to the level E_1' is forbidden and, therefore, cannot take place. As a result, the atoms accumulate at the level E_2 and the population at this level rapidly increases. Again the level E_1' is well above the ground level E_1 so that $(E_1' - E_1) >> kT$. Therefore, at normal temperature atoms cannot jump to the level E_1' from the ground level E_1 because of thermal energy. As a result, the level E_1' is almost virtually empty. The population inversion is thus maintained between the levels E_2 and E_1'.

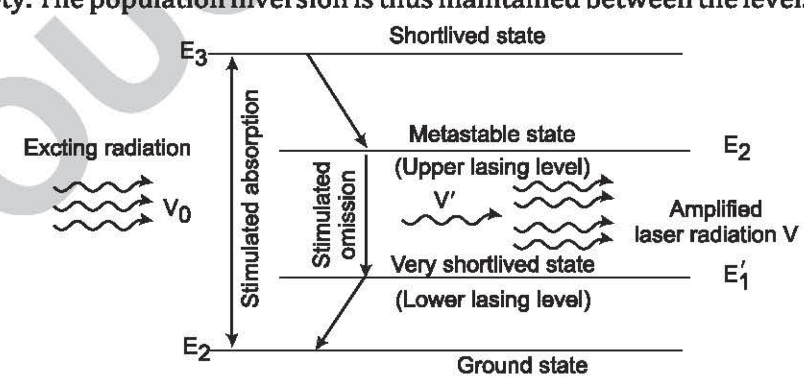

Fig.2

Further, when the atom in the state E_2 decays by spontaneous emission or stimulated emission to the state E_1', it emits a photon of frequency $\nu' = \frac{E_2 - E_1'}{h}$. This photon further produces stimulated emission from another atom and thus starts the laser action.

The level E_1' being a very short lived state, the atoms immediately relax further to the ground state, ready for being pumped to the level E_3.

Comparison of four level pumping scheme with three level pumping scheme :

1. In three level pumping scheme the atoms finally return to the ground state directly from the metastable state. Therefore, in order to achieve population inversion pumping has to go on until more than half of the ground state atoms reach the metastable state E_2 which is the actual lasing level. As number of atoms in the ground state is very large a *very high pumping* power is required for the purpose.

   But in case of four level scheme the terminal level is almost virtually empty so that the condition of population inversion is readily established even if a small number of atoms arrive at the upper lasing (or metastable) level. Consequently only a small pumping power is required to establish population inversion in four level scheme.

2. In three level pumping scheme, as soon as stimulated emissions starts, the *population inversion condition* returns to *normal population condition* and lasing stops as soon as the excited atoms drop to ground level. Again lasing beings *only when* the condition of population inversion is re-established. The laser light given out is, therefore, in the form of *pulses of short duration*. But in the case of four level scheme, the population inversion continues without interruption and laser light is continuously obtained. The laser, therefore, operates in *continuous wave (c w) mode*.

   PN junction semiconductor laser is a two level laser, Ruby (Al_2O_3 crystal) laser is a three level laser and NdYAG is a four level laser.

Q.3. Give the characteristics of laser light in detail.

Ans.

Characteristics of Laser Light

Important properties of laser light are :

(i) Directionality

Laser emits radiation in the form of a highly directional collimated beam with only a very small angle of divergence. It means that the energy carried by the laser beam can be collected easily and focussed on to a small area. The directionality of a laser beam is expressed in terms of beam divergence.

Divergence : Light from a laser diverges very little. Upto a certain distance, the beam shows little spreading and remains essentially a bundle of parallel light rays. The distance from the laser over which the light rays remain parallel is known as *Rayleigh range*. The laser beam diverges beyond the Rayleigh range as shown in Fig.1. There are two parameters which cause beam divergence. They are (i) the size of the beam waist and (ii) diffraction. The *divergence angle* is measured from the centre of the beam to the edge of the beam, where the edge is defined as the location in the beam where the intensity decreases to $1/e$ of that at the centre.

Twice the angle of divergence is known as the *full angle beam divergence*. This angle tells us how much the beam will spread as it travels through space. The full angle divergence is given by

$$2\theta = \frac{4\lambda}{\pi d_0} \quad ...(1)$$

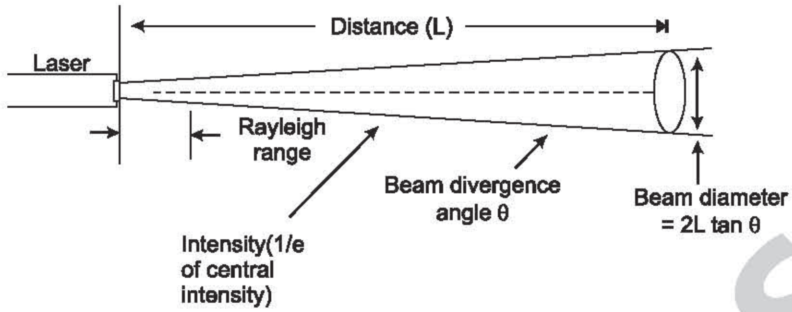

Fig.1 : Divergence of a laser beam

where $d_0 = 2W_0$ is the diameter of the beam waist (Fig.2). It is seen from Eq. (1) that the divergence is inversely proportional to d_0. Thus, divergence is large for a beam of small waist. The beam divergence due to diffraction is determined from Rayleigh's criterion.

$$d\theta = 1.22\frac{\lambda}{D} \qquad ...(2)$$

where D is the diameter of the laser's aperture.

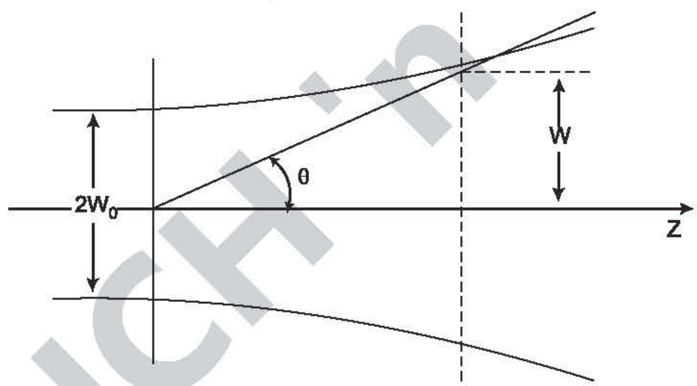

Fig.2 : Relationship between beam waist and divergence of a laser beam

In case of gas lasers, the diffraction divergence is about twice as large as the beam-waist divergence. A typical value of divergence for a He-Ne laser is 10^{-3} rad. It means that the diameter of the laser beam increases by about 1 mm for every metre it travels. Beam divergence of large lasers may be as small as a micro-dgree. As laser beam of 5 cm. diameter (divergence $= 10^{-6}$ degree) when focussed from Earth will spread to a diameter of only about 10 m on reaching the surface of the moon. This extreme collimation of the beam makes lasers a very useful tool for surveying.

(ii) Intensity

The power output of a laser may vary from a few milliwatts to few kilowatts. But this energy is concentrated in a beam of very small cross-section. The intensity of a laser beam is approximately given by

$$I = \left(\frac{10}{\lambda}\right)^2 P \ \ W/m^2 \qquad ...(3)$$

where P is the power radiated by the laser. In case of 1 m W He-Ne laser,

$$\lambda = 6328 \times 10^{-10} \text{ m and}$$

$$I = \frac{100 \times 10^{-3} W}{(6328 \times 10^{-10})^2 m^2} = 2.5 \times 10^{11} \ W/m^2$$

To obtain light of same intensity from a tungsten bulb, it would have to be raised to a temperature of 4.6×10^6 K. The normal operating temperature of a bulb is about 2000 K. Thus, the intensity of laser beam *i.e.*, power emitted per unit area per unit solid angle is *much higher* than that of light from other sources.

(iii) Monochromaticity

If light coming from a source has only one frequency of oscillation, are light is said to be monochromatic and the source a *monochromatic source*. In practice it is pt possible to produce light with only one frequency. Light coming out from any source consists of a band of frequencies closely spaced found the central frequency, v_0. The band of frequency Δv is called the line *width or band width*.

A laser beam is highly monochromatic. But the ectral content of the laser raditation extends over range equivalent to the fluorescent line width of are laser medium. Although the line width of an dividual cavity mode is extremely small, there may be many modes present in the laser output.

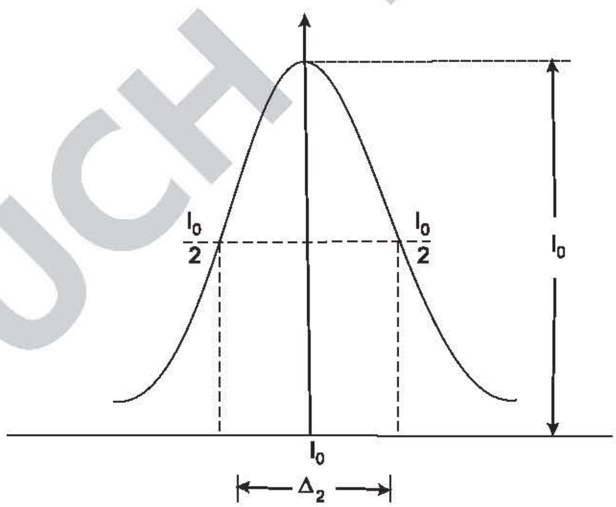

Fig.3.

The light from conventional sources has large line widths of the order of 10^{10} Hz or more. On the other hand, light from lasers is more monochromatic, having linewidth of the order of 100 Hz. From the theory of interference, the linewidth of mirrored cavity is given by

$$\Delta\gamma = \frac{c}{2\pi L}\left(\frac{1-R}{\sqrt{R}}\right) \quad ...(4)$$

where L is the length of the cavity and R is the reflectance of the output mirror.

(iv) Coherence

A laser beam is highly coherent. It is because one important property of stimulated emission is that the wave or photon produced by stimulated emission is exactly in phase with the stimulating wave or photon. As a result the spatial and temporal variation of the electric field vector of the two waves are exactly similar. For a perfect laser the electric field varies with time in an identical manner for every point on the cross-section of the beam.

Q.4. Describe a He-Ne LASER. How population inversion is achieved in type of LASER? Explain with energy level diagram.

Ans.

Helium Neon Laser

Many laser applications require a continuous wave. In 1961, a gas laser was discovered which emits light continuously rather than in pulses. He-Ne gas laser device is one of the most common continuous wave laser source with power ranging from 0.5 milliwatt to 60 milliwatt.

Construction : It is a four-level laser in which the population inversion is achieved by electric discharge. The experimental arrangement for He-Ne gas laser is shown in Fig. (1). In this laser, the active medium is a mixture of helium and neon in the ratio 9 : 1 at the pressure of about 1 mm of Hg. The mixture is contained in quartz tube.

At both ends of the tube are fitted optically plane and parallel mirrors, one of them being only partially silvered. The spacing of the mirrors is equal to an integral number of half-wavelengths of the laser light. An electric discharge is produced in the gas-mixture by electrodes connected to a high-frequency electric source.

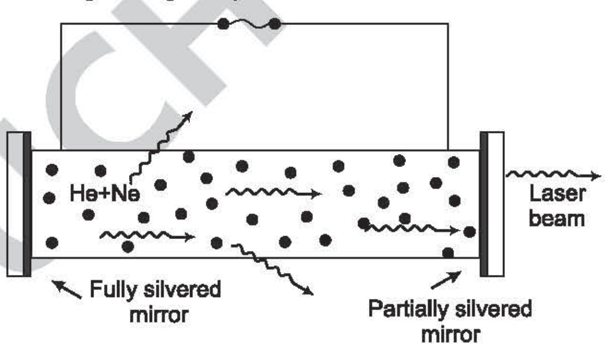

Fig.1

Working : The electrons from the discharge collide with and "pump" (excite) the He and Ne atoms to *metastale* states 20.61 and 20.66eV respectively above their ground states (Fig.2). Some of the excited. He atoms transfer their energy to ground-state Ne atoms by collisions, with the 0.05 eV of additional energy being provided by the kinetic energy of atoms. Thus, He atoms help in achieving a population inversion in the the Ne atoms.

When an excited Ne atom passes spontaneously from the metastable state at 20.66 eV to state at 18.70 eV, it emits a 6328-$\AA$ photon. This photon travels through the gas-mixture, and if it is moving parallel to the axis of the tube, is reflected back and forth by the mirror-ends until it stimulates an excited Ne atom and causes it to emit a fresh 6328-$\AA$ photon in phase with the stimulating photon. This stimulated transition from 20.66-eV level to 18.70-eV level is the laser transition. This process is continued and a beam of coherent radiation builds up in the tube. When this beam becomes sufficiently intense, a portion of it escapes through the partially-silvered end.

From the 18.70-eV level the Ne atom passes down spontaneously to a lower metastable state emitting incoherent light, and finally to the ground state through collision with the tube walls. The final transition is thus radiationless.

Obviously, the Ne atom in its ground state cannot absorb the 6328-Å photons from the laser beam, as happens in the three-level ruby laser. Also, because the electron impacts that excite the He and Ne atoms occur all the time, unlike the pulsed excitation from the xenon flash lamp in the ruby laser, the He-Ne laser operates continuously.

Further, since the laser transition does not terminate at the ground state, the power needed for excitation is less than that in a three-level laser.

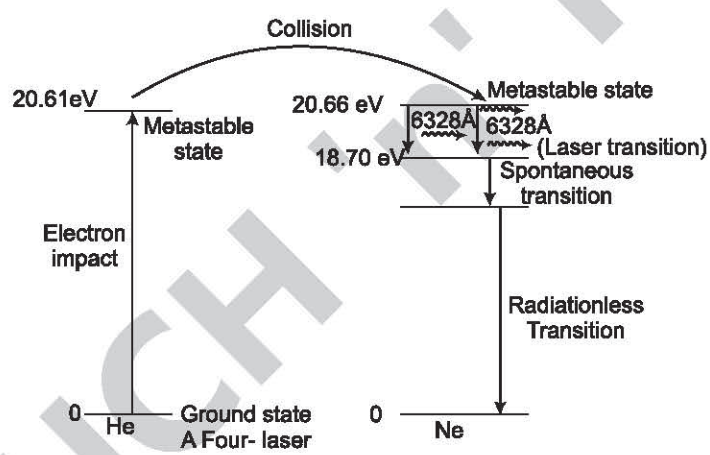

Fig.2

Q.5. Explain the construction and working of semi conductor laser.

Ans.

Semi-Conductor Laser

In a semi-conductor laser, the conduction band population of electrons is increases by injecting free electrons into the negative side of an ordinary junction diode. This ensures *population inversion* required for stimulated emission to predominate over the spontaneous emission. The recombination takes place in *active* junction region of the diode. A *Ga As p-n* junction diode is generally used. It has an operating range of 8400–8500 Å.

Construction : The basic structure of a $p-n$ junction laser is shown in Fig.1. A pair of parallel planes A and B are cleaved or polished perpendicular to the plane of the junction. A heavily doped p-region is formed on the top of the *Ga As* material by diffusing zinc atoms into it. The top and the bottom faces are metal covered and provided with metallic contacts to pass the current through the diode. The two remaining sides of the diode are made rough to eliminate laser action in a direction other than the main. The structure is called *Fabry-Perot cavity.*

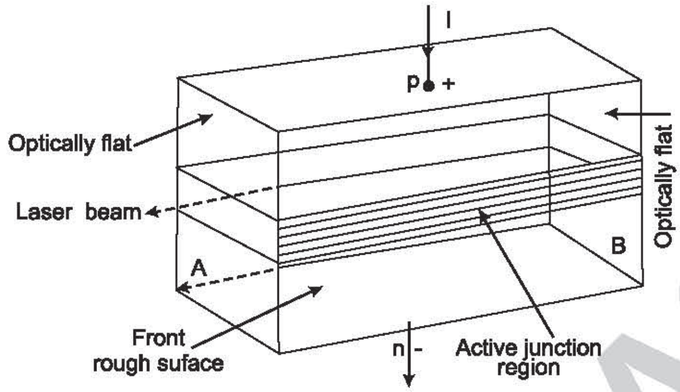

Fig.1

Working : The energy band diagram of a heavily doped $p-n$ junction is shown in Fig.2 (a). Due to heavy doping on the *n*-side the Fermi level E_F is pushed into the *conduction band*. The electrons occupy the portion of the conduction band below the Fermi level between E_F and E_C. Similarly on the heavily doped *p*-side, Fermi level E_F lies within the valence band. The holes occupy the portion of the valence band above the Fermi level between E_F and E_V. At the condition of thermal equilibrium, *Fermi level is uniform* across the junction as shown.

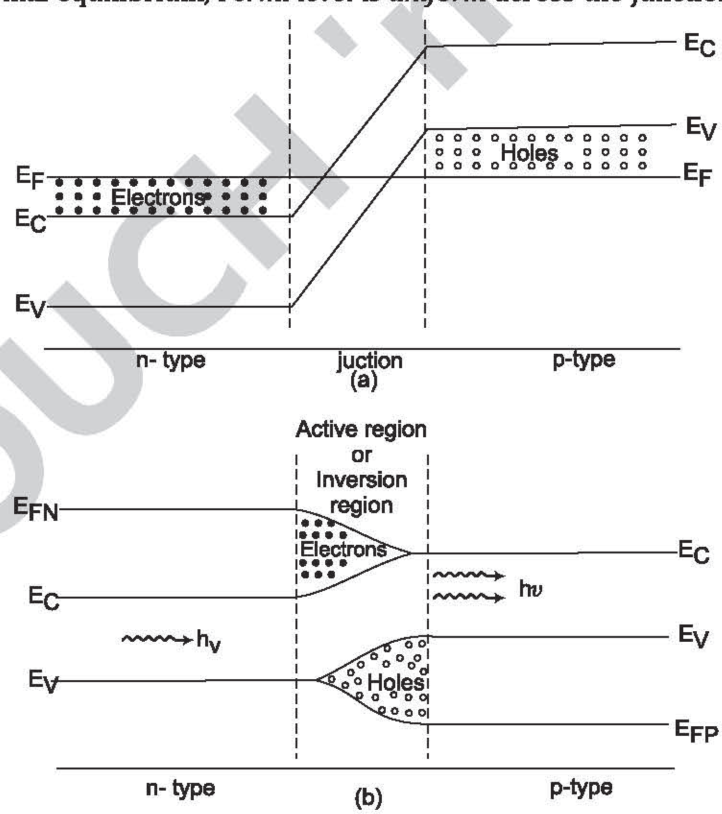

Fig.2

When the junctions is forward biased electrons from the *n* region and holes from the *p*-region are injected into the junction region. This is how *pumping action* takes place in a $p-n$ junction semiconductor laser. For small values of forward bias the electron holes recombine and the recombination brings about spontaneous, non-coherent emission of photons in all directions.

At this stage the junction acts a *light emitting* diode (LED). As the forward bias is increased the intensity of light increases but when the forward bias reaches a *threshold value* the carrier concentration in the junction region rises to a very high value. The junction region, therefore, attains a very high concentration of electrons in the conduction band and at the same time a very high concentration of holes in the valence band. As a result the upper energy level in the narrow junction region has a high electron population whereas the lower energy levels in this region are vacant, thereby giving rise to the condition of *population inversion* in the region. This narrow region in which population inversion takes place is known as "Inversion region" or '*Active region*'.

When a chance recombination of electron-hole pair takes place between the conduction and valence bands, it leads to spontaneous emission of photons. The spontaneous photons moving in the junction plane stimulate the electrons in the conduction band to jump into the vacant states of valence band (as holes means absence of electrons) and produce photons. The induced photons further produce more photons by the process of stimulated emission and thus produce coherent radiation giving rise to laser action. An highly directional beam of light is then emitted from the junction.

Semi-Conductor lasers are very compact, efficient and the be spectral purity is low and emission pattern brad. Their monochromatic and directional character are, therefore, not as good as that of other type of .

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MODEL PAPER

Electromagnetic Theory & Modern Optics

B.Sc.-II (SEM-III) [M.M. : 75

Note : Attempt all the sections as per instructions.

Section-A : Very Short Answer Type Questions

Instruction : Attempt all **FIVE** questions. Each question carries **3 Marks.** Very Short Answer is required, not exceeding 75 words. **[3 × 5 = 15]**

1. What is the relation between electric field and electric potential. Calculate the value of electric field, if potential is given by $\phi = 3x^2 yz$.
2. Define Lenz's law.
3. Write down the wave equation for electric and magnetic fields in free space.
4. What do you mean by interference of light?
5. Motor car wind screens and head lights are fitted with polaroids. Why?

Section-B : Short Answer Type Questions

Instruction : Attempt all **TWO** questions out of the following 3 questions. Each question carries **7.5 Marks.** Short Answer is required not exceeding 200 words. **[7.5 × 2 = 15]**

6. Derive the expression for potential energy of a dipole in a uniform electric field.
7. Prove the relation $\mu_r = (1 + \chi_m)$. Where μ_r is called relative permeability and χ_m is magnetic susceptibility.
8. Explain principle of dichroism.

Section-C : Long Answer Type Questions

Instruction : Attempt all **THREE** questions out of the following 5 questions. Each question carries **15 Marks.** Answer is required in detail, between 500-800 words. **[15 × 3 = 45]**

9. Using Biot-Savart law, derive expression for magnetic field at a point on the axis of a current carrying coil.
10. Write down differential equation of motion of a coil in a ballistic galvanometer and derive the condition under which its motion is oscillatory. What is logarithmic decrement?
11. Derive the electromagnetic wave equation from maxwell field equations in free space.
12. What are Fresnel's half period zones? Prove that the area of a half-period zone on a plane wavefront is independent of the order of the zone, and that the amplitude due to a large wavefront at a point infront it is just half that due to the first half-period zone acting alone.
13. What do you understand by spotaneous and induced emission? Calculate Einstein's A and B coefficients and discuss their physical significance.

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